Efficient Wide-band Analysis of GPR Antenna Around a Platform Using the Best Uniform Rational Approximation Technique

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Abstract— Evaluating wide-band performance of ground-penetrating radar (GPR) antennas is a challenging task due to the complex electromagnetic coupling effects between the GPR antennas and the platform. Typically, the efficiency of conventional methods utilized to deal with the problem is quite low. However, there are various frequency sweeping techniques which can achieve fast analysis of antennas over a broadband. Accordingly, this paper describes an efficient hybrid scheme, based upon the electric field integral equation (EFIE), for modeling a GPR antenna which is mounted on an electrically large platform.

In this paper, an antenna with two half-elliptical-shape arms, which is an improvement of bowtie antennas, is presented with resistances loaded in the terminal. A shallow rectangular conducting backed cavity is attached to the antenna. On the other hand, the best uniform rational approximation technique is applied to analyze the wide-band property of the antenna since it can avoid repeatedly solving the integral equation at each single frequency point. The main scheme of the frequency sweeping method is as follows: 1) determining the Chebyshev nodes within a given frequency range; 2) computing the equivalent surface currents of antennas at the frequency points corresponding to those Chebyshev nodes; 3) calculating the wide-band response of surface currents according to the Chebyshev series. Finally, the Maehly approximation is utilized to improve the accuracy by matching the Chebyshev series to a rational function.

In the hybrid scheme, the adaptive integral method (AIM) is applied to accelerate matrix-vector products as the IE algorithm and the impedance matrix is stored in a sparse form to facilitate analysis of large antenna-platform system. It is easy to combine the best uniform rational approximation technique with AIM but also other IE methods which are suited for modeling radiation of antennas. This hybrid method greatly extends the range of conventional numerical modeling for GPR antenna system. Furthermore, the efficiency and capability of the presented algorithm can be validated by the designed GPR antenna.

1. INTRODUCTION

As an effective instrument, ground penetrating radar (GPR) has been widely used in the geophysical research field based on a nondestructive testing (NDT) technique. It is well known that the antenna system is one of the most critical parts in the GPR equipment. To design a GPR antenna, there are several aspects which should be taken into account, such as structure, bandwidth, polarization, gain. In this paper, a half-ellipse antenna (HEA) is employed as a result of its simple structure and linear-phase characteristic in the operating frequency band. Furthermore, the radiation properties can be improved by attaching a conducting rectangular reflector above the HEA and adding loaded resistors between the antenna and the reflector.

In general, the GPR antenna is usually mounted on a platform for the convenience of motion. One of the most popular methods for antenna analysis is the frequency domain integral equation solved using method of moments (MoM) [1]. However, if MoM is utilized to analyze the antenna-platform system, the number N of unknowns resulting from MoM will be so large that excessive computer memory and long solution time are required. To facilitate the analysis of large problems, some fast numerical techniques have been proposed to reduce the memory storage and the computation complexity, such as fast multipole method (FMM) [2], multilevel fast multipole algorithm (MLFMA) [3], precorrected fast Fourier transform (p-FFT) [4], sparse-matrix/canonical-grid (SM/CG) [5] and adaptive integral method (AIM) [6]. All of these approaches reach the computational complexity and the memory requirements to $O(N^{1.5}\log(N))$ and $O(N^{1.5})$, which is less than $O(N^2)$ for the conventional MoM.

Since the GPR antenna works over a broadband, the frequency response of the antenna is concerned mostly. With the fast algorithm, it is still time-consuming due to the repeated solution of matrix equation at each frequency point. Over the past few years, there were some efforts to achieve fast frequency sweep, such as impedance matrix interpolation [7], asymptotic waveform evaluation (AWE) [8], model-based parameter estimation (MBPE) [9] and the best uniform approximation technique [10]. Among these methods, the best uniform rational approximation technique is excellent for simplicity and accuracy. To begin with, it implements the Chebyshev approximation at certain selected frequencies within the considered wideband, that is, the Chebyshev nodes. Then, the Maehly rational approximation is adopted to improve the numerical results. Compared with other approximate solutions, the best uniform rational approximation can be incorporated into the existing computer codes.

