Chaotic FM Signals for Circular SAR Imaging

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Abstract— Noise radar has low probability of intercept and high resistance to electronic countermeasures, and has the ability of non-ambiguous measurement of range and velocity. Circular SAR (CSAR) has high two-dimensional (2D) ground range resolution and a certain three-dimensional (3D) imaging capability. The combination between noise radar and CSAR can achieve the complementary strengths, which has significance for target detection and high resolution imaging. In this paper, Ulam mapping chaotic frequency modulation (FM) signal is used as the transmitted signals for 2D and 3D CSAR imaging. All imaging results are compared with other imaging results when the transmitted signals are chirp and Ulam mapping chaotic amplitude modulation (AM) signals respectively. Simulation results of point-target show that when the transmitted signal is Ulam mapping chaotic FM signal, 2D and 3D resolution are higher and side-lobes except integral sidelobe ratio in height are lower than results of other two kinds of transmitted signals, because the effective bandwidth of Ulam mapping chaotic FM signal is wider than chirp and Ulam mapping chaotic AM signals. In addition, Ulam mapping chaotic AM and FM signals for CSAR imaging have stronger anti-interference ability than chirp signal.

1. INTRODUCTION

Noise radar has low probability of intercept and high resistance to electronic countermeasures, which improves the ability of radar to survive in a complex environment. The ambiguity function of ideal noise signal has an approximate thumbtack shape, which makes noise radar with the ability of non-ambiguous measurement of range and velocity. Because of these excellent features, noise radar has broad application prospects in the military and civilian areas. In circular synthetic aperture radar (CSAR) mode, both two-dimensional (2D) high resolution image and three-dimensional (3D) image of targets can be obtained because of the complete 360 degree synthetic aperture. The combination between noise radar and CSAR can achieve the complementary strengths, which has significance for target detection and high resolution imaging.

In this paper, Ulam mapping chaotic frequency modulation (FM) signal is used as the transmitted signals for CSAR imaging. All these 2D and 3D imaging results and anti-interference ability are compared with other results when the transmitted signals are chirp and Ulam mapping chaotic amplitude modulation (AM) signals respectively. The rest of this paper is organized as follows. In Section 2, the model of Ulam mapping chaotic FM signal is introduced. In Section 3, the echo model of CSAR is illustrated. In Section 4, analysis of anti-noise performance is introduced. Simulation results are shown in Section 5. Finally, Section 6 concludes the paper.

2. THE MODEL OF ULAM MAPPING CHAOTIC FM SIGNAL

The commonly used chaotic mappings are Bernoulli mapping, Tent mapping, Logistic mapping, and so on. Here, a special case of Logistic mapping, Ulam mapping, is considered. It can be expressed by,

$$x_{n+1} = 1 - 2x_n^2, \quad x_n \in (-1, 1) \tag{1}$$

Ulam mapping signal is called modulating signal, and the expression of frequency modulation is,

$$s(t) = \exp\left(j2\pi\Delta f \int_{-\infty}^{t} \xi(\tau) d\tau\right) \tag{2}$$

where Δf is frequency offset, and $\xi(t)$ is Ulam mapping chaotic AM signal, which can be written as,

$$\xi(t) = \sum_{n=0}^{N-1} x_n g(t - nT) \tag{3}$$

where x_n is the nth of Ulam mapping sequences of length N, and $g(t)$ is pulse signal of time width T. So, the time width of $\xi(t)$ is $T_N = NT$. When $t \in [nT, (n+1)T]$, the expression of Ulam mapping chaotic FM signal is,

$$s(t) = \exp\left[j2\pi\Delta f\left(T\sum_{i=0}^{n-1} x_i + x_n(t - nT)\right)\right] \tag{4}$$

The discrete model of (4) is,

$$s(nT) = \exp\left[j2\pi\Delta fT\left(\sum_{i=0}^{n} x_i\right)\right] \tag{5}$$

