Calculation of the Reflection and Transmission of Finite Sized Beams through Layered Uniaxial Anisotropic Media Accelerated by Plane Wave Spectrum Algorithm

Shihao Ji, Ming Bai, Zhao Liu, Yao Ma, and Xiuzhu Ye
Electromagnetics Laboratory, Beihang University, Beijing 100191, China

Abstract— In this paper, a fast method for calculating full vectorial reflection and transmission near-fields of arbitrary finite sized beams through multilayer uniaxial anisotropic media is proposed. The method combines Transfer Matrix method and optical fast-Fourier-transform based angular spectrum (FFT-AS) method. This calculation method can be used in the radome design and optical design which including multilayer anisotropic media.

1. INTRODUCTION

Calculation of electromagnetic waves through layered anisotropic media has been a problem encountered in many cases, such as radome design, optical design and ferromagnetic materials. Existing research have discussed infinite plane waves through layered anisotropic media, such as the general transmitting matrix and the Green's function method [1–3]. However, in case of finite sized beam or different orientation structured beam through layered anisotropic media, such as laser beam or antenna radiation limited in size, the calculation of reflection and transmission fields has been a problem encountered. In this case, a fast calculation method is required, especially for the design process which including large amount of parameter adjustment. For layered isotropic media, there is fast calculation method accelerated by plane wave spectrum algorithm [4]. When it comes to layered anisotropic media, this is usually performed by numerical ways, such as the Finite-Difference Time-Domain (FDTD) method [5] and the method of moment (MoM) [6], which can be time and memory consuming.

This paper proposed a method for calculating arbitrary oriented finite beams through layered uniaxial anisotropic media, as shown in Figure 1. The method combines Transfer Matrix method and optical fast-Fourier-transform based angular spectrum (FFT-AS) method. It can calculate full vectorial reflection and transmission near-fields of arbitrary finite sized beams through multilayer uniaxial anisotropic media within seconds, fully taking advantage of the FFT process.

There has been discussion on the FFT-AS method before [7], which is briefly illustrated in Section 2. The angular spectrum method also converts arbitrary oriented finite beams into superposition of plane waves with different directions. Thus the Transfer Matrix method can be used to calculate the result of every single plane wave through layered anisotropic media. Finally the transmitted plane waves and the reflected plane waves can be combined at target plane. With this method, the result of arbitrary oriented finite beams through layered anisotropic media can be

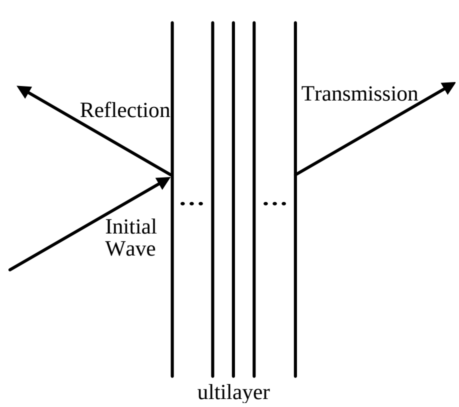

Figure 1: Schematic diagram of beam propagation through multilayer media.

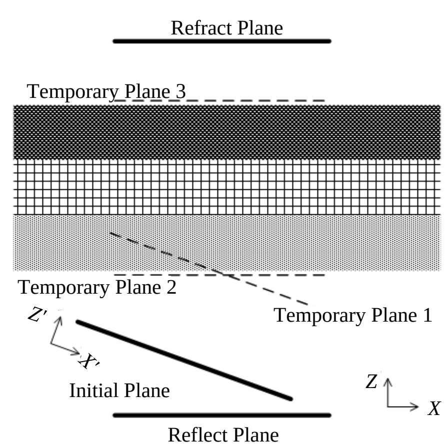

Figure 2: Model for combined method.

obtained accurately within seconds. On this basic, researchers can use this method to deal with big amount of parameter adjustment, with the complex structure and finite size of the multilayer anisotropic media involved such as radome design and optical design.

