Multi-scale Electromagnetic Modeling by Integral Equation Domain Decomposition Method with Hybrid Basis Functions

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Abstract— In this paper, a novel electromagnetic (EM) modeling method by integral equation domain decomposition method with hybrid basis functions is developed for the solution of multi-scale problems. Based on the integral equation domain decomposition method, the original multi-scale object were decomposed into several closed sub-domains and transmission condition is enforced on the touching faces to maintain the continuity of current across the interfaces. The higher order hierarchical vector basis functions and low order Rao-Wilton-Glisson (RWG) basis functions are used in different sub-domains to reduce remarkably the number of unknowns. Because the domain decomposition method provides an effective preconditioner, the convergence history will be much better than traditional method of moments (MoM). And because of the non-conformal property of the domain decomposition method, each sub-domain can be meshed independently, which enables the mesh generation be much easier than the traditional method, especially for complex multi-scale objects. Finally, some numerical results are given to validate the ability of the present method.

1. INTRODUCTION

Method of Moments (MOM) based on integral equation (IE) is one of the most well-known method for electromagnetic scattering, radiation problem. Compare to the differential equation method, finite element method (FEM) and finite difference method (FDM), the unknowns of the integral equation method is only distributed on the surface of perfect electric conductor (PEC), the number of unknowns is much less than them. And because the Green's function satisfy the radiation condition automatically, the absorption boundary condition (ABC) is not required. However, application of the MoM usually leads to a dense matrix equation. At the same time, MoM discretized with low order basis as Rao-Wilton-Glisson (RWG) basis functions or roof-top basis functions [1] often leads to a great deal of unknowns, it is difficult to solve the electrically large problems. This is because both the memory requirement and computational complexity of matrix vector multiplication is $O(N^2)$ where the N is the number of unknowns. To overcome this difficulty, two categories have been developed. One of the categories is develop the fast algorithm to accelerate the matrix-vector multiplication in iterative methods, in order to reduce the computational complexity and memory requirement, such as well-known multilevel fast multipole algorithm (MLFMA) [2] with the complexity of $O(N\log(N))$, adaptive integral method (AIM) [3], IE-FFT [4]. Another category is to discretize the IE with higher order basis functions to reduce the dimension of the matrix. The higher order basis function can be also categorized as two kinds, the interpolatory type and hierarchical type [5–7]. By using the orthogonal Legendre polynomials and modified Legendre polynomials combined with the scaling factor in the hierarchical vector basis function, a well-conditioned matrix is obtained in [6].

However, the simulation of multi-scale problems with IE method in the real world will lead to an ill-conditioned dense matrix equation. When the iterative solvers are used to deal with the matrix equation, the solvers always converge very slowly or even do not converge at all. When the high order hierarchical vector basis function were used, the convergence will be even worse. Recently, a novel non-conformal, non-overlapping integral equation domain decomposition method (IE-DDM) has been developed by Peng et al. [8, 9]. It divides the original domain into many smaller sub-domains and provides a very effective preconditioner for the multi-scale ill-conditioned dense matrix. Because the non-conformal property of the IE-DDM, each sub-domain can be meshed independently.

In these paper, a non-conformal non-overlapping integral equation domain decomposition method (IE-DDM) with hybrid basis functions is presented to simulate the EM scattering of multi-scale objects. The high order hierarchical vector basis functions and low order RWG basis functions can be chosen in different sub-domains based on its local physical and geometrical property.

2. HYBRID BASIS FUNCTIONS

2.1. High Order Hierarchical Basis Function

As shown in Fig. 1(a), the surface current defined on the higher order hierarchical vector basis function (HOHV) based on the quadrilateral which were proposed in [6] can be expressed in the following:

$$\mathbf{f}_{HO} = J_{su}\hat{a}_u + J_{sv}\hat{a}_v \tag{1}$$

where $\hat{a}_u$ and $\hat{a}_v$ are the covariant unit vectors with $\hat{a}_u = \partial\vec{r}/\partial u$ and $\hat{a}_v = \partial\vec{r}/\partial v$. $\vec{r}(u,v)$ is local coordinate the position vector corresponding to $\vec{r}(x,y,z)$ on each patch with the $-1 \le u,v \le 1$. The $u-$directed current can be expanded as:

