Time Reversal Imaging Using Minimum Norm Iterative Type Partial Noise Subspace Method

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Abstract— Orthogonality between signal subspace and noise subspace is employed to time reversal (TR) imaging. Part of the noise subspaces are used to get a projection with the signal subspace to estimating the location of targets. The proposed method is similar to the minimum norm method in form. Several numerical simulations show that the proposed method can get the exact locations of the targets with less computing resources than the conventional multiple signal classification (MUSIC) method.

1. INTRODUCTION

Orthogonality between the signal subspace and noise subspace is the foundation of the time reversal operator (TRO) based imaging techniques. The decomposition of time reversal operator (DORT) method [1] and time reversal multiple signal classification (TR-MUSIC) method [2] are the well-known two types of TRO-based imaging algorithm which are widely used in optics and electromagnetics. The TRO-based methods have better performance than the traditional time reversal wave back propagation method, which benefits from the spatial and temporal focusing properties of the time reversal waves. Time reversal imaging techniques have many applications, such as detection for buried objects, ultrasonic nondestructive testing, medical imaging [3–6], etc.. In the direction-of-arrival (DOA) estimation, the whole noise subspaces are used in the well-known MUSIC method [7], while in the minimum norm method, only one vector is used to save the computing resource [8, 9]. By analyzing the mean square error of MUSIC method and minimum norm method, the results show that the relative performance of MUSIC method and the minimum norm method depends on the ratio of their parameter sensitivities [10].

In this paper, a minimum norm iterative type partial noise subspace imaging method is proposed to estimate the locations of targets in free space. Compared with the TR-MUSIC method by using all noise subspace vectors, the imaging precision of the minimum norm method has declined in a certain degree. To solve this problem, an iterative process is introduced. The proposed method avoids the difficulty of distinguishing the signal subspace with noise subspace while providing stable imaging of the targets. In multiple scattering case, the singular value decomposition of the MSR matrix is employed to generate the signal subspace vectors (signal vectors) and noise subspace vectors (noise vectors). Actually, the tow subspaces do not need to be divided. Finally, similar to the minimum norm type method, the single noise vector in noise-well case and the partial noise subspace (bottom noise vectors) in high-level noise case are used to compute the imaging pseudo-spectrum, respectively.

2. PARTIAL NOISE SUBSPACE METHOD

Assume an analytical imaging model consist of N acoustic or electromagnetic wave transceivers (located at α_n, $n = 1, 2, \ldots, N$) and M point-like scatters (located at $\mathbf{x}_m$, $m = 1, 2, \ldots, M$) in free space. While $\mathbf{r}$ represents an arbitrary point in space, the total field Ψ_k, composed of the incident field Ψ_k^{in} and scatter field Ψ_k^{sca}, generated by the kth ($k \leq N$) transmitters at $\mathbf{r}$ and $\mathbf{x}_m$ can be expressed as follows:

$$\psi_k(\mathbf{r}, \omega) = \psi_k(\mathbf{r}, \omega)^{in} + \psi_k(\mathbf{r}, \omega)^{sca} \tag{1}$$

$$\psi_k(\mathbf{x}_m, \omega) = \psi_k(\mathbf{x}_m, \omega)^{in} + \psi_k(\mathbf{x}_m, \omega)^{sca} \tag{2}$$

Under the assumption of point scatters and receivers, the MSR matrix K under multiple scattering case can be given as:

$$K = G_0(\boldsymbol{\alpha}_j, \mathbf{x}_m)\tau_m H^{-1} G_0(\mathbf{x}_m, \boldsymbol{\alpha}_k) \tag{3}$$

where

$$H_{m,m'} = \delta_{m,m'} - (1 - \delta_{m,m'}) G_0(\mathbf{x}_m, \mathbf{x}'_m) \tag{4}$$

$G_0(\mathbf{r},\mathbf{r}',\omega)$ is the free space Green function and δ is the Kronecker delta function. Considering the singular value decomposition (SVD) of K, the following equations can be gotten:

$$\mathbf{K}\cdot\mathbf{v}_j = \sigma_j\mathbf{u}_j \tag{5a}$$

$$\mathbf{K}^H\cdot\mathbf{u}_j = \sigma_j\mathbf{v}_j, \tag{5b}$$

where σ_j is the jth singular value, $\mathbf{u}_j$ and $\mathbf{v}_j$ denote the jth column singular vectors of the unitary matrices $\mathbf{U}$ and $\mathbf{V}$, respectively. The detailed process of above is shown in [11]. In the processes of traditional TR-MUSIC method, the matrices $\mathbf{U}$ and $\mathbf{V}$ will be divided into signal subspace and noise subspace, which are spanned by the singular vectors corresponding to the nonzero and zero singular values. In the proposed method, the difficult dividing operation is omitted benefitting from the orthogonality between signal and noise subspaces. Obviously, the matrices $\mathbf{U}$ and $\mathbf{V}$ span the same space in echo-mode. For simplicity, echo-mode with matrix $\mathbf{V}$ is illustrated in the rest of this paper.