2. COMBINED METHOD OF FFT-AS AND TRANSFER MATRIX METHOD

As shown in Figure 2, the antenna in Figure 1 is replaced by an initial plane containing two orthogonal tangential field distributions. The FFT-AS method is used to calculate electromagnetic fields on temporary plane 1. Then the transfer matrix method is used to calculate transmitted beams on temporary plane 2 and reflected beams on temporary plane 3. Field distributions on arbitrary positioned target planes of the reflected and transmitted beam can be obtained by using the FFT-AS method again.

2.1. FFT-AS Method between Parallel and Tilted Planes

The Helmholtz–Kirchhoff and Rayleigh–Somerfield diffraction formulas have been widely used to analyze the propagation and diffraction of electromagnetic wave in an isotropic medium. As in most cases the formulas cannot be solved analytically, numerical calculation of the formula has been introduced, such as the angular spectrum (AS) method accelerated by fast-Fourier-transform (FFT) [7]. The AS method treats the propagation of electromagnetic fields as a superposition of plane waves with variety of wave vectors. The initial beam and its propagation are handled in the spatial-frequency domain. The scalar field distribution $U(x, y, z)$ at distance z can be calculated as

$$U(x, y, z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(\alpha, \beta, z) \cdot \exp\ [j(\alpha x + \beta y)] d\alpha d\beta \tag{1}$$

The propagation of electromagnetic fields can be given as

$$A(\alpha, \beta, z) = A(\alpha, \beta, 0) \cdot G(\alpha, \beta, z) \tag{2}$$

where α, β are the direction cosines of a plane wave, $A\ (\alpha, \beta, 0)$ is the angular spectrum of initial field $U\ (x, y, 0)$ which can be computed by two-dimensional (2D) Fourier transformation, $A\ (\alpha, \beta, z)$ is the AS of target plane at distance z, and

$$G(\alpha, \beta, z) = \exp\left(-j\frac{2\pi}{\lambda}z\sqrt{1 - \alpha^2 - \beta^2}\right) \tag{3}$$

is the transfer function of each plane wave.

Introducing the fast-Fourier-transform, the FFT-AS method can be given as

$$U(x_m, y_n, z) = \mathrm{IFFT}\left\{\mathrm{FFT}[U(x_m, y_n, 0)] \cdot \times G(\alpha_m, \beta_n, z)\right\} \tag{4}$$

where m and n mean the sampling points number of the plane, FFT2 and IFFT2 mean 2D FFT and 2D IFFT, and $\cdot\times$ stands for element-by-element multiplication.

As the wave vector of plane wave is perpendicular to electric field, the angular spectrum of electric field in z direction can be given as

$$\mathrm{AE}_z = -\frac{\mathrm{AE}_x \cdot k_x + \mathrm{AE}_y \cdot k_y}{k_z} \tag{5}$$

where AE_x, AE_y is the angular spectrum of electric field in x direction and y direction calculated above and k_x, k_y, k_z is the wave vector of plane wave. Thus the full vectorial field can be calculated by FFT-AS method.

2.2. Multilayer Uniaxial Anisotropic Structure Transfer Matrix

For a multilayer structure, the transfer matrix can be written as [8]

$$M = \prod_{i=0}^{N}\begin{bmatrix}\cos\delta_i & (j\sin\delta_i)\cdot\eta_i \\ (j\sin\delta_i)/\eta_i & \cos\delta_i\end{bmatrix} \tag{6}$$

$$\delta_i = 2\pi n_i d_i \cos\theta_i/\lambda \tag{7}$$

where d is thickness of each layer, θ is the propagating angle of each plan wave and η is the wave impedance. The transmission coefficient T and reflection coefficient Γ are given as

$$\begin{aligned} \mathrm{T} &= 1/M_{11} \\ \Gamma &= M_{21}/M_{11} \end{aligned} \tag{8}$$

As the wave impedance has different forms for TE and TM polarized plane wave, each plane wave should decompose to the TE and TM polarized plane wave and calculate separately.

