The Study on Equivalent Models of Finite-size Carbon Fiber Composite Materials

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Abstract— The purpose of this paper is to investigate the use of equivalent-layer models for the analysis of finite-size carbon-fiber composite materials. A macroscopically equivalent model based on the homogenization technique, which is a more efficient method for analyzing fiber composite is investigated. Finite size composite slabs, consist of loose carbon fibers mixed with resins, are used to validate the equivalent models. Different thicknesses and sizes of the slabs, as well as relative volume of carbon fibers are considered. It is found that the breakdown frequencies of equivalent models vary significantly with the various geometries. It is important to validate the equivalent models of each finite-size composite block before simplifying them in the whole large structure model.

1. INTRODUCTION

Carbon fiber reinforced composites (CFRC) are being used more widely in the aerospace and automotive industries as replacement for metals due to the good strength and weight properties [1]. They are realized by a matrix of resin in which high strength carbon fibers are embedded and aligned along a preferred direction of reinforcement. Despite the many mechanical advantages, composite materials are not as electrically conductive as metals such as aluminum and titanium. It has an electrical conductivity that is nearly 1000 times lower than that of most metals [2]. Therefore, Carbon composite materials usually have poor performance of shielding effectiveness which may leads to electromagnetic interference (EMI) problems under EM hazards [3].

Full numerical approaches are often used to analyze the influence of composite electrical properties on the electromagnetic performance. However, prohibitive computational time and computer memory may be required when thin panels and fibers of composite materials are spatially resolved. Thus, approximate formulas for the effective material properties are very useful. Previous work shows that equivalent models with effective electromagnetic parameters are usually proposed for the analysis of periodic carbon fiber composite materials when the period of the structure is small compared to the wavelength [4]. Basically, finite-size CFRCs are often constructed as parts of the large structure system such as aircraft in practical cases. The authors' interest stems from an analysis of aircraft structure model with many finite-size CFRC slabs. The applicable frequency range of equivalent models for finite-size CFRC may vary with different geometries.

In this paper, we firstly review close-form expressions available in the literature for approximating the effective permittivity and permeability of composite materials. A macroscopically equivalent model based on the homogenization technique, which is a more efficient method for analyzing fiber composite is investigated. Several carbon fiber composite slabs are used to validate the equivalent models. Different thicknesses and sizes of the slab, as well as relative volumes of carbon fiber are considered. It is found that the breakdown frequencies of equivalent models vary significantly with the geometries of the composite materials. It is of prime importance to validate the equivalent models of each finite-size composite block before simplifying them in the whole large structure model.

2. REVIEW OF KNOWN APPROXIMATIONS FOR EFFECTIVE PARAMETERS

In the earlier past, there has been a great deal of attention toward determining the effective properties of composite regions based on effective medium theory. Famous formulas from the historical point of view include Maxwell-Garnett, Bruggeman, and QCA-CP equations in terms of the volume fractions and properties of the constituting phases [5]. However, these formulas are oftern used to determine the effective permittivity and permeability of homogeneous and isotropic multiphase materials, such as a dispersion of dielectric spheres in a homogeneous matrix and particle medium mixtures.

The present paper consider exclusively with two-phase media. Figure 1(a) shows the geometrical structure of periodic fiber composite slab and its coordinate system. The composite layer is realized by carbon fibers with circular cross-section, having conductivity σ_f and permittivity ε_f, embedded in a matrix resin of conductivity σ_m and permittivity ε_m. The cross-section of xy-plane is shown in Figure 1(b). P is the distance between the carbon fibers, D the fiber diameter and L the overall thickness of the layer. In this paper, we only consider the two-phase dielectric materials (i.e., the permeability is assumed to be μ_0) with the appropriate invariance in the z direction.

