Optimal Waveform Design in Through-the-wall Application Based on the Information Theory

Xin Sun, Bi-Ying Lu, Peng-Fei Liu, and Zhi-Min Zhou
College of Electronic Science and Engineering
National University of Defense Technology, Changsha, Hunan 410073, China

Abstract— In this paper, the information theory is introduced to design the optimal waveform for through-the-wall application. Furthermore, the signal model and the target response are analyzed. By comparing the designed signal and the LFM signal, it is shown that the designed waveform achieves a significant improvement than the LFM signal in the aspect of obtaining mutual information of the target response.

1. INTRODUCTION

UWB through-the-wall-radar (TWR) uses the penetrable character of the low-frequency electromagnetic wave to image, detect and locate the human and targets behind the wall. As one of the hot research points, it has great significance in the military and civilian use. With the adaptive radar technology developing, waveform design is becoming one of the most important research directions.

In recent years, the waveform design methods have largely developed. The typical contributions are the method based on maximal SINR principle and the method based on the condition mutual information theory At present, the maximum SINR based method has been applied in TWI waveform design [1–3]. In these papers, the authors processed the research under the single-antenna monostatic operation and multi-antenna multistate operation, respectively. The output SINR has efficiently improved, which is very helpful for the target detection [1–3]. However, these optimize waveforms concentrate most of the energy in one or two narrow frequency bands, corresponding to the target resonant frequency or to those of highest frequency response [2]. However, such a narrow band waveform hardly provides a reasonable ability to resolve the range-to-target. As a result, more work should be done to meet this kind requirement in TWR application.

The paper is organized as follows. In Section 2 the principle of mutual information waveform design is introduced and analyzed. In Section 3, we consider the information waveform design method in TWR application based on the signal model and real requirements. In Section 4, the simulations and the comparison results are presented. Conclusions end this paper.

2. THE WAVEFORM DESIGN IN TWR APPLICATION BASED ON INFORMATION THEORY

According to the real requirements, the duration T and the total energy E_x of the transmission waveform should be finite. The transmission waveform is confined to the symmetric time interval $[-T/2\ T/2]$ and that only negligible energy resides outside the frequency interval $W = [f_0\ f_0 + W]$ for meeting the frequency needs. The mutual information waveform design method is to maximize the condition mutual information between the target response and the received signals (refer to [4]). To simplify the computation and convenience the derivation, we emphasize on the study for the single-antenna monostatic operation.

2.1. The Signal Model

The variable $x(t)$ denotes the transmission signal; the variable $w(t)$ denotes the wall impulse response; the variable $q(t)$ denotes the target impulse response; the variable $w_c(t)$ denotes the color clutter depending on the transmission waveform; the variable $c(t)$ denotes the clutter signals; the variable $n(t)$ denotes the additive Gauss noise; the variable $y(t)$ denotes the output signals. Then we get the signal model [2] in Figure 1.

So we can get the variable $s(t)$, the colored clutter $c(t)$ and the output signal $y(t)$ expression, respectively.

$$s(t) = x(t) * w(t) * q(t) * w(-t) \tag{1}$$

$$c(t) = x(t) * w_c(t) \tag{2}$$

$$y(t) = s(t) + c(t) + n(t) \tag{3}$$

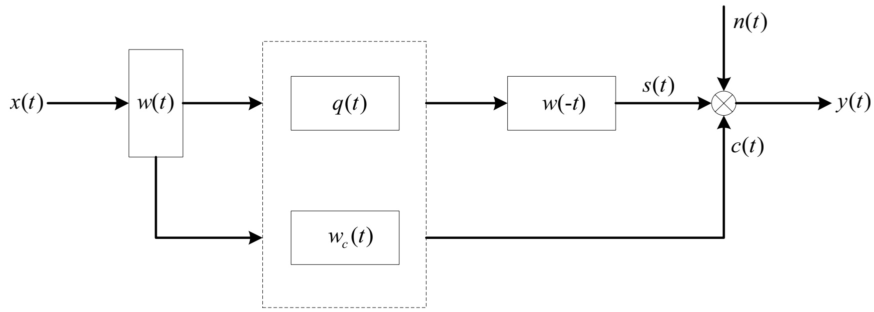


Figure 1: The signal model of TWR.