In this paper, AIM is combined with the best uniform rational approximation technique to analyze the antenna-platform system over a wide frequency band. To achieve the analysis fast, AIM is employed to store all the full matrices in a sparse form and accelerate matrix-vector products. Furthermore, the best uniform rational approximation is applied to implement fast frequency sweeping by avoiding repeatedly solving integral equation at each frequency point. Finally, numerical results show that the HEA has a good radiation property and the proposed hybrid method is valid and efficient.

2. METHOD DESCRIPTION

2.1. Adaptive Integral Method (AIM)

To analyze the antenna above the platform with integral equation approach, AIM is employed to reduce the matrix storage and to accelerate the matrix-vector multiplications.

The procedures of AIM can be summarized as follows:

1) enclose the antenna and platform in a cube and partition the cube uniformly into Cartesian cells;
2) establish auxiliary basis functions and calculate the translation coefficient;

   The RWG basis function [11] $\mathbf{f}_n(\mathbf{r})$ and its convergence $\nabla\cdot\mathbf{f}_n(\mathbf{r})$ can be approximated as linear combinations of Dirac delta functions:

$$\begin{aligned} &\psi_{\alpha,n}(\mathbf{r}) \cong \sum_{u=1}^{(p+1)^3} \mathbf{\Lambda}_{\alpha,nu}\delta(\mathbf{r}-\mathbf{r}_{nu}) \;\; \alpha = x, y, z, d \\ &\psi_{\alpha,n}(\mathbf{r}) \in \{\mathbf{f}_{x,n}(\mathbf{r}), \mathbf{f}_{y,n}(\mathbf{r}), \mathbf{f}_{z,n}(\mathbf{r}), \nabla\cdot\mathbf{f}_n(\mathbf{r})\} \end{aligned} \tag{1}$$

   where $\mathbf{\Lambda}_{\alpha nu}$ are the translation coefficients for the x, y, z components and the divergence of basis function, $\mathbf{r}_{nu} = (x_{nu},\, y_{nu},\, z_{nu})$ is the coordinate of the grid, and p is the order of the translation. Therefore, the translation coefficient can be obtained via matching the multipole moments to basis functions.
3) By using the near-zone threshold d_{near}, the impedance matrix can be divided into near-zone group and far-zone group, which can be expressed as follows,

$$\mathbf{Z} = \mathbf{Z}_{near} + \mathbf{Z}_{far} \tag{2}$$

   The element of near-zone interaction matrix $\mathbf{Z}_{near}$ can be calculated by the conventional MoM, and the element of far-zone interaction matrix $\mathbf{Z}_{far}$ can be calculated as follows:

$$\mathbf{Z}_{far} = \mathbf{\Lambda}\mathbf{G}\mathbf{\Lambda}^T - \tilde{\mathbf{Z}} \tag{3}$$

   where $\mathbf{\Lambda}$ denote the translation coefficients; $\mathbf{G}$ is the Green's function matrix; and $\tilde{\mathbf{Z}}$ the inaccurate contributions from the near-zone grid sources which should be removed.
4) Since $\mathbf{G}$ is a Toeplitz matrix, this fact can make use of FFT to accelerate the matrix-vector multiplication as follows,

$$\begin{aligned} \mathbf{ZI} &= (\mathbf{Z}_{near} + \mathbf{Z}_{far})\mathbf{I} \\ &= \left(\mathbf{Z}_{near} - \tilde{\mathbf{Z}}\right)\mathbf{I} + \mathbf{\Lambda} IFFT\left\{FFT(\mathbf{G})\, FFT\left(\mathbf{\Lambda}^T\mathbf{I}\right)\right\} \end{aligned} \tag{4}$$

   Due to that $\mathbf{Z}_{near}$, $\mathbf{\Lambda}$ and $\mathbf{G}$ are all sparse matrice, the memory requirements can be reduced drastically.