$$J_{su} = \frac{1}{\eta_S(u,v)} \sum_{m=0}^{M_u} \sum_{n=0}^{N_v} b_{mn}^u \tilde{C}_m P_m(u) C_n P_n(v) \tag{2}$$

where b_{mn}^u are the unknown coefficients, $\eta_S(u,v)$ is the Jacobian factor, which is defined as $\eta_S(u,v) = |\hat{a}_u \times \hat{a}_v|$ and M_u, N_v are the basis order along the current flow direction and transverse direction. The v-directed current can be also obtained just need to interchange the u and v in (2). $P_m(u)$ and $P_n(v)$ are the Legendre polynomials, $\tilde{P}_m(u)$ and $\tilde{P}_n(v)$ are the modified Legendre polynomials, which can be defined as:

$$\tilde{P}_m(u) = \begin{cases} 1-u, & m=0 \\ 1+u, & m=1 \\ P_m(u) - P_{m-2}(u), & m \ge 2 \end{cases} \tag{3}$$

$\tilde{C}_m, C_n$ are the scaling factor, defined as:

$$\tilde{C}_m = \begin{cases} \frac{\sqrt{3}}{4}, & m = 0,1 \\ \frac{1}{2}\sqrt{\frac{(2m-3)(2m+1)}{(2m-1)}}, & m \ge 2 \end{cases} \tag{4}$$

$$C_n = \sqrt{n + \frac{1}{2}} \tag{5}$$

Obviously, (2) will be degenerated to the well-known roof-top basis function, when the order of the function equals to one. Only this order's basis function has the nonzero value across the edge of the adjacent patches.

2.2. RWG Basis Function

As shown in Fig. 1(b), the well-known RWG basis function based on triangular can be expressed as following:

$$\mathbf{f}_{\text{RWG}} = \begin{cases} l_m \boldsymbol{\rho}_m^+ / 2A_m^+, & r \in T_m^+ \\ l_m \boldsymbol{\rho}_m^- / 2A_m^-, & r \in T_m^- \\ 0, & \text{others} \end{cases} \tag{6}$$

where T_m^+ and T_m^- are the adjacent triangles that support the mth RWG basis function, l_m is the length of common edge, A_m^+ and A_m^- are the areas of the triangles T_m^+ and T_m^-, respectively. $\boldsymbol{\rho}_m^+$ and $\boldsymbol{\rho}_m^-$ are a vector pointing from a free vertex of triangle T_m^+ to a point $\mathbf{r}$ in to it, and a vector pointing from a point $\mathbf{r}$ in T_m^- to its free vertex, respectively.

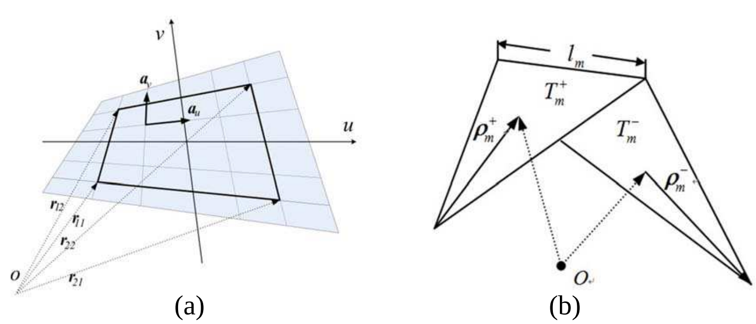

Figure 1: (a) The high order hierarchical basis function defined on the quadrilateral. (b) The RWG basis function associated with the m-th edge.

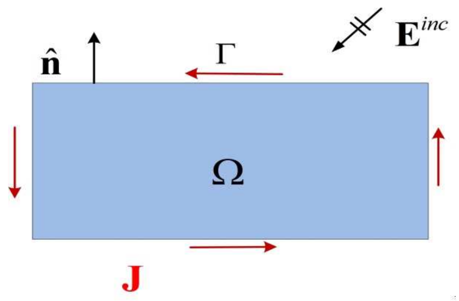

Figure 2: EM scattering from a PEC object.