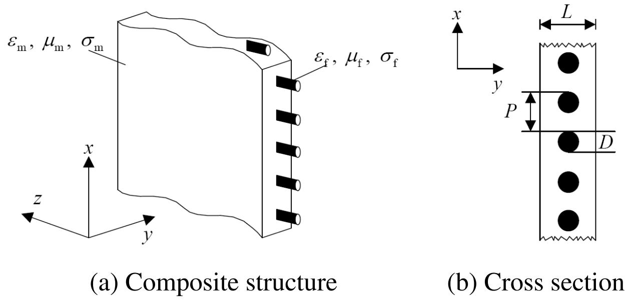


Figure 1: Carbon fiber composite structure and coordinate system.

As to the inhomogeneous and anisotropic composite materials, an efficient method known as homogenization has been used to solve problems of this periodic type in reasonable low frequency ranges. Homogenization technique allows the separation of the average field from the microstructure. If the period of a periodic structure is small compared to a wavelength, the EM waves see the composite as an effective anisotropic homogeneous region with tensor permittivity $[\varepsilon^h]$ given by [4]

$$\left[\varepsilon^h\right] = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}. \tag{1}$$

Among many approximation techniques, the most widely known were the Hashin-Shtrikman (HS) upper and lower bounds, which were derived in terms of the material parameters and the volume fraction [6]. They limit ε_x and ε_y to a region between two bounds defined as

$$\varepsilon_{HS}^L = \varepsilon_m \frac{(1+g)\varepsilon_f + (1-g)\varepsilon_m}{(1-g)\varepsilon_f + (1+g)\varepsilon_m}, \tag{2}$$

$$\varepsilon_{HS}^U = \varepsilon_f \frac{(2-g)\varepsilon_m + g\varepsilon_f}{g\varepsilon_m + (2-g)\varepsilon_f}, \tag{3}$$

where g is the relative volume of space occupied by the material with carbon fibers,

$$g = \pi D^2/(4PL) \tag{4}$$

Another approximation technique is the Lichtenecker (Li) bounds depend on the specific geometry of the unit cell and can be written as

$$\varepsilon_{Li}^L = \int_0^P \frac{dy}{\int_0^P \frac{dx}{\varepsilon(x,y)}}, \tag{5}$$

$$\varepsilon_{Li}^U = \frac{1}{\int_0^P \frac{dx}{\int_0^P \varepsilon(x,y)dy}}. \tag{6}$$

Previous work has shown that, in general, the HS lower bound is a good approximation for the effective properties when ε_f is larger than ε_m [4]. Similarly, HS upper bound is better when resin is a denser medium. For more complicated composites, Li bounds are more accurate. However, these bounds are recommended to be used for composite material with real permittivity. Besides these bounds, [7] has proposed formulas for the components in (1).

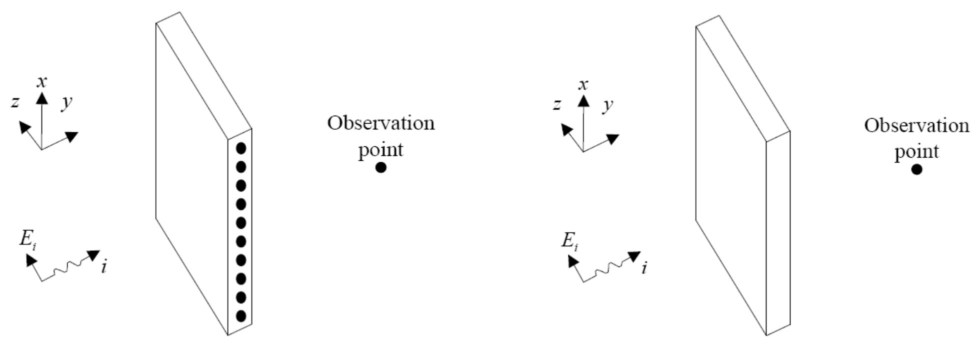
$$\varepsilon_x^{-1} = (1-g)\varepsilon_m^{-1} + g\varepsilon_f^{-1}, \tag{7}$$

