Implementation of Three-dimensional Diffractive Gaussian Beam Analysis Method

Fangyuan Cheng[1], **Zejian Lu**[1], **Xiaoming Liu**[1], **Hai Wang**[1], **Junsheng Yu**[1], **and Xiaodong Chen**[2]

[1]School of Electronic Engineering, Beijing University of Posts and Telecommunications
279 Box, 10 Xi Tu Cheng Road, Haidian District, Beijing 100876, China
[2]School of Electronic Engineering and Computer Science, Queen Mary University of London
Mile End Road, London El 4NS, UK

Abstract— This paper developed a 3-dimensional Diffractive Gaussian Beam Analysis (DGBA) method for the analysis of electrically large reflector antennas. Unlike a symmetrical 2-D structure, it adopts the approach of full plane expansion, where the illuminating field over the whole input plane is decomposed into a set of elementary Gaussian beams. These Gaussian beams are propagated to the reflector. Reflected beams are treated with Geometric Optics, while edge diffraction is modeled using the canonical problem of a 3D Gaussian beam incident upon an opaque Kirchhoff half-screen. The output beam is a superposition of all reflected and diffracted elementary beams. The proposed analysis method is verified against a GRASP Physical Optics.

1. INTRODUCTION

In millimeter wave and sub-millimeter wave radiometry and radio astronomy applications, analysis technology for quasi-optical (QO) system is the research focus for the analysis of electrically large systems [1, 2]. Many numerical calculation methods can be employed to predict the diffracted fields of reflector antennas such as Physical Optics (PO) and Geometrical Optics (GO). Unfortunately, GO does not account for the effects of boundary diffraction. PO is accurate but expensive in terms of computation time and storage for electrically large system. In this connection, a faster method, namely diffracted Gaussian beam analysis, combines the numerical efficiency of GO and the rigor of PO, has been developed The current 2-Dimensional DGBA requires the incident wave and reflector to be symmetrical about a certain plane. To overcome this limitation, this paper introduces the approach of full plane expansion, where the field over the whole input plane is decomposed to a series of GBs. This proposed method can be applied to stereoscopic system in some practical applications.

2. THEORY

2.1. The Theory of DGBA

The process of DGBA method follows fours steps [3]: (1) The incident fields that could be either the near field of a feed or the radiated field from a previous reflector are first expanded to Gaussian beams by windowed Fourier transform on the input plane; (2) These Gaussian beams will be propagated to the reflector to be analyzed according to Gaussian beam propagation theory; (3) The reflected Gaussian beams can be processed by Geometric Optic, and the diffraction coefficients of Gaussian beams can be modeled by an equivalent Kirchhoff half plane; (4) The reflected and diffracted Gaussian beams will be superposed on an output plane, which forms the input plane to the next reflector or other Quasi-Optical components. If this reflector is the last one in the system, far fields can be calculated by superposing the far field of each Gaussian beam.

2.2. Gaussian Beam Diffraction

2.2.1. Field Representation in the Forward-scattering Region

Diffraction of a Gaussian beam with a circular spot size normally incident upon an opaque Kirchhoff half-screen was investigated based on the boundary-diffraction wave (BDW) theory [4, 5]. The incident beam is assumed to propagate in z-direction and can be written as

$$U_i(x, y, z) = \frac{q(-z_0)}{q(z)} \exp(ik_0\phi(x, y, z)) \tag{1}$$

$P(x, y, z)$ is the observation point and $Q(x_0, y_0, z_0)$ is a source point of boundary-diffraction located on the boundary of the half-screen. Following the BDW theory, we can represent the total diffracted field $U_K(P)$ in the forward scattering region ($z > 0$) at the observation point P as

$$U_k(P) = \sum_j F_j(P) + U_B(P) \tag{2}$$

In the formula, $U_B(P)$ and $\sum_j F_j(P)$. Separately represent Geometrical Optics incident field and diffraction part.

$$U_B(P) = \int_\Gamma \vec{W}(P,Q)d\vec{l} = \int_\Gamma U_i(Q)\frac{\exp(jk_0 s)}{4\pi s}\frac{\hat{s}\times\nabla_Q\phi}{1+\hat{s}\cdot\nabla_Q\phi}\, d\vec{l} \tag{3}$$

$$\sum_j F_j(P) = E(x_s - x)U_i(P) \tag{4}$$

The total diffracted field can be written as a superposition in terms of complementary error functions:

$$U(P) = \begin{cases} U_i(P) - \frac{q(0)}{2q(z)}\mathrm{erfc}\left(j\left[jk_0\left(d(y_s) - d\left(y_p^1\right)\right)\right]^{1/2}\right) & \\ \exp\left(-jk_0(d(y_s) - d\left(y_p^1\right) - s(Q_d))\right)U_i(Q_d) & x \le x_s \\ \frac{q(0)}{2q(z)}\mathrm{erfc}\left(-j\left[jk_0\left(d(y_s) - d\left(y_p^1\right)\right)\right]^{1/2}\right) & \\ \exp\left(-jk_0(d(y_s) - d\left(y_p^1\right) - s(Q_d))\right)U_i(Q_d) & x \ge x_s \end{cases} \tag{5}$$

