# Minimum Variance Variable Constrain DOA Algorithm

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**Abstract**— This work proposes a new direction-of-arrival (DOA) estimation algorithm for Smart antenna. The proposed algorithm is based on MVDR algorithm through the introduction of a new optimization problem, which aims to maintain the array gain in the 'look direction', so that the DOA can be estimated correctly using Smart antenna array. To verify the performance of the proposed algorithm, computer simulations are performed and then the results obtained are compared with the MVDR algorithm. The results show that the proposed algorithm outperforms the MVDR algorithm.

## 1. INTRODUCTION

The use of electrostatic MEMS switches is attractive because of its advantages, such as very low power consumption and high isolation. However, MEMS switches have their share of problems, such as, high driving voltage, relatively low speed and low power handling. Many previous researches focus on RF MEMS switch, which presents a good performance at microwave frequency. This result an increasing use of RF MEMS switches for telecommunication purpose in the last ten years [1]. This paper purposes is to use RF MEMS series switch for Switched beam array creates a group of overlapping beams that together result in 0° to 180° coverage The beam pattern is generated using combines of MEMS phase shifters that feed networks into a highly integrated multifunctional chip, which can then be connected to the antenna array and T/R modules, Adaptive arrays allow the antenna to steer the beam to any direction of interest while simultaneously nulling interfering signals [2]. Beamdirection can be estimated using direction-of-arrival (DOA) estimation methods.

## 2. MEMS SWITCHES

MEMS refer to a 21th century technology named micro-electromechanical systems. MEMS are processes technology used to create tiny integrated devices or systems that combine mechanical and electrical components. They are fabricated using integrated circuit (IC) batch processing techniques and can range in size from a few micrometers to millimeters. These devices (or systems) have the ability to sense, control and actuate on the micro scale, and generate effects on the macro scale. In the most general form, MEMS consist of mechanical microstructures, micro-sensors, micro-actuators and microelectronics, all integrated onto the same silicon chip. Nowadays, MEMS have a diversity of applications across multiple markets like automotive, electronics, medical, communications, and defense weapons. MEMS switches are surface-micro machined devices which use a mechanical movement to achieve a short circuit or an open circuit in the RF transmission-line. RF MEMS switches are the specific micromechanical switches which are designed to operate at RF to mm-wave frequencies form 0.1 to 100 GHz [3–7]. With the recent exciting advancements in the field of microelectromechanical systems (MEMS) and micromachining technology, miniaturized MEMS phase shifters have shown to offer a superior RF performance in comparison to their semiconductor counterparts. The main objective of this research is to develop a new technique for integration at the module level that combines the MEMS phase shifters and feed networks into a highly integrated multifunctional chip, which can then be connected to the antenna array and T/R modules, RF switches are used in a wide array of commercial, aerospace and defense application areas including satellite communications systems, wireless communications systems and radar systems. The circuits are modeled and optimized using Agilent's HP ADS circuit simulator [8]. Fig. 1 shows the equivalent circuit model of the switched Phase Shifter set up in ADS in which the "on" and "off" states of the switch obtained from decoder are used in the ADS circuit simulator to optimize and obtain the simulated response One of the key elements in the array design is the MEMS phase shifter. RF MEMS phase shifters possess many advantages such as low loss, improved power handling capability and can also be used to realize high density packages to reduce the size and weight components. 50–75% of the T/R modules can be eliminated in large phased arrays by using MEMS phase shifters. This research proposes a high resolution wide band tunable MEMS phase shifters for radar phased array antenna to steer the beam. The proposed circuit is based on dividing the tuning range which change the phase by 22.5° steps for course tuning to achieve the desired phase. A microstrip phase shifters have been designed at 5 GHz. The phase shifter is designed, simulated and tested using Advanced Design System (ADS) software. This type of phase shifters can give high performance and resolution. Switched line phase shifters schematic diagram are illustrated in Fig. 1. It consists of nine coarse delay transmission line elements from, 0 to 180 degrees with 22.5 degrees step. A digital control circuit plays an important function during the phase change mechanism. The control circuit is a decoder circuit converts the code word to control signals open and close the MEMS switches. If the required phase is 135 degree then the control circuit chooses to close the MEMS switches of the 90 degree element and the other switch of the 45 degree element, then the total phase shift satisfied is 135 degree.

