Adaptive Optimal Polarization Detection of Target in Clutter Background Based on Generalized Rayleigh Quotient

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Abstract— Most current methods on polarization targets detection assume the target echoes to be ascertained in signal model during the detection period. But in fact, the target echo may obey a kind of distribution and the distribution relates with the polarizations. Aiming at improving the performance of polarization detection, an adaptive generalized likelihood ratio (AGLR) polarization detector is proposed. The detector assumes the target echo to be a non-zero mean Gaussian distribution with respect to the polarization states of transceiver. It adaptively adjusts the polarization states of transceiver according to the variation of the polarization characteristics of target and clutter to maximize signal to clutter and noise ratio (SCNR). The problem of maximizing the SCNR is converted to a problem of maximizing a generalized Rayleigh quotient (GRQ). Based on the GRQ, the AGLR detector is proposed. The performance of the AGLR detector is theoretically analyzed and numerically validated through some numerical experiments. Compared with two conventional GLR detectors, the AGLR detector achieves a 3 dB to 10 dB improvement.

1. INTRODUCTION

Detection of a target in clutter background is a challenging problem for target detection system when the target and the clutter are closely spaced in angle and Doppler domains. Polarization information has been widely used to improve the detection performance [1–3]. It has been demonstrated that polarization information is as important as amplitude, phase and frequency. Especially, when the polarization states of transceiver match target scattering characteristics well and mismatch clutter scattering characteristics, detection performance would be substantially improved.

The polarization information has been applied to improve detection performance since 1950s [4]. At the early stage, the researchers focused on the optimal polarization state selection, assuming the clutter and noise were given. A practicable polarization state selection method in an ideal environment was proposed by W. M. Boerner et al. [5]. L. M. Novak et al. extended this idea into a more realistic case where a target was embedded in a deterministic distributed noise background [6]. Gaussian distributed clutter model in [7] and non-Gaussian distributed clutter model [8, 9] were utilized to construct polarization detectors. For more generalized clutter and noise models, D. Pastina et al. used training data to estimate clutter [10]. A. Nehorai et al. applied compound-Gaussian distributed clutter models in different applications [11, 12].

In optimal polarization detection, the selection of a proper target model is also a critical issue. In recent years, a general framework for adaptive polarization waveform design was established in [13–17]. This framework obtained target information by estimating scattering parameters based on estimation theories. Methods based on the framework investigated the improvement of detection performance with fixed transceiver polarization states during every detection period. However, since the target echo obeys a kind of distribution that relates with the polarizations, receiver with a fixed polarization state may not receive the strongest echo.

Different from the condition of fixed transceiver polarization states [13–17], a variable transceiver polarization states in the detection period is studied to optimize the echo reception in this paper. The target echo is assumed to be a non-zero mean Gaussian distribution as in [18]. The clutter is estimated by training data. We propose an adaptive Generalized Likelihood Ratio (AGLR) detector by adaptively adjusting the polarization states of transceiver according to the variation of the polarization characteristics of the target, clutter and noise. In the design of the optimal detector, the optimization polarization detection problem is converted to a generalized Rayleigh quotient (GRQ) problem. Based on the GRQ, the detector achieves maximum signal to clutter and noise ratio (SCNR) and optimal polarization states of the transceiver. The detector is theoretically analyzed and numerically validated by several numerical experiments. Compared with two conventional generalized likelihood ratio (GLR) detectors, the proposed detector is always better in different clutter backgrounds.

The remainder of the paper is organized as follows. In Section 2, a measurement model is introduced. The optimization polarization detection problem is outlined. In Section 3, the optimization detection problem is induced to a GRQ problem. The AGLR detector is proposed by seeking of a maximum GRQ. Several numerical experiments are given in Section 4. Conclusions and remarks are drawn in the final section. Notations: the superscript "H" denotes the Hermitian transpose conjugate; "$|\mathbf{A}|$" denotes the determinant of $\mathbf{A}$; $\|x\|$ denotes the II-Norm of x. the brackets "$\langle\cdot\rangle$" denotes ensemble averaging.