For homogeneous and isotropic media, the signal subspace also spanned by the conjugation of Green function $\mathrm{g}_0^*(\mathbf{x})$ vector, where

$$g_0(\mathbf{x}) = [G_0(\mathbf{x},\alpha_{\mathbf{1}},\omega), G_0(\mathbf{x},\alpha_2,\omega),\ldots,G_0(\mathbf{x},\alpha_N,\omega)]^T \tag{6}$$

is the background Green's function vector. Because of the existed orthogonality between signal subspace and noise subspace, the inner product of noise and signal vectors is zero in theory. The locations of the scatters can be determined by the pseudo-spectrum computed by any singular vector. In other words, the signal and noise subspaces do not need to be distinguished actually. The imaging pseudo-spectrum can be written as:

$$P_1(\mathbf{r}) = \frac{1}{|< v_{N-p+1}, g_0^*(\mathbf{r}) >|^2}, \tag{7}$$

where p denotes the last pth singular vector. For more stable imaging in high-noise case, more bottom singular vectors can be used by an iterative strategy written as:

$$P_2(\mathbf{r}) = \frac{1}{\sum\limits_{i=N-q+1}^{N} |< v_i, g_0^*(\mathbf{r}) >|^2}, \tag{8}$$

where q denotes the bottom qth singular vectors. In the subsequent simulations, the proposed method is simulated in different level noise cases.

3. NUMERICAL VALIDATION AND EVALUATIONS

In this section, simulations of two targets (M=2) embedded in the free space in different SNRs are presented to show the performance of proposed method. The configuration of simulation model is set as Fig. 1(a). The targets are placed at (1.5λ, 4.5λ, -6λ) and (4.5λ, 1.5λ, -9λ) in the "$6\lambda\times6\lambda\times6\lambda$" 3D cubic imaging domain. The operation frequency is set as 300 MHz and the imaging resolution is set as $\lambda/4$. All the imaging pseudo-spectrums are normalized as dB. The transmitters and receivers use the same array with the number $N = 32$ in echo-mode. The transceivers are distributed as a planar array located at the plane $z = 0$, the space between two adjacent transceivers is set as 1λ. The simulations are carried out on a PC with quad-core 3.5 GHz CPU and 4GB RAM, the programs are compiled by MATLAB 2012b.

When $SNR = 60$ dB, Fig. 1(b) shows the normalized singular values of the MSR matrix. Fig. 1(c) shows the imaging result in Eq. (13) with $p = 1$, which means that the last vector can get a good imaging of the targets. The proposed method employs only one noise vector while the conventional TR-MUSIC method employs N noise vectors (whole noise subspace). Thus, our method needs much less computing time and memory than TR-MUSIC method.

Figures 2(a)–(c) present the imaging results of Eq. (13) with $SNR = 35$ dB for $p = 1, 2, 3$, respectively. All the three vectors can get the accurate locations of the targets independently. Figs. 2(d)–(f) is the results with $SNR = 25$ dB while the vectors is the same as (a)–(c). In the high-level noise cases, a single noise vector is not enough to get a clear imaging. While the noise is getting high, the imaging performance is getting worse. Then Eq. (14) is employed to get a

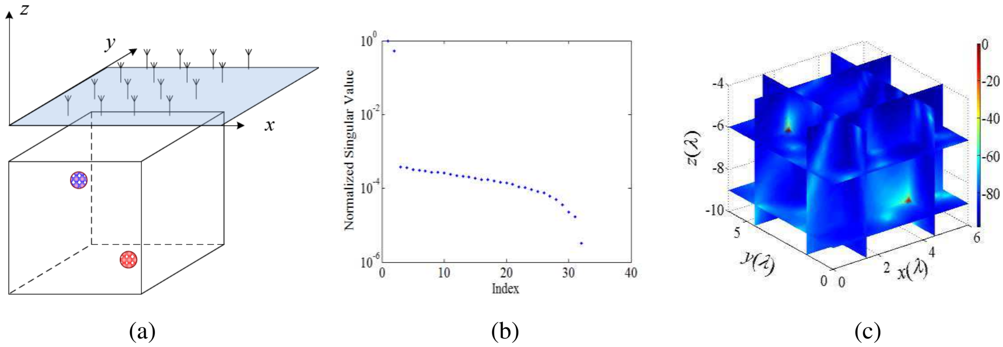

Figure 1: Three-Dimension imaging results with $SNR = 60\,\mathrm{dB}$. (a) Configuration of 3D imaging. (b) Normalized singular values of the MSR matrix K. (c) The imaging pseudo-spectrum of the proposed method with $p = 1$.