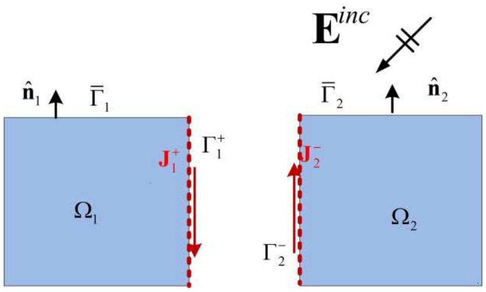

Figure 3: The notation of the decomposition of the original PEC object.

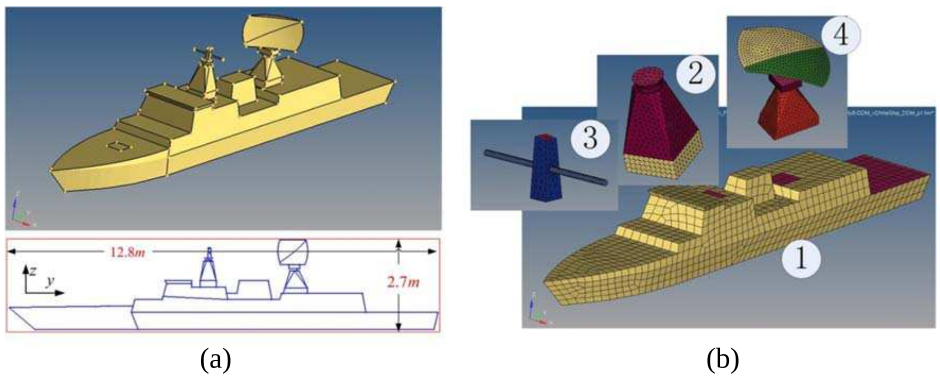

Figure 4: (a) The Geometry and dimension of the simplified ship. (b) Domain partition by using IE-DDM with hybrid basis functions to solve the EM scatteing of the simplified ship.

3. FORMULATION

Considering the electromagnetic scattering of a PEC object Ω, as shown in Fig. 3. It can be decomposed into two non-overlapping objects Ω_1 and Ω_2, as shown in Fig. 4. The HOHV basis functions $\mathbf{f}_{HO}$ based on quadrilateral are used to discrete the current on the surface of Ω_1, the RWG basis functions $\mathbf{f}_{\mathrm{RWG}}$ are used to discrete the current on on the surface of Ω_2. The system matrix equations can be written as:

$$[\mathbf{M}]\,\mathbf{x} = [\mathbf{b}] + [\mathbf{N}]\,\mathbf{x} \tag{7}$$

where

$$[\mathbf{M}] = \begin{bmatrix} \mathbf{A}_{\bar{\Gamma}_1\bar{\Gamma}_1} & \mathbf{A}_{\bar{\Gamma}_1\Gamma_1^+} & & \\ \mathbf{A}_{\Gamma_1^+\bar{\Gamma}_1} & \mathbf{A}_{\Gamma_1^+\Gamma_1^+} & & \\ & & \mathbf{A}_{\Gamma_2^-\Gamma_2^-} & \mathbf{A}_{\Gamma_2^-\bar{\Gamma}_2} \\ & & \mathbf{A}_{\bar{\Gamma}_2\Gamma_2^-} & \mathbf{A}_{\bar{\Gamma}_2\bar{\Gamma}_2} \end{bmatrix} \tag{8}$$

The matrix bocks $\mathbf{A}_{\bar{\Gamma}_j\bar{\Gamma}_j}, \mathbf{A}_{\bar{\Gamma}_j\Gamma_j^+}$, $\mathbf{A}_{\Gamma_j^+\bar{\Gamma}_j}, \mathbf{A}_{\Gamma_j^+\Gamma_j^+}$, stand for the self-coupling of each sub-domain $\Omega_j, j = 1, 2$ itself.

