Accurate Numerical Solutions for Electromagnetic Scattering by Strongly Anisotropic Structures

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Abstract— Solving volume integral equations (VIEs) with anisotropic media usually relies on the method of moments (MoM) with the divergence-conforming Schaubert-Wilton-Glisson (SWG) basis function or curl-conforming edge basis function. However, the basis functions may not be suitable to represent unknown functions when the anisotropy of media is strong. In this work, we replace the MoM with a point-matching method to solve such problems and the method characterizes the unknown functions at discrete points with directional components and more degrees of freedoms. Also, the method allows the use of JM-formulation which does not explicitly include material property in the integral kernels. A typical numerical example is presented to demonstrate the approach.

1. INTRODUCTION

Electromagnetic (EM) analysis for anisotropic structures relies on the accurate solution of volume integral equations (VIEs) in the integral equation approach [1]. Traditionally, the VIEs are solved by the method of moments (MoM) with the divergence-conforming Schaubert-Wilton-Glisson (SWG) basis function [2] or curl-conforming edge basis function [3]. These basis functions are defined over a pair of tetrahedral elements with a common face or common edge and require conforming meshes in geometric discretization, resulting in a higher preprocessing cost. Also, the basis functions have to assume a homogeneous material in each tetrahedron and disregard the possible directional difference of material, so they may not be suitable for strongly anisotropic materials [4]. In this work, we present a point-matching method which may be more appropriate to solve such problems because of its different features. The method uses a collocation procedure to transform integral equations into matrix equations and possesses several merits, i.e., the simple mechanism of implementation, use of nonconforming meshes, and removal of basis and testing functions [5]. The method represents unknown functions at discrete points with directional components and usually includes more degrees of freedoms than the MoM. Furthermore, the method allows the use of JM-formulation which does not explicitly include material property in the integral kernels in the VIEs [6]. These characteristics indicate that it is very suitable for solving anisotropic problems and also very friendly to the incorporation of fast algorithms like multilevel fast multipole algorithm (MLFMA) [7]. Although the method has to handle the hypersingular kernels resulting from the dyadic Green's function, the robust treatment techniques developed in recent years [8, 9] make it become a good alternative to the MoM, especially for highly anisotropic problems. A typical numerical example is presented to demonstrate the method and good performance can be observed.

2. VOLUME INTEGRAL EQUATIONS

Consider the EM scattering by an anisotropic object embedded in the free space whose permittivity and permeability are ϵ_0 and μ_0, respectively, the VIEs for governing the problem can be written as [1]

$$\mathbf{E}^{inc}(\mathbf{r}) = \mathbf{E}(\mathbf{r}) + \int_V \nabla g(\mathbf{r},\mathbf{r}') \times \mathbf{M}_V(\mathbf{r}')d\mathbf{r}' - i\omega\mu_0 \int_V \bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{J}_V(\mathbf{r}')d\mathbf{r}', \quad \mathbf{r} \in V \tag{1}$$

$$\mathbf{H}^{inc}(\mathbf{r}) = \mathbf{H}(\mathbf{r}) - \int_V \nabla g(\mathbf{r},\mathbf{r}') \times \mathbf{J}_V(\mathbf{r}')d\mathbf{r}' - i\omega\epsilon_0 \int_V \bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{M}_V(\mathbf{r}')d\mathbf{r}', \quad \mathbf{r} \in V \tag{2}$$

where $\mathbf{E}^{inc}(\mathbf{r})$ and $\mathbf{H}^{inc}(\mathbf{r})$ are the incident electric field and magnetic field, respectively, while $\mathbf{E}(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$ are the total electric field and total magnetic field inside the object, respectively. Also, $\bar{\mathbf{G}}(\mathbf{r},\mathbf{r}')$ is the three-dimensional (3D) dyadic Green's function defined by

$$\bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') = \left(\bar{\mathbf{I}} + \frac{\nabla\nabla}{k_0^2}\right) g(\mathbf{r},\mathbf{r}') \tag{3}$$

where $\bar{\mathbf{I}}$ is the identity dyad and $g(\mathbf{r}, \mathbf{r}') = e^{ik_0 R}/(4\pi R)$ is the scalar Green's function in which $k_0 = \omega\sqrt{\mu_0\epsilon_0}$ is the free-space wavenumber and $R = |\mathbf{r} - \mathbf{r}'|$ is the distance between an observation point $\mathbf{r}$ and a source point $\mathbf{r}'$. In addition,