$$\varepsilon_y = \varepsilon_z = (1-g)\varepsilon_m + g\varepsilon_f. \tag{8}$$

This approximation approach is a better estimate for single-panel carbon fiber composite materials. In general, this homogenous model can be accurate for periods as large as half wavelength [8]. Once either the frequency of operation or the material of the structures becomes too highly conductive the homogenization technique begins to break down. Meanwhile, different from infinite periodic structures, the size of composite materials would influence the applicable frequency range of the equivalent model. Actually, finite-size composite models are often built for large structures on analysis of the interaction of EM field in practical cases.

3. DISCUSSION OF EQUIVALENT MODEL FOR FINITE-SIZE COMPOSITE SLABS

In this section, an equivalent model based on approximated formulas (7) and (8) is investigated. Four configurations with different geometrical parameters of carbon fiber composite slabs are used for validating the equivalent model. The EM parameters are: $\varepsilon_m = 5\varepsilon_0$, $\varepsilon_f = 2\varepsilon_0$, $\sigma_f = 70000\,\mathrm{S/m}$, $\sigma_m = 0\,\mathrm{S/m}$. The geometrical parameters of the slabs are given in Table 1. They are illuminated by an incident plane wave ($+y$ direction) with electric field parallel to z direction, as shown in Figure 2(a). The magnitude of E-field of plane wave is 1 V/m. An observation point (0 cm, 2 cm, 0 cm) is set to get the transmitted electric field strength on the other side of the slabs. By using the formulas (7) and (8), effective permittivity $[\varepsilon^h]$ can be calculated and an equivalent homogenous anisotropic model is given in Figure 2(b).



(a) Actual slab (b) Equivalent slab

Figure 2: Setup of finite size slab models.

Table 1: Geometric structures of finite-size carbon fiber composites.

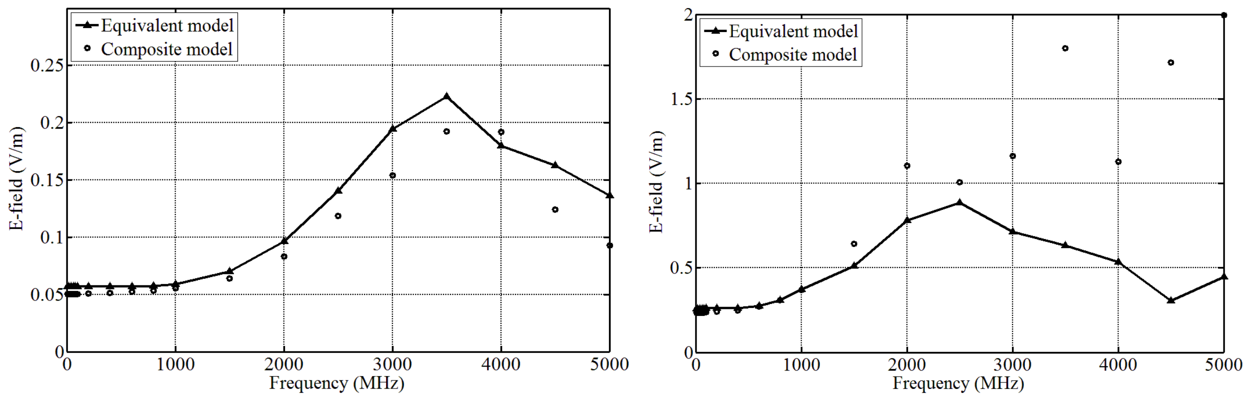
No.	Geometry
1	Dimension: 10 cm × 10 cm×2 cm, $P = 2$ cm, $D = 1$ cm, $L = 2$ cm
2	Dimension: 20 cm × 20 cm×2 cm, $P = 2$ cm, $D = 1$ cm, L =2 cm
3	Dimension: 20 cm × 20 cm×3 cm, $P = 2$ cm, $D = 1$ cm, $L = 3$ cm
4	Dimension: 20 cm × 20 cm×2 cm, $P = 1.25$ cm, $D = 1$ cm, $L = 2$ cm