2.2. The Best Uniform Rational Approximation Technique

When the frequency response of the antenna is of interest, the matrix equation can be written as follows,

$$\mathbf{Z}(k)\,\mathbf{I}(k) = \mathbf{V}(k) \tag{5}$$

where $\mathbf{Z}(k)$, $\mathbf{I}(k)$ and $\mathbf{V}(k)$ are the impedance matrix, unknown coefficients, and excitation vector, respectively. And all of them are frequency dependent. It can be seen that Equation (5) must be solved repeatedly for different frequency points within the interest band, which must be very time-consuming.

To alleviate this problem, the best uniform rational approximation technique is introduced as follows:

1) For a given frequency band $f \in [f_a, f_b]$ and the wave-number $k \in [k_a, k_b]$, the coordinate transform is expressed as:

$$\tilde{k} = \frac{2k - (k_a + k_b)}{k_b - k_a} \quad \tilde{k} \in [-1, 1] \tag{6}$$

2) The Chebyshev nodes within the band $[k_a, k_b]$ can be obtained as follows,

$$k_i = \frac{\tilde{k}_i\,(k_b - k_a) + (k_a + k_b)}{2} \tag{7}$$

$$\text{where} \quad \tilde{k}_i = \cos\left(\frac{i - 0.5}{n}\pi\right), \quad i = 1, 2, \ldots, n \tag{8}$$

3) Based on the Chebyshev approximation theory, the unknowns coefficients vector $\mathbf{I}(k)$ can be expressed as,

$$\mathbf{I}(k) = \mathbf{I}\left(\frac{\tilde{k}\,(k_b - k_a) + (k_a + k_b)}{2}\right) \approx \sum_{l=1}^{n} c_l T_l\left(\tilde{k}\right) - \frac{c_1}{2} \tag{9}$$

where c_l is the coefficient and given by

$$c_l = \frac{2}{n}\sum_{i=1}^{n} \mathbf{I}(k_i)\, T_l\left(\tilde{k}_i\right) \tag{10}$$

Furthermore, $T_l(x)(l = 1, 2, \ldots, n)$ is lth order of the Chebyshev polynomial, and the recursion formula about $T_l(x)$ is concluded below,

$$\begin{cases} T_0(x) = 1 \\ T_1(x) = x \\ \ldots \\ T_{l+1}(x) = 2xT_l(x) - T_{l-1}(x) \end{cases} \tag{11}$$

4) The Maehly approximation is applied to improve the accuracy of the numerical solution by matching the Chebyshev series to a rational function as follows,

$$\mathbf{I}(k) \approx R_{LM}\left(\tilde{k}\right) = \frac{P_L\left(\tilde{k}\right)}{Q_M\left(\tilde{k}\right)} = \frac{a_0T_0\left(\tilde{k}\right) + a_1T_1\left(\tilde{k}\right) + \ldots a_LT_L\left(\tilde{k}\right)}{b_0T_0\left(\tilde{k}\right) + b_1T_1\left(\tilde{k}\right) + \ldots b_MT_M\left(\tilde{k}\right)} \tag{12}$$

In generally, b_0 is set to be 1. Substitute (12) into (9) and use the identity

$$T_p(x)T_q(x) = \frac{1}{2}(T_{p+q} + T_{|p-q|}(x)) \tag{13}$$

the unknown coefficients a_i (i=0,1,..., L) and b_j (j=1,2,..., M) can be obtained as:

$$\begin{cases} a_0 = \frac{1}{2}b_0c_0 + \frac{1}{2}\sum_{j=1}^{M} b_jc_j \\ a_i = c_i + \frac{1}{4}b_ic_0 + \frac{1}{2}\sum_{j=1}^{M} b_j(c_{j+i} + c_{|j-i|}) \\ i = 1, 2, \ldots, L \end{cases} \tag{14a}$$