What we considered is to get more information of the targets behind the wall in TWR. So in the information aspect, it is to make the transmission waveform $x(t)$ maximize the mutual information between the receive signals $y(t)$ and the target response $q(t)$, namely maximizing the variable $I(y(t); q(t)|x(t))$. It is very difficult to get the expression. For convenience in mathematical reasoning, here we resort to a lemma in [4] to proof that the mutual information between the receive signals $y(t)$ and the target response $q(t)$ equals the mutual information between the receive signals $y(t)$ and the "wall-target" total response $g(t)$, namely

$$I(y(t); g(t)|x(t)) = I(y(t); q(t)|x(t)) \tag{4}$$

Proof:

[**Lemma 1**] Let $a(t)$ and $b(t)$ be finite-energy random processes and let D be a reversible transformation of $a(t)$ to a finite-energy random process $c(r)$ (where r is a new independent variable, but r could equal t). Then

$$I(a(t); b(t)) = I(c(r); b(t)) \tag{5}$$

Here, we define the variable D as follow expression,

$$D: \; q(t) \rightarrow w(t) * q(t) * w(-t) \tag{6}$$

It is obvious that the variable D is a reversible transmission, then define the following expression.

$$g(t) = w(t) * q(t) * w(-t) \tag{7}$$

So we can conclude that the mutual information between the receive signal $y(t)$ and the target response $q(t)$ equals the mutual information between the receive signal $y(t)$ and the "wall-target" total response $g(t)$, namely

$$I(y(t); g(t)|x(t)) = I(y(t); q(t)|x(t)) \tag{8}$$

Thus, according to the lemma, we determine the principle of the TWR application is to maximize the mutual information between the receive signal $y(t)$ and the "wall-target" total response $g(t)$. Then get the expression of the mutual information between the receive signal $y(t)$ and the "wall-target" response $g(t)$ via the above method.

$$I(y(t); g(t)|x(t)) = T \int_W \ln \left[1 + \frac{2|X(f)|^2 \sigma_G^2(f)}{TP_{nn}(f) + 2|X(f)|^2 \sigma_C^2(f)} \right] df \tag{9}$$

where, $|X(f)|^2$ is the optimal waveform magnitude squared spectrum with $\int_W |X(f)|^2 df = E_x$. $\sigma_G^2(f)$ is the "wall-target" response spectrum variance, and the variable $\sigma_c^2(f)$ is the colored clutter spectrum variance.

Resolving for the $|X(f)|^2$ by Lagrange method, get the expression of the variable $|X(f)|^2$, for details expression reasoning resort to [5].

$$|X(f)|^2 = \max \left[0, -R(f) + \sqrt{R2(f) + S(f)(A - D(f))} \right] \tag{10}$$

where,

$$D(f) = TP_{nn}(f)/\left(2\sigma_G^2(f)\right) \tag{11}$$

$$R(f) = \frac{TP_{nn}(f)\left(2\sigma_C^2(f)+\sigma_G^2(f)\right)}{4\sigma_G^2(f)\left(\sigma_C^2(f)+\sigma_G^2(f)\right)} \tag{12}$$

$$S(f) = \frac{TP_{nn}(f)\sigma_G^2(f)}{2\sigma_G^2(f)\left(\sigma_C^2(f)+\sigma_G^2(f)\right)} \tag{13}$$

Here $P_{nn}(f)$ is the power density of the Gaussian noise $n(t)$. The constant A is determined by the following expression.

$$E_x = \int_W \max\left[0, -R(f) + \sqrt{R2(f) + S(f)(A - D(f))}\right] df \tag{14}$$

2.2. The Wall Impulse Response

Once the scattered field is calculated, an approximation of the impulse response of the wall can be obtained according to the convolution relationship between the incident and the response. The matrix-vector expressions are given for the convenience of mathematical reasoning [2].

$$\mathbf{ZW} = \mathbf{S} \tag{15}$$

where, the variable $\mathbf{Z}$ is the M-by-N convolution matrix denotes to the incident waveform; $\mathbf{W}$ is the unknown wall's impulse response and the variable $\mathbf{S}$ is the scatter field vector. The subscript M and N denote the length of the incident waveform and the scatter field, respectively. Let $\mathbf{Z}$ be decomposed using the SVD, we get the expression

$$\mathbf{Z} = \mathbf{U\Sigma V^T} \tag{16}$$

where $\mathbf{U}$ and $\mathbf{V}$ are $M \times M$ orthogonal matrices and $\mathbf{\Sigma}$ is an $M \times M$ diagonal matrix whose entries $(\mathbf{\Sigma})_{ij} = \sigma_i$, $\sigma_i \geq \sigma_{i+1}$ are called the "singular values" of $\mathbf{Z}$. By introducing the vector $\boldsymbol{\eta}$ with the entries

$$\boldsymbol{\eta}_i = \left(\mathbf{U^T s}\right)_i / \sigma_i \tag{17}$$

Then, we can get the matrix-vector expression of the wall impulse response.

$$\mathbf{W} = \mathbf{V}\boldsymbol{\eta} \tag{18}$$

It is mentioned that, some methods or techniques, such as regularization and iterative, are using in the above calculation for resolve the ill-conditioned problem of the expression (15).