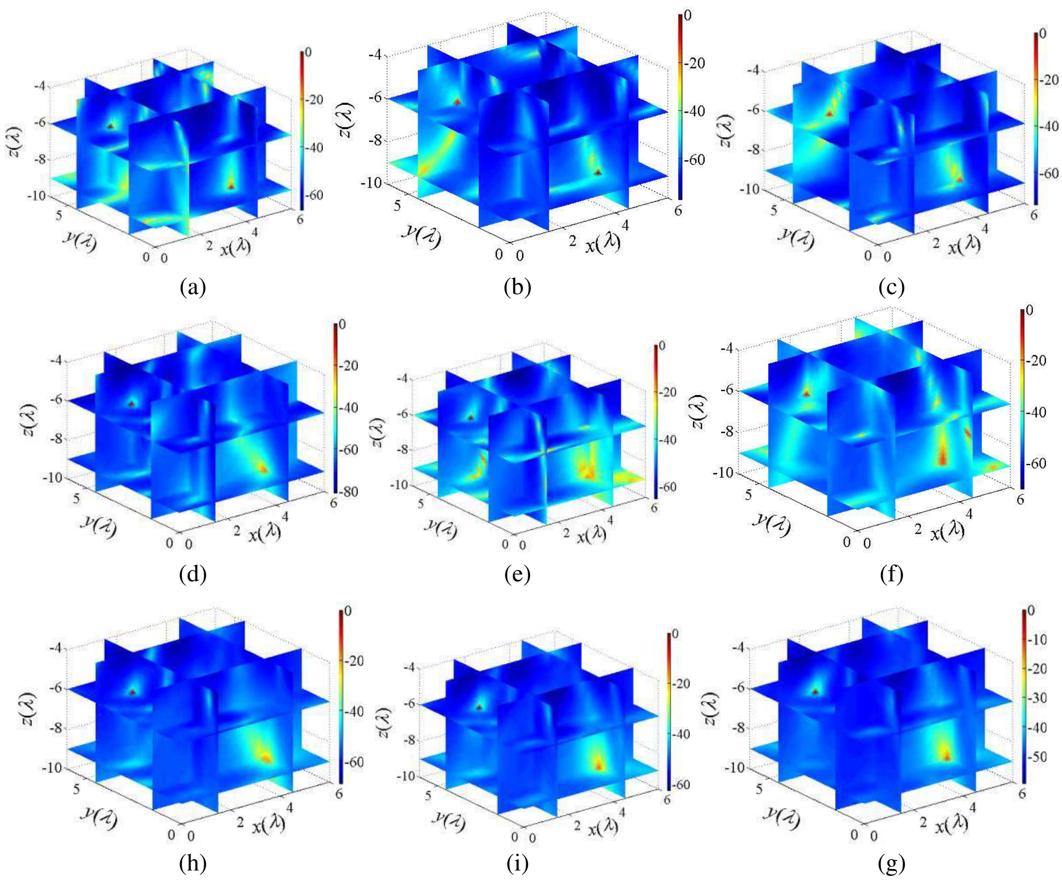

Figure 2: Images obtained using single vector method and partial noise subspace method. (a) The last noise vector ($p = 1$) with $SNR = 35\,\mathrm{dB}$. (b) The penultimate noise vector ($p = 2$) with $SNR = 35\,\mathrm{dB}$. (c) The 3rd last noise vector ($p = 3$) with $SNR = 35\,\mathrm{dB}$. (d) The last noise vector ($p = 1$) with $SNR = 25\,\mathrm{dB}$. (e) The penultimate noise vector ($p = 2$) with $SNR = 25\,\mathrm{dB}$. (f) The 3rd last noise vector ($p = 3$) with $SNR = 25\,\mathrm{dB}$. (g) The bottom two noise vector ($q = 2$) with $SNR = 25\,\mathrm{dB}$. (h) The bottom three noise vector ($q = 3$) with $SNR = 25\,\mathrm{dB}$. (i) The bottom four noise vector ($q = 4$) with $SNR = 25\,\mathrm{dB}$.

better imaging in such cases. Figs. 2 (g)–(i) show the results with $SNR = 25\,\mathrm{dB}$ while $q = 2$, 3, 4, respectively. It is shown that while q is getting larger, the imaging performance is becoming better. Thus, in the noise-well case, Eq. (13) can be used as the imaging pseudo-spectrum to save computing resources. Whereas, in the high-level noise case, Eq. (14) can be an alternative method to get better imaging while costing relatively more computing resources than Eq. (13). Obviously, all these two methods employ less singular vectors than the TR-MUSIC method, so the proposed method is suited for the fast target imaging and detection.

4. CONCLUSION

In multiple scattering case, the minimum norm iterative type partial noise subspace method is developed for target imaging. The proposed method provides stable imaging and reduces the computing resources than the TR-MUSIC method, while avoiding the cutoff between signal and noise subspaces without the loss of imaging precision. The partial noise subspace method can replace the TR-MUSIC method in many cases without any loss. It is a good method for fast detection and imaging of targets.

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