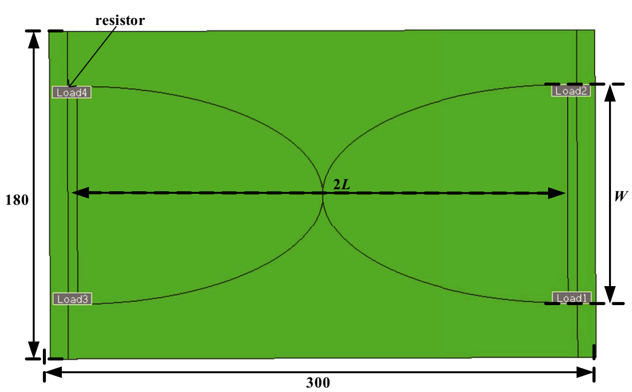
$$\begin{bmatrix} c_{L+2}+c_L & c_{L+3}+c_{L-1} & \cdots & c_{L+M+1}+c_{L-M+1} \\ c_{L+3}+c_{L+1} & c_{L+4}+c_L & \cdots & c_{L+M+2}+c_{L-M+2} \\ \vdots & \vdots & \vdots & \vdots \\ c_{L+M+1}+c_{L+M-1} & c_{L+M+2}+c_{L+M-2} & \cdots & c_{L+2M}+c_L \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_M \end{bmatrix} = -2 \begin{bmatrix} c_{L+1} \\ c_{L+2} \\ \vdots \\ c_{L+M} \end{bmatrix} \tag{14b}$$

Once the coefficients of the rational function are calculated, the induced current distribution can be obtained at any frequency within the whole frequency band.

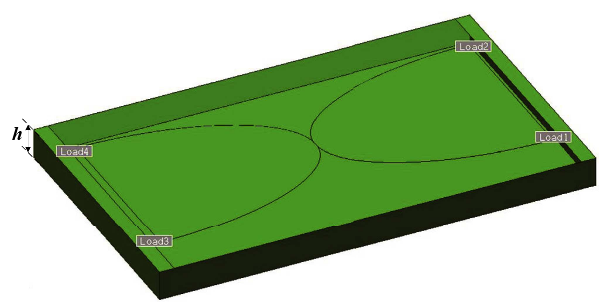
3. HALF-ELLIPSE ANTENNA AND NUMERICAL ANALYSIS

3.1. Half-Ellipse Antenna

The geometry of the half-ellipse antenna is shown in Fig. 1. The half-ellipse antenna was mounted at the open side of the cavity. The length and the width of the cavity are 300 mm and 180 mm, respectively. Furthermore, two 100-Ω resistors were used to connect each end of the antenna to the cavity. The dimensions of the antenna are as follows: $2L = 270$ mm, $W = 120$ mm, and $h = 20$ mm, where $2L$ is the length of the antenna, W is the antenna width, and h is the height of the cavity. As seen in Fig. 2, the GPR antenna was located above the top of the conducting platform, whose size is 800 mm × 600 mm × 260 mm.



(a)



(b)

Figure 1: Geometry of the half-ellipse antenna. (a) 2-D view. (b) 3-D view.

3.2. Numerical Analysis

In this section, numerical results will ben presented to show the accuracy and efficiency of AIM in conjunction with the best uniform rational approximation ($L = M = 4$) for solving the wideband problem of the half-ellipse antenna on the platform. All the computation is performed on a PC with Intel Core i5-2400 CPU 3.10 GHz and 2.0G RAM. Furthermore, all the variables used in the program code are double precision.

In the simulation, the frequency response of the half-ellipse antenna over 0.1 to 1 GHz is analyzed and the interval is 9 MHz. The whole structure is modeled by triangular meshes. And there are 1546 basis functions on the surface of antenna while 8342 basis functions are on the surface of platform. Firstly, the conventional MoM is employed to calculate the normalized radiation pattern of the half-ellipse antenna, as shown in Fig. 3. It can be seen that the antenna has a smooth main lobe and good directivity. Furthermore, the higher the frequency, the smaller the backward radiation. Fig. 4 shows the input impedance of the antenna mounted on the platform over the frequency band. The results computed by the direct AIM agree very well with that obtained by AIM combined with the Maehly approximation technique, which shows the proposed algorithm is valid. On the other hand, it is indicated that the real part of HEA's impedance is around 100-Ω from 100 MHz to 750 MHz. Table 1 shows some computational characteristics of the two methods. The presented algorithm can reduce the solution time drastically without significant memory increase. Therefore, it is an efficient approach to solve the wideband problem of antennas on a large conducting platform.