From (5), the instantaneous frequency is $f(n) = x_n \cdot \Delta f$. Because $x_n \in (-1, 1)$, the instantaneous frequency range is $-\Delta f \leq f \leq \Delta f$. According to Nyquist sampling theorem, $1/T \geq 2\Delta f$ is obtained. Suppose $1/T = 2\Delta f$, then Equation (5) is simplified as,

$$s(n) = \exp\left[j\pi\left(\sum_{i=0}^{n} x_i\right)\right] \tag{6}$$

3. THE ECHO MODEL OF CSAR

In CSAR mode, radar platform travels around targets in a circular trajectory, which is different from the traditional linear SAR mode. The CSAR system model for data acquisition is shown in Fig. 1(a). Its top view and side view are shown in Figs. 1(b) and (c), respectively.

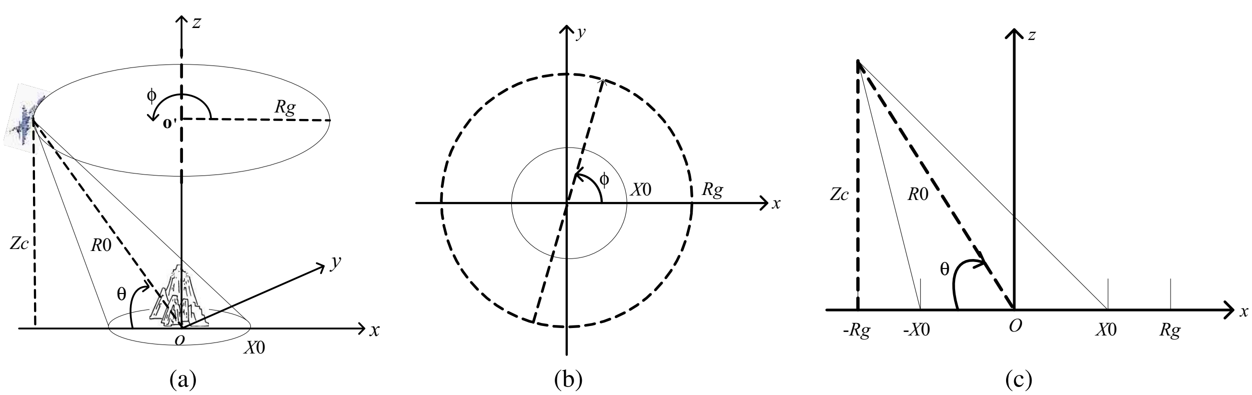


Figure 1: (a) Geometry of CSAR system. (b) Top view. (c) Side view.

From Fig. 1, the radar height is Zc, and the flight radius is Rg. Denote radar azimuthal angle as $\phi \in [0, 2\pi)$, and denote the transmitted radar signal as $p(t)$. The reflectivity function of 3D target is defined as $f(x, y, z)$. Then the received echo signal can be given by,

$$s(t, \phi) = \int_z\int_y\int_x f(x, y, z)p\left[t - 2\sqrt{(x - R_g\cos\phi)^2 + (y - R_g\sin\phi)^2 + (z - Z_c)^2}/c\right]dxdydz \tag{7}$$

where t is fast-time, and c is light speed. For 2D target, we only need to set $z = 0$ in Equation (7).

4. ANALYSIS OF ANTI-NOISE PERFORMANCE

In order to describe noise intensity, the input signal to noise ratio is defined as $\mathrm{SNR}_{in} = 10\log_{10}(P_{S_{in}} / P_{N_{in}})$, where $P_{S_{in}}$ is the power of echo signal, and $P_{N_{in}}$ is the power of noise. Noise interference belongs to barrage jamming. The equivalent number of looks (ENL) and correlation coefficient of SAR image are adopted in order to evaluate barrage jamming.