$$[\mathbf{N}] = \begin{bmatrix} & & \mathbf{C}_{\bar{\Gamma}_1\Gamma_2^-} & \mathbf{C}_{\bar{\Gamma}_1\bar{\Gamma}_2} \\ \mathbf{B}_{\Gamma_1^+\bar{\Gamma}_1} & \mathbf{B}_{\Gamma_1^+\Gamma_1^+} & \mathbf{D}_{\Gamma_1^+\Gamma_2^-} & \\ & \mathbf{D}_{\Gamma_2^-\Gamma_1^+} & \mathbf{B}_{\Gamma_2^-\Gamma_2^-} & \mathbf{B}_{\Gamma_2^-\bar{\Gamma}_2} \\ \mathbf{C}_{\bar{\Gamma}_2\bar{\Gamma}_1} & \mathbf{C}_{\bar{\Gamma}_2\Gamma_1^+} & & \end{bmatrix} \tag{9}$$

$$\mathbf{b} = \left[\mathbf{b}_{\bar{\Gamma}_1}, \mathbf{b}_{\Gamma_1^+}, \mathbf{b}_{\Gamma_2^-}, \mathbf{b}_{\bar{\Gamma}_2}\right]^T \tag{10}$$

$$\mathbf{x} = \left[\mathbf{J}_{\bar{\Gamma}_1}, \mathbf{J}_{\Gamma_1^+}, \mathbf{J}_{\Gamma_2^-}, \mathbf{J}_{\bar{\Gamma}_2}\right]^T \tag{11}$$

The matrix blocks $\mathbf{C}_{\bar{\Gamma}_1\Gamma_2^-}$, $\mathbf{C}_{\bar{\Gamma}_1\bar{\Gamma}_2}$, $\mathbf{C}_{\bar{\Gamma}_2\bar{\Gamma}_1}$, $\mathbf{C}_{\bar{\Gamma}_2\Gamma_1^+}$ stand for coupling among each sub-domain, and the $\mathbf{D}_{\Gamma_1^+\Gamma_2^-}$, $\mathbf{D}_{\Gamma_2^-\Gamma_1^+}$ are the cement matrixes that enforcing the transmission condition.To realize a fast solution of Equation (7), a preconditioner is necessary. Here, the inverse of matrix $\mathbf{M}$ is used as the preconditioner, as follows,

$$\mathbf{M}^{-1}[\mathbf{M}-\mathbf{N}]\,\mathbf{x}=\mathbf{M}^{-1}\mathbf{b} \tag{12}$$

Because it is difficult to calculate the inverse of the matrix $\mathbf{M}$, directly, especially for the electrically large problem, the inner-outer iteration based on the Krylov subspace method can be used to solve the equation.

4. NUMERICAL RESULT

In the numerical example, the EM scattering from a simplified PEC ship model is investigated. The incident plane wave illuminate the model along the negative of the z axis, at 0.6 GHz. The dimension and geometry of the ship is shown in Fig. 4(a). This ship is decomposed into 4 closed sub-regions, the domain partition of this ship and the mesh of the IE-DDM with hybrid basis functions are shown in the Fig. 4(b). The second order HOHV basis functions are used in the first sub-region, the RWG basis functions are used in the other sub-regions. Traditional higher order MoM can also be used to solve this problem, but because of the multi-scale property of the geometry, it will lead to an ill-conditioned matrix. The maximum and minimum size of HOHV basis functions are 0.5λ, 0.01λ, if the mesh size less than 0.1λ the first order are used and the others use the second order. The bistatic RCS for H-H polarization of the IE-DDM with hybrid basis functions compare with traditional MoM discretized with RWG basis functions are shown in the Fig. 5. Obviously, the result of the IE-DDM with hybrid basis functions agrees with the result of the MoM discretized with RWG basis functions. The computational statistics of these three methods are shown in the Table 1. The GMRES [10] method is used both in the inner and outer iteration, the convergence tolerance of outer iteration is 0.01, and the convergence tolerance of inner iteration is 0.001.