2. POLARIZATION DETECTION METHOD

In this paper, the clutter and the noise are assumed to be polarized and unpolarized, respectively. In this section, a measurement model will be described. The optimization detection problem is formulated based on the SCNR.

2.1. The Optimization Detection Problem

Considering a detection system, a transmitter emits polarization electromagnetic wave. The illuminated target may change the polarization state of the incident wave. As a result, the scattering waves propagate to different directions with different polarization states. Therefore, an ideal detection system should be the one which could adaptively adjust the polarization states of the transceiver with respect to the polarization scattering matrix (PSM) of the object.

Let $\mathbf{S}$ represent the PSM, which is expressed as

$$\mathbf{S}(t)=\begin{bmatrix} s_{hh}(t), & s_{hv}(t)\\ s_{vh}(t), & s_{vv}(t)\end{bmatrix}, \tag{1}$$

where t denotes time, s_{ij} $(i,\, j=h,\, v)$ stands for polarization scattering coefficient with transmitting in i state and receiving in j state. As in [3], the transmitter emits a polarized waveform which can be written as

$$\mathbf{s}(t)=\sqrt{P_0}\boldsymbol{\xi}(t)s(t), \tag{2}$$

where $P_0\in R^+P_0$ is transmitted power. $\boldsymbol{\xi}(t)=[\xi_h(t),\xi_v(t)]^T$ is transmitted polarization state vector and $\|\boldsymbol{\xi}(t)\|=1$ ($\|\cdot\|$ denotes II-Norm). $s(t)$ is transmitted pulse waveform which has a complex envelope. The received signal is expressed as

$$x(t)=\sqrt{P_0}g\left[\boldsymbol{\eta}(t)\right]^T\mathbf{S}(t)\boldsymbol{\xi}(t)s(t-t_0)\exp(j2\pi f_d t)+n_0(t); \tag{3}$$

where g is a constant depending on wavelength, characteristic impedance of free space, distance between the target and the transceiver, gains, etc. $\boldsymbol{\eta}(t)=[\eta_h(t),\eta_v(t)]^T$ is received polarization state vector and $\|\boldsymbol{\eta}(t)\|=1$. t_0 is delay time. f_d is target Doppler frequency and $n_0(t)$ is noise. Since parameters $s(t-t_0)$ and f_d would not affect the received power, filtering technology is utilized to normalize (3) for simplicity

$$x(t)=\sqrt{P}g[\boldsymbol{\eta}(t)]^T\mathbf{S}(t)\boldsymbol{\xi}(t)+n(t); \tag{4}$$

where $\sqrt{P}=\sqrt{P_0}s(t-t_0)\exp(j2\pi f_d t)$ is normalized received power, and, $n(t)=\sqrt{P_0/P}n(t)$ denotes normalized noise. When transmitted beam and received beam illuminate target and clutter at the same time, the $\mathbf{S}$ consists of the $\mathbf{S}_t$ of target and the $\mathbf{S}_c$ of clutter; hence, (4) is rewritten as

$$x(t)=\sqrt{P}g[\boldsymbol{\eta}(t)]^T\mathbf{S}_t(t)\boldsymbol{\xi}(t)+\sqrt{P}g[\boldsymbol{\eta}(t)]^T\mathbf{S}_c(t)\boldsymbol{\xi}(t)+n(t), \tag{5}$$

We rearrange the scattering matrix and the antenna polarization states as

$$\begin{aligned}\mathbf{A}&=\boldsymbol{\eta}(t)\bigotimes\boldsymbol{\xi}(t)=[\eta_h(t)\xi_h(t),\eta_h(t)\xi_v(t),\eta_v(t)\xi_h(t),\eta_v(t)\xi_v(t)]^T,\\ \mathbf{S}&=[s_{hh}(t),s_{hv}(t),s_{vh}(t),s_{vv}(t)]^T,\end{aligned} \tag{6}$$

where $\mathbf{A}$ is a compound coherent vector of the polarization states of transceiver. $\bigotimes$ is the Kronecker product. Omitting the time index for notational simplicity, (5) is rewritten as

$$x=\sqrt{P}\mathbf{A}^T\left(\mathbf{S}_t+\mathbf{S}_c\right)+n=\sqrt{P}\mathbf{A}^T\mathbf{S}_t+\sqrt{P}\mathbf{A}^T\mathbf{S}_c+n \tag{7}$$