(1) ENL

$$\mathrm{ENL} = \frac{\mu}{\sigma} \tag{8}$$

where μ is the statistical average value of a gray image, and σ is the standard deviation. ENL reflects contrast of a gray image, and large ENL means small image contrast and ambiguous image.
(2) Correlation coefficient

Denote a gray image without noise as $f(m,n)$, where m and n are respectively the numbers of row and column of the image. If it is interrupted by noise, then the image with noise is denoted as $g(m,n)$. The correlation coefficient ρ of these two images is,

$$\rho = \frac{\sum_m \sum_n f(m,n)g(m,n)}{\left[\sum_m \sum_n f^2(m,n) * \sum_m \sum_n g^2(m,n)\right]^{1/2}} \tag{9}$$

The correlation coefficient reflects the degree of deterioration of the image before and after interference. The large ρ means small noise interference. The value range of ρ is $0 \sim 1$.

5. SIMULATION

(1) 2D point target imaging

The CSAR system parameters are listed in Table 1. A 2D point target is located in the center of the scene. The back projection algorithm is used in imaging. When three kinds of signals including chirp, Ulam mapping chaotic AM, and Ulam mapping chaotic FM signals are transmitted, 2D imaging results are respectively shown in Figs. 2(a)–(c), and the x and y profiles of 2D imaging results are respectively shown in Figs. 3(a)–(b).

From Fig. 2 and Fig. 3, we can know that 2D imaging results are regular when the transmitted signal is chirp, while 2D imaging results are stochastic when the transmitted signal are Ulam mapping chaotic AM and FM. However, experiments can be repeated many times to obtain statistical values of resolution and sidelobes for Ulam mapping chaotic AM and FM signals. Imaging results show that when the transmitted signal is Ulam mapping chaotic FM, higher resolution and lower peak to sidelobe ratio (PSLR) and integrated sidelobe ratio (ISLR) in x and y profiles can be obtained than other two cases. The main reason is that the bandwidth of Ulam mapping chaotic FM signal is wider than other two kinds of signals.

Table 1: The CSAR system parameters.

Carrier frequency	10 GHz
Bandwidth of Chirp and chaos AM signal	4 GHz
Radius of the flight track	200 m
Height of the radar	200 m
Radius of imaging area	1 m