$$\mathbf{J}_V(\mathbf{r}') = i\omega\left[\epsilon_0\bar{\mathbf{I}} - \bar{\boldsymbol{\epsilon}}(\mathbf{r}')\right]\cdot\mathbf{E}(\mathbf{r}') \tag{4}$$

$$\mathbf{M}_V(\mathbf{r}') = i\omega\left[\mu_0\bar{\mathbf{I}} - \bar{\boldsymbol{\mu}}(\mathbf{r}')\right]\cdot\mathbf{H}(\mathbf{r}') \tag{5}$$

are the induced volumetric electric current density and magnetic current density inside the object, respectively, and $\bar{\boldsymbol{\mu}}_r(\mathbf{r}') = \bar{\boldsymbol{\mu}}(\mathbf{r}')/\mu_0$ and $\bar{\boldsymbol{\epsilon}}_r(\mathbf{r}') = \bar{\boldsymbol{\epsilon}}(\mathbf{r}')/\epsilon_0$ are the relative permeability tensor and relative permittivity tensor, respectively.

3. POINT-MATCHING DISCRETIZATION

The above VIEs can be solved with the point-matching method. We choose the current densities $\mathbf{J}_V(\mathbf{r}')$ and $\mathbf{M}_V(\mathbf{r}')$ inside the anisotropic media as unknowns to be solved, leading to the use of JM-formulation. The choice is more suitable than the other two choices, i.e., the DB-formulation using $\mathbf{D}(\mathbf{r}')$ and $\mathbf{B}(\mathbf{r}')$ as unknowns and EH-formulation using $\mathbf{E}(\mathbf{r}')$ and $\mathbf{H}(\mathbf{r}')$ as unknowns, for anisotropic problems [6], especially when the anisotropy is strong. Another advantage for such a choice is that we can avoid the involvement of material property, i.e., the permittivity tensor and permeability tensor inside the integrands as can be seen from Eqs. (1) and (2) since the current densities have accounted for the material property as shown in Eqs. (4) and (5), leading to much convenience in numerical implementation for anisotropic media. The material property will appear in the total fields when they are expressed in terms of current densities through Eqs. (4) and (5), namely,

$$\begin{bmatrix} E^x \\ E^y \\ E^z \end{bmatrix} = \mathbf{E}(\mathbf{r}') = \frac{1}{i\omega}\left[\epsilon_0\bar{\mathbf{I}} - \bar{\boldsymbol{\epsilon}}(\mathbf{r}')\right]^{-1}\cdot\mathbf{J}_V(\mathbf{r}') = \frac{1}{i\omega}\begin{bmatrix} \epsilon_0-\epsilon_{11} & -\epsilon_{12} & -\epsilon_{13} \\ -\epsilon_{21} & \epsilon_0-\epsilon_{22} & -\epsilon_{23} \\ -\epsilon_{31} & -\epsilon_{32} & \epsilon_0-\epsilon_{33} \end{bmatrix}^{-1}\cdot\begin{bmatrix} J_V^x \\ J_V^y \\ J_V^z \end{bmatrix} \tag{6}$$

$$\begin{aligned}\begin{bmatrix} H^x \\ H^y \\ H^z \end{bmatrix} &= \mathbf{H}(\mathbf{r}') = \frac{1}{i\omega}\left[\mu_0\bar{\mathbf{I}} - \bar{\boldsymbol{\mu}}(\mathbf{r}')\right]^{-1}\cdot\mathbf{M}_V(\mathbf{r}') \\ &= \frac{1}{i\omega}\begin{bmatrix} \mu_0-\mu_{11} & -\mu_{12} & -\mu_{13} \\ -\mu_{21} & \mu_0-\mu_{22} & -\mu_{23} \\ -\mu_{31} & -\mu_{32} & \mu_0-\mu_{33} \end{bmatrix}^{-1}\cdot\begin{bmatrix} M_V^x \\ M_V^y \\ M_V^z \end{bmatrix}.\end{aligned} \tag{7}$$

Note that the VIEs are changed into a scalar or component form in the point-matching method. Although the permittivity tensor and permeability tensor appears in the VIEs, they are outside the integrals and only related to self-interaction terms or diagonal elements in the impedance matrix. This can greatly facilitate the implementation of fast algorithms like the MLFMA since the fast algorithms are usually designed for far-interaction terms and the self-interaction terms will be handled in a traditional way [7].