Hence, the signal to clutter and noise ratio (SCNR) can be defined as

$$\mathrm{SCNR} = \frac{\left\|\sqrt{P}\mathbf{A}\mathbf{S}_t\right\|}{\left\|\sqrt{P}\mathbf{A}\mathbf{S}_c\right\| + \|n\|^2} = \frac{P\mathbf{A}^H[\mathbf{C}_t]\mathbf{A}}{P\mathbf{A}^H[\mathbf{C}_c]\mathbf{A} + [\mathbf{C}_n]}, \tag{8}$$

where $[\mathbf{C}] = \|\mathbf{S}\|^2$. Let $\sigma_n^2\mathbf{I}_4 = [\mathbf{C}_n]/P$, the SCNR is rewritten as

$$\mathrm{SCNR} = \frac{\mathbf{A}^T[\mathbf{C}_t]\mathbf{A}}{\mathbf{A}^T[\mathbf{C}_c]\mathbf{A} + \sigma_n^2\mathbf{I}_4}. \tag{9}$$

The objective of target detection is to maximize the SCNR. Therefore, the optimization problem is formulated as

$$\underset{\|\mathbf{A}\|=1}{\arg\max}\ \mathrm{SCNR} = \frac{\mathbf{A}^T[\mathbf{C}_t]\mathbf{A}}{\mathbf{A}^T[\mathbf{C}_c]\mathbf{A} + \sigma_n^2\mathbf{I}_4}, \tag{10}$$

2.2. Adaptive Optimal Polarimetric Detector

We define a matrix function $f(\mathbf{A})$ as

$$f(\mathbf{A}) = \frac{\mathbf{A}^T[\mathbf{C}_t]\mathbf{A}}{\mathbf{A}^T[\mathbf{C}_c]\mathbf{A} + \sigma_n^2\mathbf{I}_4}, \tag{11}$$

Since matrices $[\mathbf{C}_t]$, $[\mathbf{C}_c]$ and $\sigma_n^2\mathbf{I}_4$ are all positive semi-definite ones, is bigger or at least equal to zero. Compared (11) with (8), the optimization problem in (10) is equivalence to a problem of maximizing $\sigma_n^2\mathbf{I}_4$. Since $\sigma_n^2\mathbf{I}_4 = \mathbf{A}^T(\sigma_n^2\mathbf{I}_4)\mathbf{A}$, problem (10) can be rewritten as

$$\underset{\|\mathbf{A}\|=1}{\arg\max}\ f(\mathbf{A}) = \underset{\|\mathbf{A}\|=1}{\arg\max}\ \frac{\mathbf{A}^T[\mathbf{C}_t]\mathbf{A}}{\mathbf{A}^T\left([\mathbf{C}_c] + \sigma_n^2\mathbf{I}_4\right)\mathbf{A}}. \tag{12}$$

Since matrix $[\mathbf{C}_c]$ is a Hermitian positive semi-definite one and matrix $\sigma_n^2\mathbf{I}_4$ is a Hermitian positive definite one, the summation of these two matrices is a Hermitian positive definite one. Let $[\mathbf{C}_{c+n}] = [\mathbf{C}_c] + \sigma_n^2\mathbf{I}_4$. The optimization problem is simplified as

$$\underset{\|\mathbf{A}\|=1}{\arg\max}\ f(\mathbf{A}) = \underset{\|\mathbf{A}\|=1}{\arg\max}\ \frac{\mathbf{A}^T[\mathbf{C}_t]\mathbf{A}}{\mathbf{A}^T[\mathbf{C}_{c+n}]\mathbf{A}}. \tag{13}$$