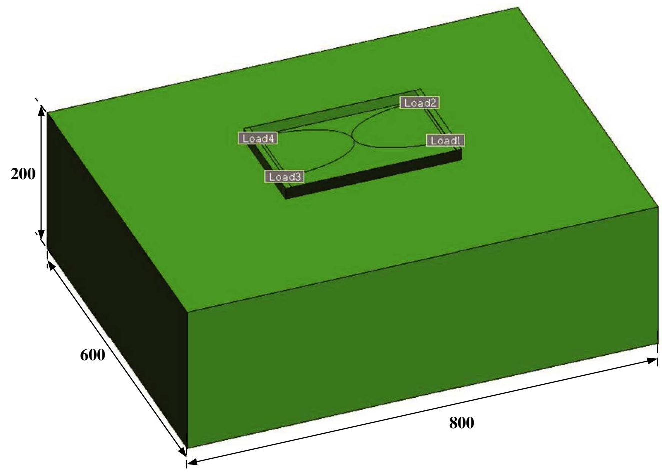


Figure 2: HEA on the top of the platform.

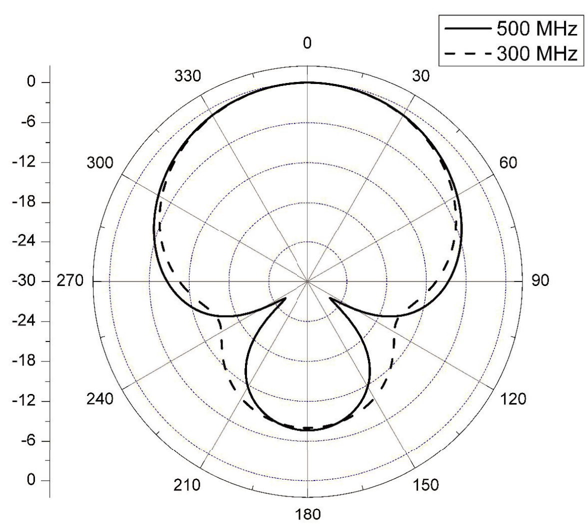


Figure 3: Normalized radiation pattern of the HEA.

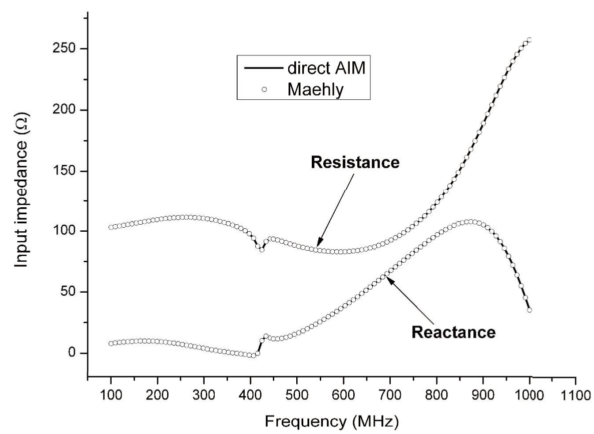


Figure 4: Frequency response of input impedance.

Table 1: Computational characteristics of the two methods for Fig. 4.

Method	Memory (MB)	Total CPU time (h)
Direct AIM	142	35.2
Maehly	147	4.5

4. CONCLUSION

AIM in conjunction with the best uniform rational approximation technique has been presented to achieve fast frequency sweep analysis of a half-ellipse antenna mounted on a conducting platform. Numerical results show that the HEA has a smooth main lobe, good directivity, and small backward radiation. The real part of HEA's impedance is around 100-Ω from 100 MHz to 750 MHz. On the other hand, the proposed algorithm in this paper can offer saving in terms of CPU time without significant memory increase than the conventional one.

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