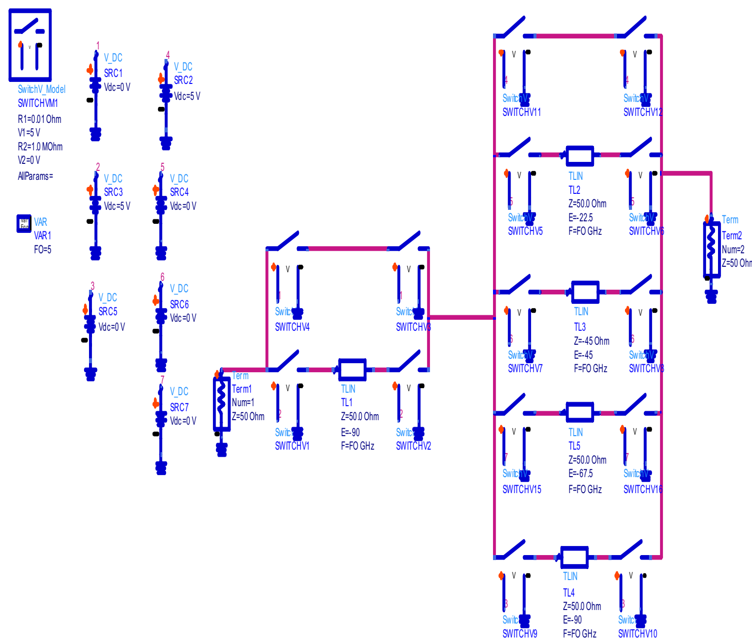


Figure 1: Switched line phase shifter schematic diagram.

## 3. SWITCHING-BEAM ARRAY

In the smart antenna systems, the Switching-Beam Array approach forms multiple fixed beams with enhanced sensitivity in specific area. These antenna systems will detect signal strength, and select one of the best, predetermined, fixed beams for the subscribers as they move throughout the coverage sector. Instead of modeling the directional antenna pattern with the metallic properties and physical design of a single element, a Switching-Beam Array system couple the outputs of multiple antennas in such a manner that it forms a finely directional beams with spatial selectivity [2].

### 3.1. Data Model

We considered an m-element array with an arbitrary geometry. The array receives narrow band signals from M spatially separated users. We ignore the multipath signals. Assume that the array response vector to a transmitted signal source from a DOA $\theta$. Calculation of the DOA of a signal is required to track the location of the signal. We have simulated study for Capon and MUSIC DOA estimation algorithms, since those are suitable for circular array or any arbitrary shape array. Define array signal [9–12]:

$$x(t) = \sum_{i=1}^{M} a(\theta)s_i(t) + n(t) = As(t) + n(t) \tag{1}$$

where

$$A = [a(\theta_1), a(\theta_2), \ldots, a(\theta_M)] \tag{2}$$

is signal spatial signature and:

$$s(t) = [s_1(t), s_2(t), \ldots, s_M(t)]^T \tag{3}$$

is signal vector.

The covariance matrix of array signal is:

$$R = E\left\{x(t)x(t)^H\right\} = AE\left\{s(t)s(t)^H\right\}A^H + E\left\{n(t)n(t)^H\right\} = APA^H + \sigma^2 I \tag{4}$$

where $P$ is received signal power matrix, and $\sigma^2 I$ is noise power matrix.

### 3.2. MVDR DOA

Minimum Variance Distortionless Response algorithm (MVDR). The key is to minimize the output power of the system except the one that pointing to the desired signal direction:

$$\min_w w^H R w \quad \text{subject to} \quad (w^H a(\theta) = 1)$$

to estimate the optimal weight vector. Solution of this minimization problem gives the optimal weight vector as [13, 14]:

$$w_{mvdr} = \frac{R^{-1}a(\theta)}{a^H(\theta)R^{-1}a(\theta)} \tag{5}$$

The MVDR minimizes the total output power while keeping the output signal constant, thus the MVDR output power spectrum is:

$$P = \frac{g^H(\theta)g(\theta)}{a^H(\theta)R^{-1}a(\theta)} \tag{6}$$

### 3.3. Minimum Variance Variable Constrain

The proposed algorithm is based on modified MVDR algorithm that is targeted to work with the directional antenna array. In general, the steering vector can be expressed as follows:

$$A(\theta) = [g(\theta_1)a(\theta_1) \;\; g(\theta_2)a(\theta_2) \;\; \dots \;\; g(\theta_N)a(\theta_N)]^T \tag{7}$$

where $g(\theta_i)$ is the array gain in a specific direction $\theta$. For antenna arrays with isotropic elements, $g(\theta_i)$ is equal to 1 for $i = 1, 2, \dots, N$. However, in the case of antenna arrays with directional elements, $g(\theta_i)$ is determined from the array gain pattern and thus significantly affect the values of $a(\theta_i)$. Furthermore, since the array response is affected by the gain pattern in a 'look direction', the constraint in Minimum Variance Variable Constrain MVVC can be given by:

$$\min W^H RW \quad \text{subject to} \quad w^H a(\theta) = g(\theta) \tag{8}$$

Applying Lagrange optimisation technique on the constraint in (8) yields the Lagrange multiplier, $\lambda$, and weight $w$ given by (9) and (10) respectively.