The evaluation of breakdown frequency of equivalent model is carried out by computing the relative differences in the E-field strength at the observation point. The error is defined as the ratio of E-field strength different over the E-field strength of actual carbon fiber composite model. The equation for the error is indicated below:

$$Error = \frac{|E_{act} - E_{eff}|}{E_{act}} \times 100\%, \tag{9}$$

where E_{act} is the E-field strength at the observation point for actual composite model. E_{eff} is for equivalent anisotropic model. The equivalent model fails when the error is larger than 25%.

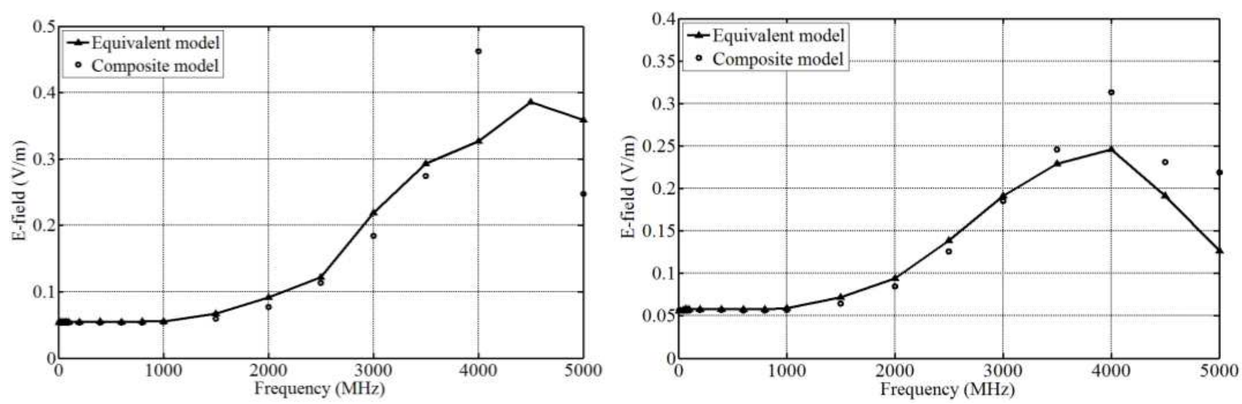
Numerical results are obtained with a full-wave tool (HFSS) for these four finite-size composite slabs. The results are shown Figure 3 and Figure 4. It can be seen that the E-field results of equivalent model agree well with that of actual composite model in the low frequency. When frequency of operation becomes too high, the equivalent model begins to break down. The breakdown frequency is 2.9 GHz in Geometry 1 (i.e., the error is larger than 25% after 2.9 GHz). Similarly,



(a) Geometry 1

(b) Geometry 2

Figure 3: E-field strength at observation point in Geometry 1 and Geometry 2.



(a) Geometry 3

(b) Geometry 4

Figure 4: E-field strength at observation point in Geometry 3 and Geometry 4.

the breakdown frequencies are 1.8 GHz in Geometry 2, 3.9 GHz in Geometry 3, 4.6 GHz in Geometry 4. Different configurations of finite-size composite slabs results in different frequency limits for equivalent models.

4. CONCLUSION

Equivalent model based on homogenization technique can be used efficiently to analyze the interaction of EM field with carbon fiber composite structures. A macroscopically equivalent model derived under the assumption of infinite periodic structures is investigated for analysis of finite-size structure. Several carbon fiber composite slabs are used to validate the equivalent models. Different thicknesses and sizes of the slab, as well as relative volume of carbon fiber are considered. The breakdown frequencies of equivalent models vary significantly with the geometries of the composite materials. It is of prime importance to validate the equivalent models of each finite-size composite block before simplifying them in the whole large structure model.

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