Hence, (13) is converted to a problem of maximizing a general Rayleigh quotient (GRQ). Let $\mathbf{P}$ be a non-singular matrix, such that $[\mathbf{C}_{c+n}] = \mathbf{P}\mathbf{P}^H$. Let $\mathbf{Z}^T = \mathbf{A}^T\mathbf{P}$ and $\mathbf{Q} = \mathbf{P}^{-1}[\mathbf{C}_t]\mathbf{P}^{-T}$, (13) is rewritten as

$$\underset{\|\mathbf{Z}\|=\|[\mathbf{C}_{c+n}]\|}{\arg\max}\ f(\mathbf{A}) = \underset{\|\mathbf{Z}\|=1}{\arg\max}\ \frac{\mathbf{Z}^T\mathbf{Q}\mathbf{Z}}{\mathbf{Z}^T[\mathbf{C}_{c+n}]\mathbf{Z}}. \tag{14}$$

Assuming λ_i $(i = 1, 2, \ldots, n)$ are the eigenvalues of matrix $\mathbf{Q}$ and $\lambda_1 \geqslant \lambda_2 \geqslant \ldots \geqslant \lambda_n$. Applying the results of [19], we obtain

$$\underset{\|\mathbf{Z}\|=\|[\mathbf{C}_{c+n}]\|}{\arg\max}\ f(\mathbf{A}) = \lambda_1. \tag{15}$$

Let $\mathbf{Z}_1$ represent the eigenvector corresponding to the maximum eigenvalue λ_1 of matrix $\mathbf{Q}$. The optimum polarization state of transceiver obeys the following condition,

$$\mathbf{A}_{opt} = \mathbf{P}^{-T}\mathbf{Z}_1. \tag{16}$$

3. NUMERICAL SIMULATIONS

In this section, several numerical experiments are exploited to validate the detection performance of the AGLR detector in comparison with two conventional detectors, i.e., HH-GLR detector and HV-GLR detector. The general form of covariance matrix is

$$[\mathbf{C}] = \beta\begin{bmatrix} 1 & 0 & 0 & \delta \\ 0 & \alpha_1 & \phi & 0 \\ 0 & \phi & \alpha_2 & 0 \\ \delta & 0 & 0 & 1 \end{bmatrix}, \tag{17}$$

where β is the scaling factor to fulfill the given SCNR. δ is the correlation factor between co-polarized channels. ϕ is the correlation factor between cross polarized channels. α_1 and α_2 are the is the power ratio of the two cross polarized channels to the HH channel.

For comparison purposes, the following two fixed polarization detectors, i.e., HH and HV, is computed. They polarization states $\mathbf{A}$:

$$\begin{aligned}\mathbf{A}_{hh} &= [1, 0, 0, 0]^T, \\ \mathbf{A}_{hv} &= [0, 1, 0, 0]^T.\end{aligned} \tag{18}$$

The parameters of the target are assumed to be: $\delta_t = 0.99$, $\phi_t = 0.1$ and $\alpha_1 = \alpha_2 = 0.1$. The constant parameters of the clutter are $\delta_c = 0.9$ and $\phi_c = 0.1$. In the following experiments, we assume $\mathrm{CNR} = 10\,\mathrm{dB}$, the number of adjacent cells and pulses in one scan period are both 15; i.e., $M = 10$ and $N = 10$. The signal has a dimension of $D = 1$.

Figures 1(a)–(c) show the detection probability P_d of three detectors, i.e., the AGLR detector, the HH-GLR detector and the HV-GLR detector, as functions of the signal to clutter ratio (SCR). Results shows: 1) The AGLR detector has the best performance in different clutter scenarios. 2) When the clutter is stronger in co-polarized channel, i.e., $\alpha_1 = \alpha_2 = 0.1$, the HH-GLR detector outperforms the HV-GLR detector. 3) When the clutter is the same, i.e., $\alpha_1 = \alpha_2 = 1$, the HH-GLR detector and the HV-GLR detector have the similar performance. 4) When the clutter is weaker in the co-polarized channel, i.e., $\alpha_1 = \alpha_2 = 10$, the HH-GLR detector is worse than the HV-GLR detector.

Figures 2(a)–(c) illustrate the detection probabilities of the AGLR detector, the HH-GLR detector and the HV-GLR detector as functions of the false alarm probability at different signal to clutter ratios (SCRs). The figures show that the AGLR detector always outperforms the two conventional GLR detectors. It means that the AGLR detector has the highest detection probability for the same false alarm probability or the AGLR detector has a lower false alarm probability for the same detection probability.