2.3. Spectrum Variance Estimate

Some simplification should be done for getting the estimation of the spectrum variance. Assuming the transmission wall illuminating the wall vertically, then the wall response $h(t)$ can be divided into two parts, the forward response and the back response, according to the angle between the receive signal and the wall.

$$h(t) = \sum_{\phi_1}^{h_1(t,\phi_1)} + \sum_{\phi_2}^{h_2(t,\phi_2)} \tag{19}$$

where, the variable h_1 denotes the forward response and the variable h_2 denotes the backward response. Their values depend on the angle ϕ_1 and ϕ_2. It is inevitable that the problem of the direct wave and multipath components are presence in the receive signals. While, there are some techniques and methods can remove these components, such as the time-gating method. So in this paper, we don't consider these problems. It can be concluded that we choose the forward response of the wall in this paper.

$$w(t) = h_1(t, \phi_1) \tag{20}$$

In the single-antenna monostatic operation, the simplest scene is the line which connects the antenna and the target vertical to the wall. At this situation, we choose the right forward response as the wall response.

$$w(t) = h_1(t, 0) \tag{21}$$

Then, we get the expression of the variable $g(t)$.

$$g(t) = h_1(t,0) * q(t) * h_2(t,\pi) \tag{22}$$

For symmetric walls such as homogeneous ones, $h_1(t,0) = h_2(t,\pi)$. A substitution of this expression into (22) yields the variable $g(t)$ and makes the Fourier transform.

$$G(\omega) = H_1(\omega,\pi) \cdot Q(\omega) \cdot H_1(\omega,\pi) = [H_1(\omega,\pi)]^2 \cdot Q(\omega) \tag{23}$$

Attributing to the randomicity of the targets presence, a common way to deal with the targets spectrum variance $\sigma_Q^2(\omega)$ is assumed to obey the gauss distribution. Assuming the target response $Q(\omega)$ also obeys the gauss distribution in the fixed frequency, because $[H_1(\omega,\pi)]^2$ is a fixed coefficient in a certain fixed frequency, then we can get spectrum variance of the "wall-target" response.

$$\sigma_G^2(\omega) = |H_1(\omega,\pi)|^4 \, \sigma_Q^2(\omega) \tag{24}$$

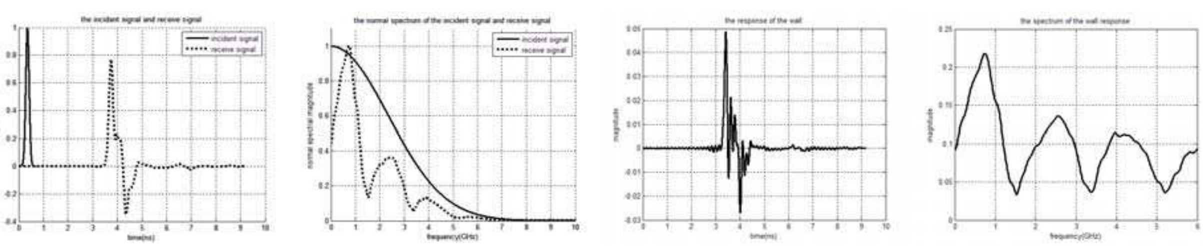
There are some other methods to estimate the spectrum variance, for example, the Monte Carlo analysis method [6]. It estimates the target spectrum variance via the probability and statistical theories after doing much Monte Carlo simulation. Using this method, the spectrum variance express is,

$$\sigma_Q^2(\omega) = \sum_{i=1}^{m} P_i \,|Q_i(\omega)|^2 - \left|\sum_{i=1}^{n} P_i Q_i(\omega)\right|^2 \tag{25}$$

where, the targets response q_i is a random variable and the variable p_i is the probability of respective response.