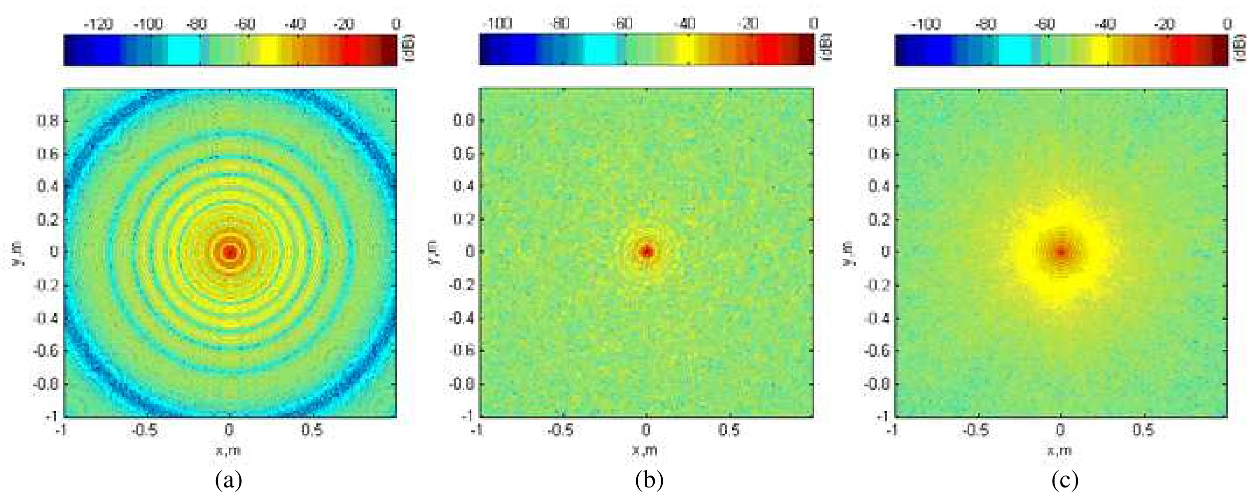
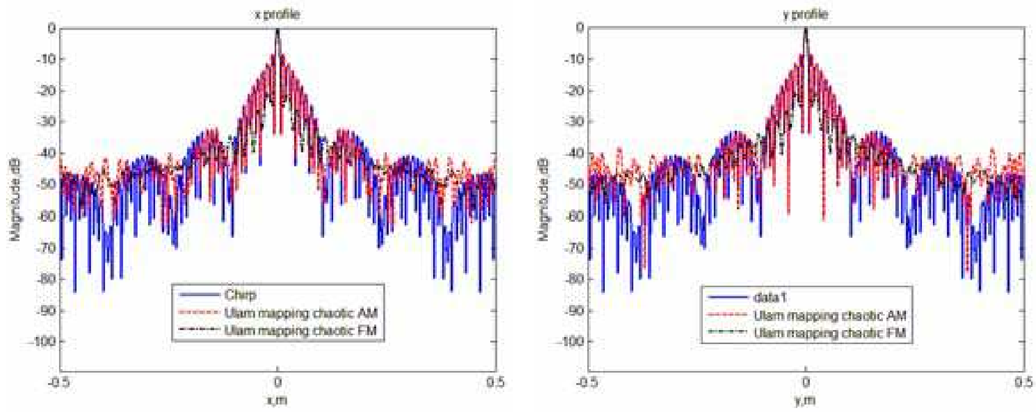
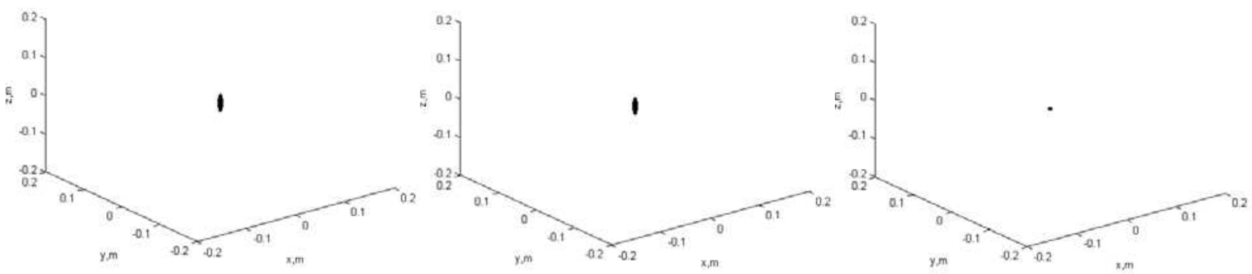


Figure 2: 2D imaging results of three kinds of transmitted signals. (a) Chirp. (b) Ulam mapping chaotic AM. (c) Ulam mapping chaotic FM.



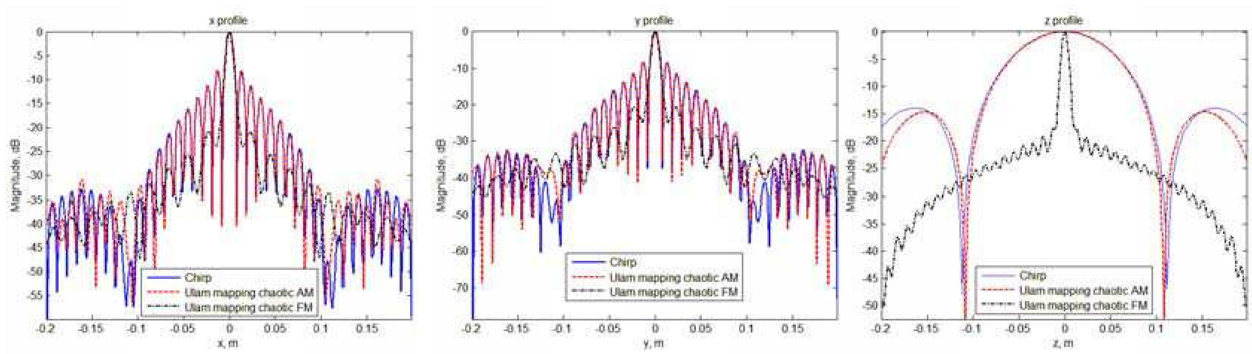
(a) (b)