$$\lambda = \frac{g(\theta)}{a^H(\theta)R^{-1}a(\theta)} \tag{9}$$

$$w = \frac{R^{-1}a(\theta)g(\theta)}{a^H(\theta)R^{-1}a(\theta)} \tag{10}$$

Finally, the power spectrum of the MVVC algorithm is given as:

$$P = \frac{g^H(\theta)g(\theta)}{a^H(\theta)R^{-1}a(\theta)} \tag{11}$$

## 4. RESULTS

Computer simulations using Matlab are conducted to evaluate the performance of the MVVC algorithm. This section compares the performance of MVVC and MVDR algorithms. The simulation results presented in this section use the Smart antenna array shown in Fig. 2. Eight uniform linear array (ULA).

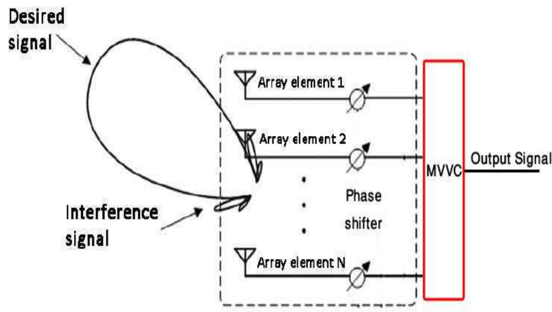


Figure 2: Switching-beam array, phase shifter and MVVC schematic diagram.

### 4.1. Simulation of DOA Estimation

Using the elements array shown in Fig. 2, both MVDR and MVVC algorithms are applied to estimate the DOA. Suppose there is a tin signal impinging the antenna array at 5°, 20°, 45°, 60°, 70°, 80°, 90°, 100°, 130° and 140°, signal-to-noise ratio (SNR) is fixed at 10 dB, and the number of snapshots is 10000. Fig. 3 shows a comparison of DOA estimation between the MVDR and MVVC algorithms. Clearly the MVDR algorithm yields too many spurious peaks in spatial power spectrum that could lead to inaccurate estimation. Although there is a peak appearing at 130°, but the MVVC will be estimated based on the highest peak strength, which appears at 45°. On the other hand, the MVVC algorithm manages to estimate the DOA correctly and suppress any spurious peaks. The simulation is repeated for multiple signals with the same SNR and snapshot value, Therefore, the observations in results show that the MVVC algorithm has better performance compared with the MVDR algorithm.

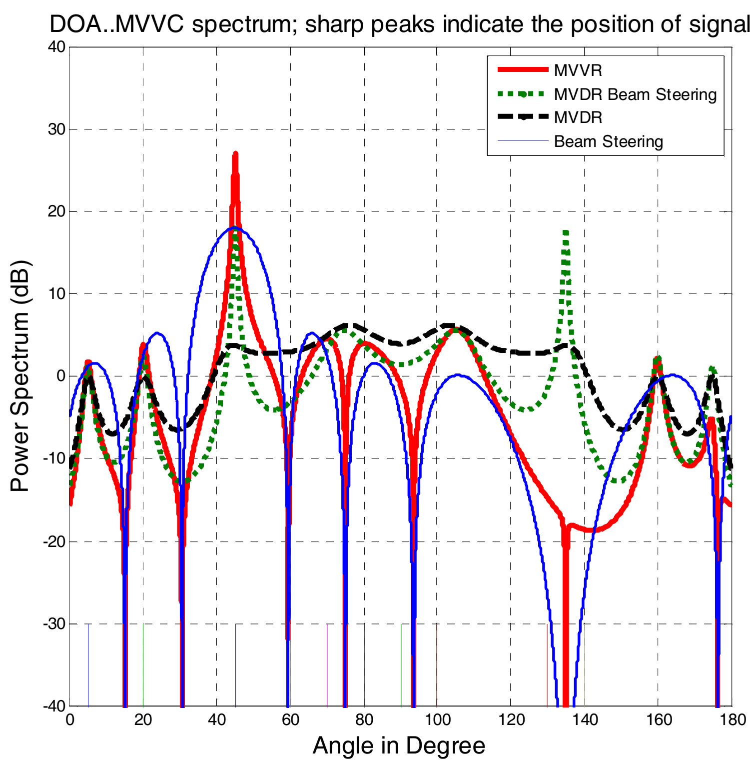


Figure 3: DOA estimation comparison between MVVC and MVDR algorithms for single signal estimation.