2.2.2. Field Representation in the Backward-scattering Region

We can find an equivalent geometry, as valid in the forward-scattering, to describe the diffracted in the backward-scattering region. A new coordinate system with the transformed (x, y, z) pointing in $(-x, y, -z)$ as shown in Fig. 2. The equivalent half-screen must be complementary to the true half-screen to generate a shadow region in the lower half space. The BDW does only depend on the location of the edge and not on the orientation of the half screen.

2.2.3. Generalization to Oblique Incidence

The transmitted beam-related co-ordinate system (x_t, y_t, z_t), the reflected beam related co-ordinate system (x_r, y_r, z_r), the half-screen based co-ordinate system (x, y, z) and the observation point is being described in the half-screen based (ρ, ϕ, y). The co-ordinate systems are expressed in terms of the half-screen co-ordinate system by simple translation and rotation around the y-axis:

$$\begin{pmatrix} z_t \\ x_t \end{pmatrix} = \rho\begin{pmatrix} \cos(\varphi - (\pi + \varphi_0)) \\ \sin(\varphi - (\pi + \varphi_0)) \end{pmatrix} + \begin{pmatrix} z_e\cos(\varphi) \\ 0 \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} z_r \\ x_r \end{pmatrix} = \rho\begin{pmatrix} \cos(\varphi - (\pi - \varphi_0)) \\ \sin(\varphi - (\pi - \varphi_0)) \end{pmatrix} + \begin{pmatrix} z_e\cos(\varphi_0) \\ 0 \end{pmatrix} \tag{7}$$

Since the BDW is obtained as a line integral along the edge and does not depend on the orientation of the half-screen, we are now in a position to apply the formulas for normal incidence. The

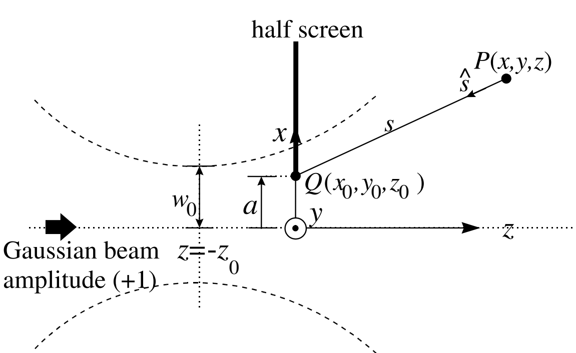

Figure 1: Geometry of the problem (forward-scattering region).

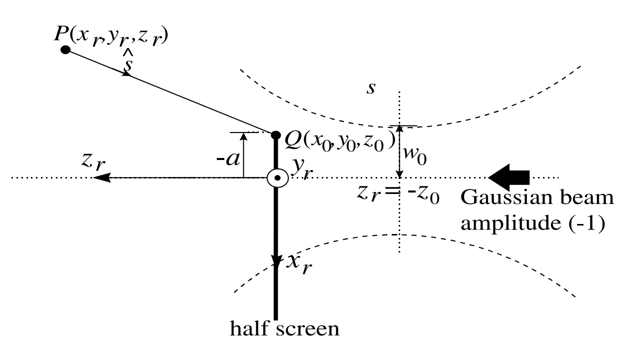

Figure 2: Mirror-symmetric geometry for determining the backward- scattered field.

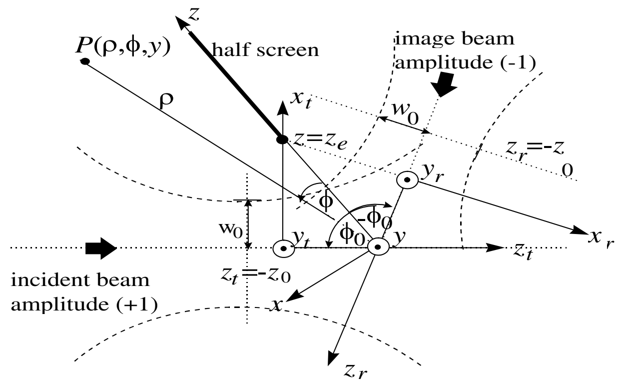

Figure 3: Equivalent geometry for determining the backward-scattered field at oblique incidence.