For uniaxial anisotropic media, the permittivity tensor ε can be written as $\begin{bmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{bmatrix}$ in the principle coordinate system. For TE and TM polarized plane wave, the refractivity is given as

$$n_{\mathrm{TE}} = \frac{c}{\sqrt{\frac{\varepsilon_\perp}{\varepsilon_0}}}, \quad n_{\mathrm{TM}} = \frac{c}{\sqrt{\frac{\sin^2\theta}{\mu\varepsilon_\parallel} + \frac{\cos^2\theta}{\mu\varepsilon_\perp}}} \tag{9}$$

As discussed above, the initial wave is decompose to the TE and TM polarized plane wave, thus the plane of incidence is required to be parallel to the coordinate plane when using the transfer matrix method. To obtain this, a coordinate transformation matrix is adopted for each plane wave individually. The matrix $R_{i,j}$ is given as

$$R_{i,j} = \begin{bmatrix} \cos\phi_{i,j} & \sin\phi_{i,j} & 0 \\ -\sin\phi_{i,j} & \cos\phi_{i,j} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{10}$$

$$\phi_{i,j} = \cos^{-1}(sign(\beta_{i,j})\frac{\alpha_{i,j}}{\sqrt{\alpha_{i,j}^2 + \beta_{i,j}^2}} \tag{11}$$

The AS method decomposes the initial wave to a series of plane wave with different directions on temporary plane 1. The AS matrices of temporary plane 2 and 3 can be obtained using the transfer matrix method. Thus the full vectorial fields on any plane can be obtained using the FFT-AS method.

3. EXAMPLES AND DISCUSSION

To verify the validity of this calculation method, an example is given. The structure of the example is shown in Figure 3 and the parameters of the structure is given in Table 1.

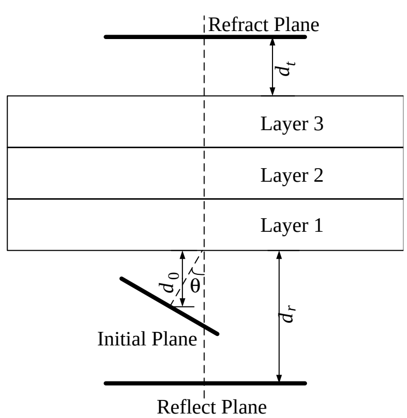

Figure 3: Simulation model of Gaussian beam propagation through layered anisotropic media.

Table 1: Structure parameters of Figure 4.

d_0	θ	d_r	d_t
45 mm	20°	75 mm	150 mm

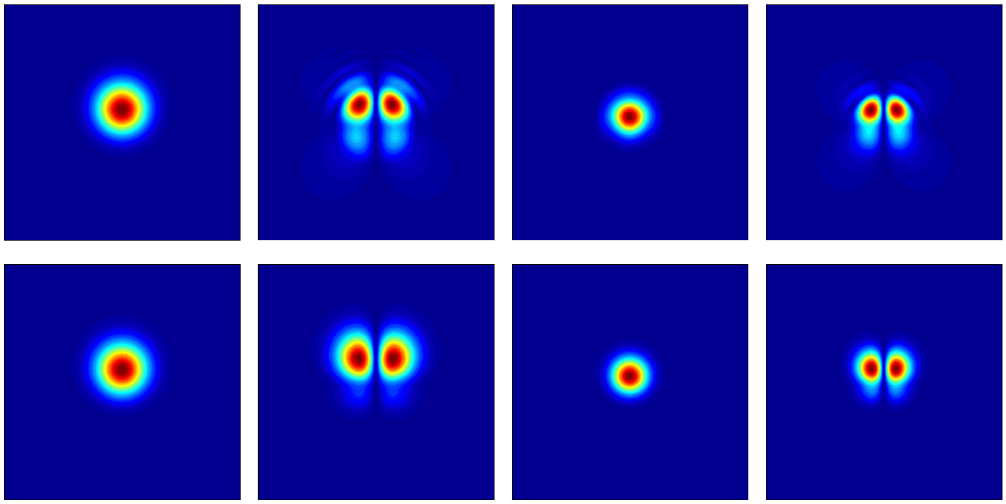

Figure 4: Amplitude of reflection and transmission fields on target plane. The first row is anisotropic media and the second row is isotropic media. The first column is main-polarization of transmitted fields. The second column is cross-polarization of transmitted fields. The third column is main-polarization of reflected fields. The forth column is cross-polarization of reflected fields.