### 4.2. Accuracy

Accuracy of algorithms was tested as a function of Signal to Noise Ratio (SNR) and number of samples. Fig. 4 depicts the results obtained when RMSE is computed when SNR is varied for the algorithms of interest. Results were averaged over 200 runs and 1000 sample were used. The number of elements used in computing the pseudo-spectrum in MVVC was 8. The results clearly show that MVVC and MVDR have comparable performance.

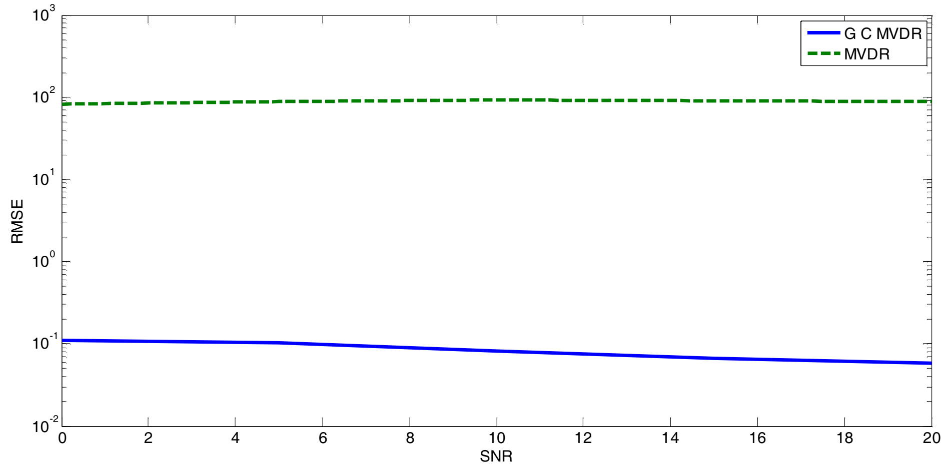


Figure 4: The convergence of estimation RMSE versus SNR for MVVC and MVDR algorithms.

Accuracy was also test as number of samples or shots were varied with a fixed SNR (10 dB). It was concluded that as the number of samples increase, RMSE was computed while varying the SNR from 0 to 20 dB in 2 dB increments. The RMSE tends to zero. MVVC RMSE seems to be slightly higher compared to MVDR but the increase is not sufficiently significant to cause a dramatic error in the DOA estimates. In a non-ideal setting, where mutual coupling was considered, RMSE was calculated for a varied mutual coupling effect (only adjacent element coupling was considered). MVVC seems to be slightly more affected. Magnitude and phase errors effects on accuracy were also investigated for the various algorithms. Results concluded that MVVC achieves a similar performance as far as robustness is concerned compared to other subspace methods.

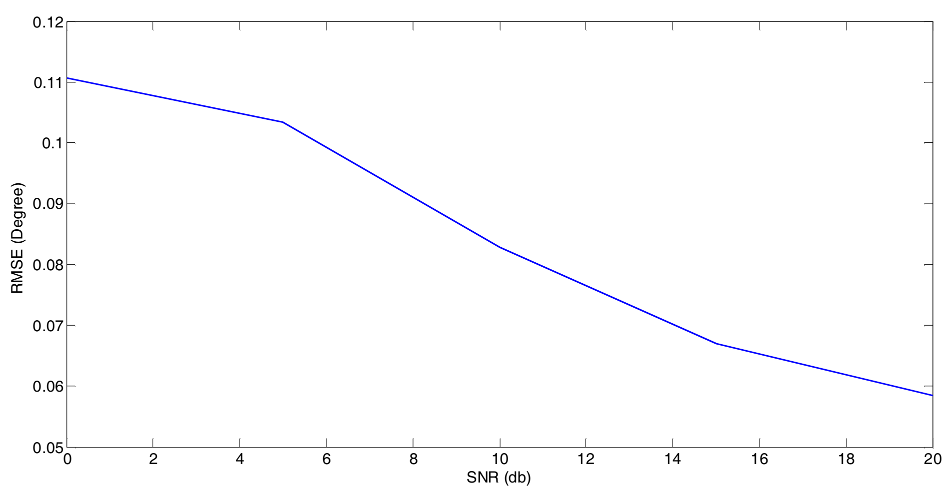


Figure 5: The convergence of estimation RMSE versus SNR for MVVC.

## 5. CONCLUSION

In this paper, a new algorithm (named Minimum Variance Variable Constrain) is proposed for antenna array with antenna elements. The MVVC algorithm is similar to the MVDR algorithm in one aspect, which is to minimize power from all directions subject to specific gain in the 'look direction'. However, the main difference is that while MVDR algorithm maintains unity gain at 'look direction', MVVC algorithm aims to maintain the corresponding array gain at 'look direction'. Simulations have demonstrated that the MVVC algorithm provides more accurate DOA estimations when used with antenna arrays.

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