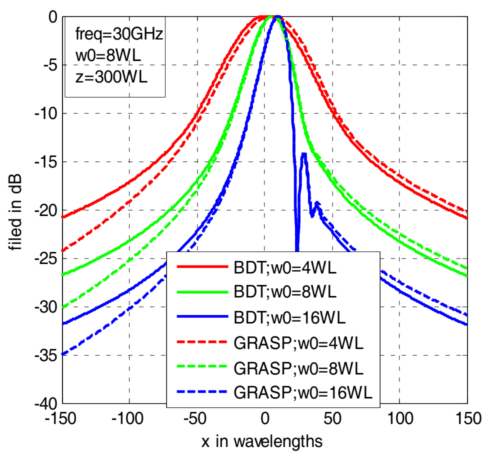

Figure 4: Comparison of the backscattered field with PO for various beam spot sizes.

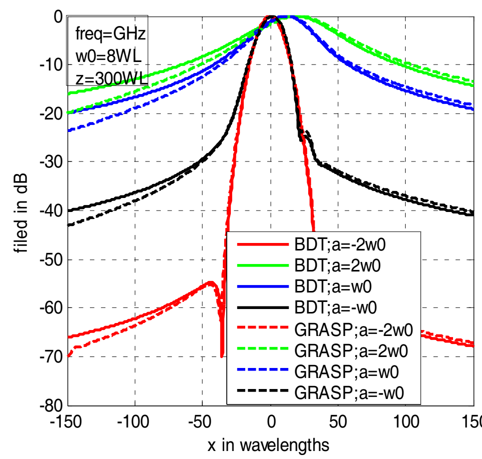

Figure 5: Comparison of the backscattered field with PO for various values of the parameter a.

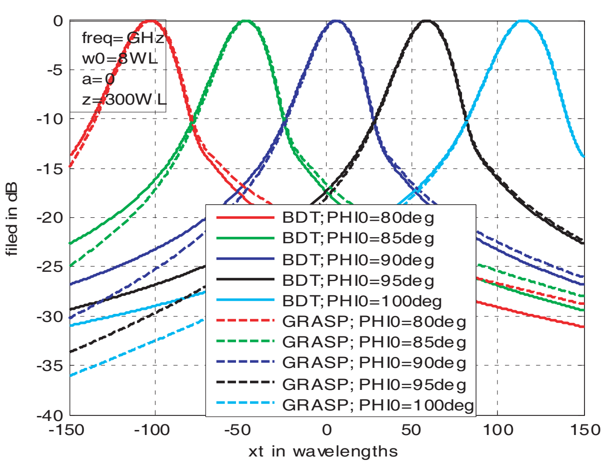

Figure 6: Comparison of the backscattered field with PO for various orientations of the half-screen.

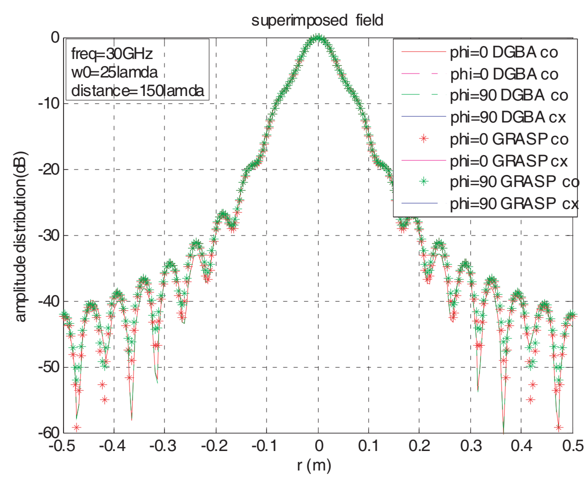

Figure 7: Comparison of the near field from main reflector with PO at 30 GHz.

offset distance a from the edge is related to the axial position z_e of the edge in the half-screen by $a = z_e \sin(\phi_0)$.

We consider the case of azimuthally normal incidence, but oblique incidence with respect to the polar angle. For incidence angles close to 90, the formulas (5) can still be applied. The beam direction of the transmitted and reflected beam changes, however according to GO and the transmitted and reflected related co-ordinate system are rotated by $(\pi/2 - \theta)$ around the x-axis compared to the case with normal incidence.

2.2.4. Simulation and Numerical Verification

To check the accuracy of the asymptotic method, the problem of an incident Gaussian beam upon a Kirchhoff half-screen has been analyzed with the commercial reflector analysis tool GRASP [6], using Physical Optics.

3. NUMERICAL RESULTS

The Gaussian beams normally incident upon the the paraboloidal reflector operate at 30 GHz. There are the following geometrical characteristics: the beam waist is 0.25 m, reflector diameter is 0.5 m, reflector focal length is 4 m, the distance from the feed to the reflector edge is 0.1 m and the output expansion plane is at $zt = 1.5\,\mathrm{m}$. The results to be shown is of comparative field plots between GRASP and the DGBA.

4. CONCLUSION

The proposed 3-Dimensional DGBA procedure with the approach of full plane expansion has been presented. This method can be applied to 3D system, where the optical centers of the feed and all of the reflectors can lie on the different planes. The field in the result section demonstrates the effectiveness of the DGBA in comparison to GRASP.

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