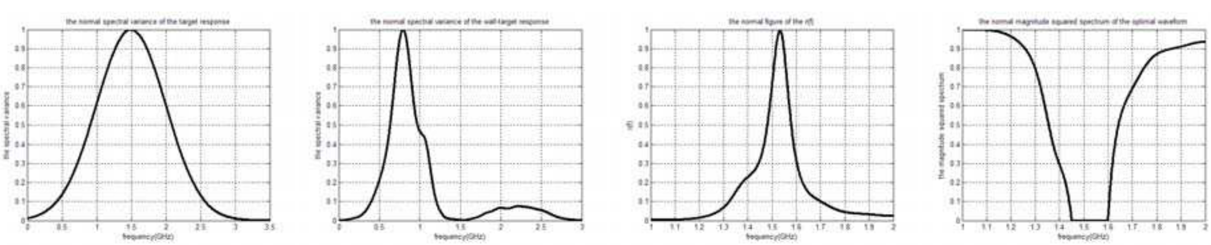
3. SIMULATION

Considering the real requirement in the TWR application, we determine the ultra-wild-band waveform's parameters as follows. The frequency range is 1G $\sim$ 2G, and the waveform duration is 1 µs. The power is 15 mw. Construct the electromagnetic model via CST 2008 software. In this model, the thick of the wall is 20 cm, the conductance of the wall is 0.03 S/m and dielectric permittivity is 4.2.



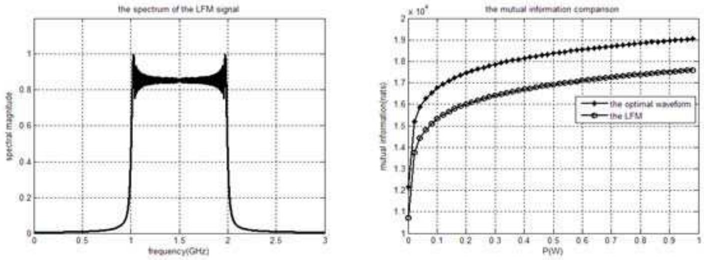
(a) (b) (c) (d)

Figure 2: (a) Time dimension of the incident and forward received signals. (b) Spectrum of the incident and forward received signals. (c) Right forward response. (d) Spectrum of the right forward response.



(a) (b) (c) (d)

Figure 3: (a) Normal spectrum variance of the target response. (b) Normal spectrum variance of the "wall-target" response. (c) Normal figure of the middle variable $r(f)$. (d) Normal magnitude squared spectrum of the optimal waveform.



(a) (b)

Figure 4: (a) LFM signal spectrum. (b) Mutual information comparison of the two signals.

The incident waveform is the Gauss signal. Figures 2(a) and (b) are the time dimension and spectral of the incident signal and received signal. Using the expression (22) and FFT transform, we get the time dimension and frequency dimension results of the right forward, shown in Figures 2(c) and 2(d). Assume the spectrum variance of the target as the Figure 3(a) showing, and calculate the spectrum of the "wall-target" response in Figure 3(b). Using the above analysis, we get the magnitude squared spectrum of the optimal waveform, shown in Figure 3(d). By analyzing the results of Figures 3(b) and 5(d), we can get the conclusion that, the optimal waveform is to concentrate most of the energy in the frequency bands, corresponding to the large value of the whole spectrum variance. It doesn't concentrate most of the energy in one or some narrow frequency bands as the maximum SINR method does, which is one of the differences and advantages of this method. The reason why appearing the phenomenon can be explained by the entropy theory. Because the larger of the variance, the more of the entropy value. And larger entropy value includes more information of the target. The advantage is shown in Figure 4(b) by comparing with the LFM signals whose result is in accord on the above analysis.

4. CONCLUSION

From the above theory analysis and simulation comparison, we can conclude that the information waveform design method can be fully applied into the TWR application and get the satisfied result according to signal model and the real requirement. The optimal waveform can be propitious to get more information of the target, which is very important in further process. And this method concentrates most of the energy into the frequency band, corresponding to the large value of the whole spectrum variance, rather than some narrow band. So it eliminates the narrow-band problem of the maximal SINR method at a certain extent. Furthermore, the comparison between the optimal waveform and the LFM waveform shows that the mutual information by the optimal waveform is obviously better than the LFM signal, which has the same duration, frequency interval and energy as the optimal waveform.

It is noted that the more perfect we get the target spectrum variance, the more accurate optimal waveform can be designed and the more target information can be got. Aiming at the problem, we resort to the [4] and adapt the Gauss spectrum variance. And we can also via the Monte Carlo simulation method in [6] to get the spectrum variance. So in the further research, lots of measurements should be done to summarize the spectrum variance in the real scene for getting more perfect result.

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