4. NUMERICAL EXAMPLE

To demonstrate the point-matching method, we consider the EM scattering by a uniaxially anisotropic sphere with a radius a as shown in Figure 1. The uniaxially anisotropic material is characterized by the following permittivity tensor and permeability tensor

$$\bar{\boldsymbol{\epsilon}} = \epsilon_0\begin{bmatrix} \epsilon_r & 0 & 0 \\ 0 & \epsilon_t & 0 \\ 0 & 0 & \epsilon_t \end{bmatrix}\cdot\bar{\mathbf{I}} \tag{8}$$

$$\bar{\boldsymbol{\mu}} = \mu_0\begin{bmatrix} \mu_r & 0 & 0 \\ 0 & \mu_t & 0 \\ 0 & 0 & \mu_t \end{bmatrix}\cdot\bar{\mathbf{I}} \tag{9}$$

with $\bar{\mathbf{I}} = \hat{r}\hat{r} + \hat{\theta}\hat{\theta} + \hat{\phi}\hat{\phi}$ in a spherical coordinate system. It is assumed that the incident wave is a plane wave with a frequency $f = 300\,\text{MHz}$ and is propagating along $+z$ direction in free space. We select $a = 0.5\lambda$, $\epsilon_r = 2.0$, $\epsilon_t = 10.0$, $\mu_r = 9.0$, and $\mu_t = 3.0$, respectively, for the object, resulting in $\epsilon_t/\epsilon_r = 5.0$ and $\mu_r/\mu_t = 3.0$. We calculate the bistatic radar cross section (RCS) observed along

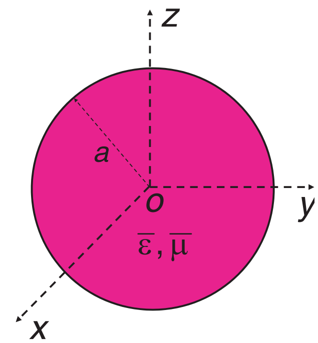

Figure 1: Geometry of a highly anisotropic object with a radius a.

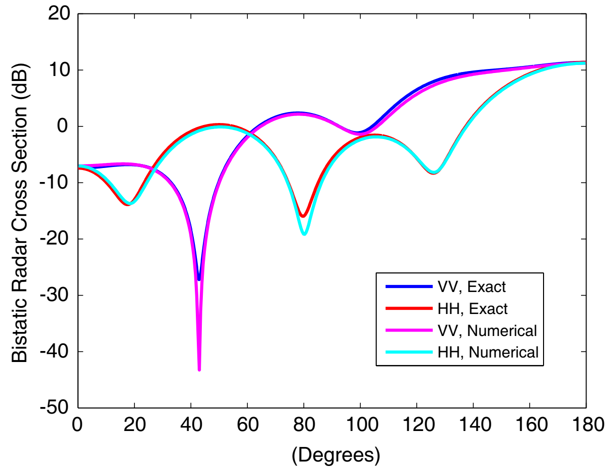

Figure 2: Bistatic RCS solutions for EM scattering by a strongly anisotropic object with a radius $a = 0.5\lambda$.

the principal cut ($\phi = 0°$ and $\theta = 0° - 180°$) for the scatterer in both vertical polarization (VV) and horizontal polarization (HH). Figure 2 plots the bistatic RCS solutions and they are in good agreement with the corresponding analytical solutions which can be obtained using the formulas in [10].

5. CONCLUSION

The traditional MoM based on the divergence-conforming SWG basis function or curl-conforming edge basis function may not be suitable for solving the EM problems with highly anisotropic media because of the features of the basis functions. In this work, we present a point-matching method to replace the MoM for resolving such problems and the method is more flexible in implementation. In addition to its well-known merits, the method can better characterize the unknown functions at discrete points with directional components and more degrees of freedom. Furthermore, the method allows the use of the JM-formulation which can avoid the involvement of material parameters in the integral kernels. The typical numerical example has been presented to demonstrate the robustness of the method.

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