Figure 3: The x and y profiles of 2D imaging results of three kinds of transmitted signals. (a) x profile. (b) y profile.



(a) (b) (c)

Figure 4: 3D imaging results of three kinds of transmitted signals. (a) Chirp. (b) Ulam mapping chaotic AM. (c) Ulam mapping chaotic FM.



(a) (b) (c)

Figure 5: The x, y and z profiles of 3D imaging results of three kinds of transmitted signals. (a) x profile. (b) y profile. (c) z profile.

(2) 3D point target imaging

The CSAR system parameters are listed in Table 1. A 3D point target is located in the center of the scene. The back projection algorithm is used in imaging. When three kinds of signals including chirp, Ulam mapping chaotic AM, and Ulam mapping chaotic FM signals are transmitted, 3D imaging results are respectively shown in Figs. 4(a)–(c). The x, y and z profiles of 3D imaging results are respectively shown in Figs. 5(a)–(c).

The same as the 2D imaging, experiments are repeated many times to obtain statistical values of resolution and sidelobes for Ulam mapping chaotic AM and FM signals. Imaging results show that when the transmitted signal is Ulam mapping chaotic FM, higher resolution and lower PSLR and ISLR in x and y profiles can be obtained than other two cases. Higher resolution and lower

PSLR can also be obtained except higher ISLR than other two cases. The main reason is also the wide bandwidth of Ulam mapping chaotic FM signal.

(3) Analysis of anti-noise performance

Based on 2D point target imaging, Figs. 2(a)–(c) are changed into gray images, and the corresponding results are noiseless. If Gauss white noise is added to three kinds of echo signals, and the input signal to noise ratio SNR_{in} is equal to $-40\,\text{dB}$, then 2D imaging results with this noise interference are obtained.

Repeat experiments 50 times. The statistical values of ENL and correlation coefficients for three kinds of echo signals without and with noise jamming are listed in Table 2. It is obvious that when the transmitted signals are Ulam mapping chaotic AM and FM, the change of ENL is smaller than the case of chirp signal. On the other side, when the transmitted signals are Ulam mapping chaotic AM and FM, the correlation coefficients are larger than the case of chirp signal. Small change of ENL and large correlation coefficients mean that when the transmitted signals are Ulam mapping chaotic AM and FM, the corresponding 2D imaging results are not sensitive to noise. It proves that noise radar has high resistance to noise jamming.

Table 2: The statistical values of ENL and correlation coefficients.

		Chirp	Ulam mapping chaotic AM	Ulam mapping chaotic FM
ENL	Without noise	0.201	0.538	0.615
	With noise	1.889	1.903	1.908
ρ		0.247	0.457	0.491

6. CONCLUSION

In this paper, Ulam mapping chaotic FM signal is presented for CSAR imaging. In order to illustrate the performance of this transmitted signal, Ulam mapping chaotic AM signal and chirp signal are used for comparison. Simulation results of point-target show that when the transmitted signal is Ulam mapping chaotic FM signal, 2D and 3D resolution are higher and side-lobes except integral sidelobe ratio in height are lower than results of other two kinds of transmitted signals, because of the wider effective bandwidth. In addition, Ulam mapping chaotic AM and FM signals for CSAR imaging have stronger anti-interference ability than the case of chirp signal.

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