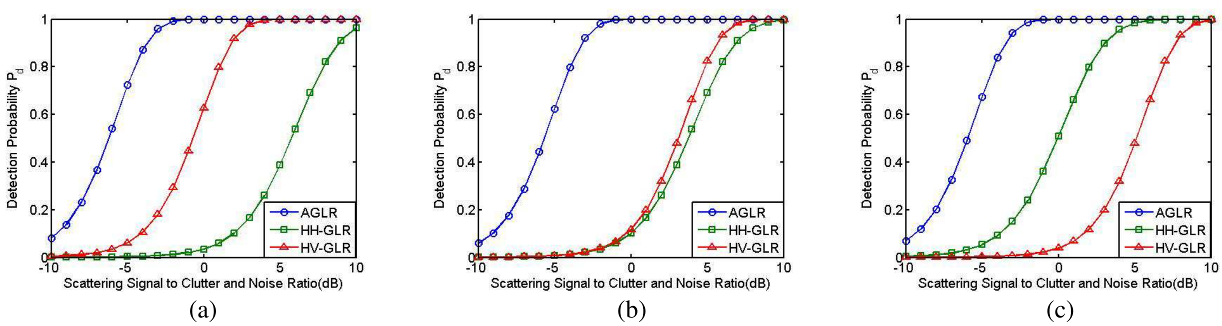

Figure 1: Detection probabilities of the 3 detectors as functions of SCNR with clutter to noise ratio (CNR) $\mathrm{CNR} = 10\,\mathrm{dB}$, $P_{fa} = 10^{-4}$, $M = 10$, $N = 10$ and $D = 1$: (a) $\alpha_1 = \alpha_2 = 0.1$, (b) $\alpha_1 = \alpha_2 = 1$ and (c) $\alpha_1 = \alpha_2 = 10$.

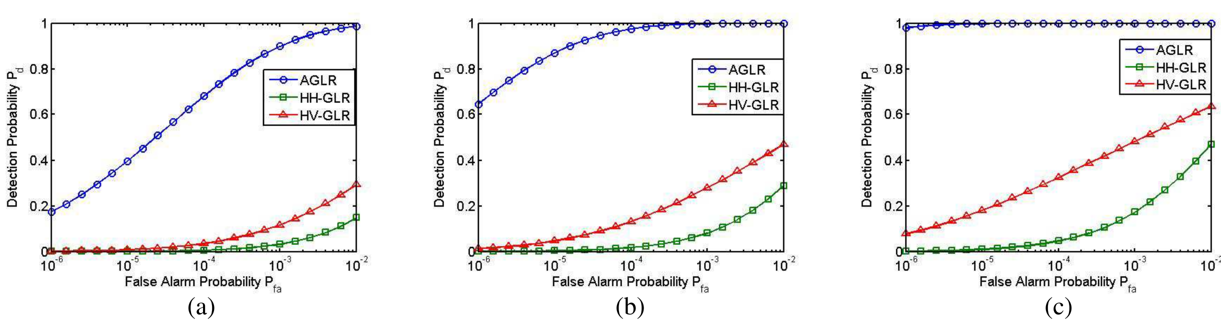

Figure 2: Detection probabilities of the 3 detectors as functions of false alarm probability for $\mathrm{SCR} = 0\,\mathrm{dB}$, $\mathrm{CNR} = 10\,\mathrm{dB}$, $M = 10$, $N = 10$ and $D = 1$: (a) $\alpha_1 = \alpha_2 = 0.1$, (b) $\alpha_1 = \alpha_2 = 1$ and (c) $\alpha_1 = \alpha_2 = 10$.

4. CONCLUSION

In this paper, the problem of detecting target in clutter background was addressed. Target was assumed to be a non-zero Gaussian distribution based on the fact that different transceiver polarization states would obtain different target echoes. We proposed an AGLR detector for adaptively adjusting the polarization states of transceiver so as to maximize the SCNR. Clutter and noise were not required to be known a prior. Training data were used to acquire the information of clutter and noise. Compared with two conventional GLR detectors, i.e., the HH-GLR detector and the HV-GLR detector, the AGLR detector always performs the best when dealing target detection in different clutter backgrounds.

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