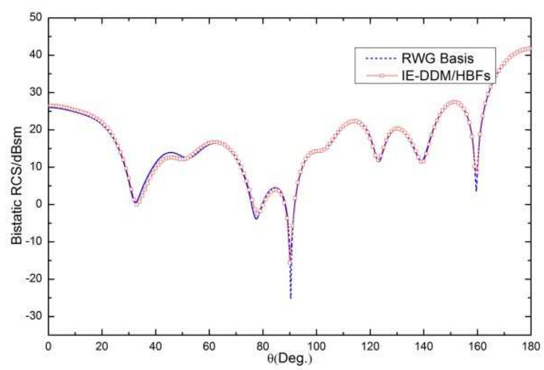

Figure 5: The result of the ship of IE-DDM with hybrid basis functions compare with the result of MoM discreted with RWG basis functions.

Table 1: Computational statistics of the ship.

Method	**Unknowns**	**Mem. (GB)**	**Time Filling/Iteration (hh:mm)**	**Iter. Num.**
MoM-RWG	89,739	60	121:00	86
HO-MoM	25,034	4.7	03:00 / 01:07	2420
IEDDM/HBFs	21,421	3.5	02:45 / 00:08	9

5. CONCLUSION

The integral equation domain decomposition method with hybrid basis functions is developed to solve time harmonic multi-scale EM scattering problems. By using this method, the HOHV basis and RWG basis can be chosen in different sub-regions and several multi-scale problems can be solved, successfully.

ACKNOWLEDGMENT

This work is supported partly by NSFC under Grant 61271033.

REFERENCES

1. Rao, S. M., D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat.*, Vol. 30, No. 5, 409–418, 1982.
2. Song, J. M., C. C. Lu, and W. C. Chew, "Multilevel fast multipole algorithm for electromagnetic scattering by large complex objects," *IEEE Trans. Antennas Propagat.*, Vol. 45, No. 10, 1488–1493, 1997.
3. Ling, F., C.-F. Wang, and J.-M. Jin, "Efficient algorithm for analyzing large-scale microstrip structures using adaptive integral method combined with discrete complex-image method," *IEEE Trans. Microwave Theory Tech.*, Vol. 48, No. 5, 832–839, 2000.
4. Seo, S. M. and J.-F. Lee, "A fast IE-FFT algorithm for solving PEC scattering problems," *IEEE Trans. Magn.*, Vol. 41, No. 5, 1476–1479, 2004.
5. Graglia, R. D., D. R. Wilton, and A. F. Peterson, "Higher order interpolatory vector bases for computational electromagnetics," *IEEE Trans. Antennas Propagat.*, Vol. 45, No. 3, 329–342, 1997.
6. Jorgensen, E., J. L. Volakis, P. Meincke, and O. Breinbjerg, "Higher order hierarchical Legendre basis functions for electromagnetic modeling," *IEEE Trans. Antennas Propagat.*, Vol. 52, No. 11, 2985–2995, 2004.
7. Nie, Z., W. Ma, Y. Ren, Y. Zhao, J. Hu, and Z. Zhao, "Wideband electromagnetic scattering analysis using MLFMA with higher order hierarchical vector basis functions," *IEEE Trans. Antennas Propagat.*, Vol. 57, No. 10, 3169–3178, 2009.
8. Peng, Z., X.-C. Wang, and J.-F. Lee, "Integral equation based domain de-composition method for solving electromagnetic wave scattering from non-penetrable objects," *IEEE Trans. Antennas Propagat.*, Vol. 59, No. 9, 3328–3338, 2011.
9. Peng, Z., K.-H. Lim, and J.-F. Lee, "Computations of electromagnetic wave scattering rom penetrable composite targets using a surface integral equation method with multiple traces," *IEEE Trans. Antennas Propagat.*, Vol. 61, No. 1, 256–270, 2013.
10. Saad, Y. and M. H. Schultz, "GMRES: A generalized minimal residual algorithm for solving nonsymmetric linear systems," *SIAM J. Sci. Stat. Comput.*, Vol. 7, No. 3, 1986.