Table 2: Multilayer parameters of Figure 4.

	$\varepsilon_\perp$	$\varepsilon_\parallel$	**Thickness**
Layer 1	2.8	3.5	5 µm
Layer 2	1	1	15mm
Layer 3	2.8	3.5	5 µm

We assume the layered media be uniaxial anisotropic media such as liquid crystal. Liquid crystals are dielectric materials with anisotropic characteristics [9]. The permittivity of liquid crystal can be tuned through an external electric field. The parameters of each media is given in Table 2. The initial field is the waist of a Gaussian beam at 20 GHz. The waist of the beam is 5λ. To compare the result with isotropic media, we assume the media in layer 1 and layer 3 be an isotropic media with the permittivity tensor 2.8. The simulation process is fast and costs less than 5 minutes. The result is shown in Figure 4.

The result implies that the cross-polarization of reflection and refraction fields of the anisotropic media is different from the isotropic media. The amplitude of cross-polarization of anisotropic media is higher than isotropic media and the amplitude of main-polarization of anisotropic media is lower than isotropic media. This is because the anisotropic media has the depolarization characteristic. This characteristic can be useful in antenna design and microwave devices design.

4. CONCLUSION

A fast way to calculate reflection and refraction fields of infinite beam through multilayer uniaxial anisotropic media is proposed in this letter. This method combines FFT-AS method and transfer matrix method and can avoid the time consuming numerical solution, full vectorial near fields of reflected and transmitted can be calculated within seconds. The method provides a way for large amount parameters adjustment in design process. An example of layered liquid crystal is given and implies that the cross-poloarizion difference between anisotropic media and isotropic media.

ACKNOWLEDGMENT

This work was supported by the Chinese National Programs for Fundamental Research and Development (Project 2012CB315601).

REFERENCES

1. Løseth, L. O. and B. Ursin, "Electromagnetic fields in planarly layered anisotropic media," *Geophysical Journal International*, Vol. 170, No. 1, 44–80, 2007.

2. Elshafiey, T. F. and J. T. Aberle, "Green's function for multilayer arbitrarily biased anisotropic structures — Application to phase shifters, transducers, and magnetization angle effect," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 54, No. 2, 513–521, 2006.
3. Titchener, J. B. and J. R. Willis, "The reflection of electromagnetic waves from stratified anisotropic media," *IEEE Transactions on Antennas and Propagation*, Vol. 39, No. 1, 35–39, 1991.
4. Liang, B., M. Bai, N. Ou, M. Jin, and J. Miao, "Fast analysis of arbitrary beam through planar multilayer structures," *Microwave and Optical Technology Letters*, Vol. 54, No. 9, 2185–2190, 2012.
5. Oh, C. and M. J. Escuti, "Time-domain analysis of periodic anisotropic media at oblique incidence: An efficient FDTD implementation," *Optics Express*, Vol. 14, No. 24, 11870–11884, 2006.
6. Kobidze, G. and B. Shanker, "Integral equation based analysis of scattering from 3-D inhomogeneous anisotropic bodies," *IEEE Transactions on Antennas and Propagation*, Vol. 52, No. 10, 2650–2658, 2004.
7. Shen, F. and A. Wang, "Fast-Fourier-transform based numerical integration method for the Rayleigh-Sommerfeld diffraction formula," *Applied Optics*, Vol. 45, No. 6, 1102–1110, 2006.
8. Macleod, H. A., *Thin-film Optical Filters*, CRC Press, 2001.
9. Yaghmaee, P., O. H. Karabey, B. Bates, C. Fumeaux, and R. Jakoby, "Electrically tuned microwave devices using liquid crystal technology," *International Journal of Antennas and Propagation*, 2013.