Self-reconstruction and Rectification of Non-diffracting Beams after Focusing

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Abstract— Based on Huygens-Fresnel diffraction integral, the expression of intensity distribution of the self-reconstruction beam is obtained and used to study the intensity evolution of self-reconstruction beams. It is shown that the intense central spot size of the self-reconstruction beam is larger than that of the non-diffracting beam before focusing. This property limits the application of the self-reconstruction beams.

1. INTRODUCTION

In 1998, the self-reconstruction of the non-diffracting beam in free space was fully demonstrated and explained by Bouchal et al. [1]. In 2002, Garces-Chavez et al utilized the self-reconstruction property of the beam in the application of manipulation [2]. Furthermore, the self-reconstruction of the non-diffracting beam in a nonlinear medium has also been examined [3, 4]. Recently, some researchers have further studied the self-reconstruction phenomenon of the non-diffracting beam [5–9]. In previous studies, the people seldom considered the variable properties of the parameters of reconstruction beam. In this paper, using Huygens-Fresnel diffraction integral, we discuss the self-reconstruction of focused non-diffracting beam generated by axicon. Especially, the effect of parameters of the reconstruction beams on the intensity evolution of self-reconstruction beams are considered. We numerically calculate the intensity distribution of the reconstruction beam. In this way, the beam can be used to micro-manipulate different size particles.

2. THEORETICAL FORMULATION

The electric field amplitude of a zero-order Bessel beam which illuminates on focal lens is given by

$$E_1(r_1) = A_0 J_0(k_r \cdot r_1), \tag{1}$$

where J_0 is the zero-order Bessel function of the first kind, k_r is the radial wave vector, and r_1 is the radial coordinate.

Using the generalized Huygens-Fresnel diffraction integral [10], the reconstructing field $E_2(r_2, \psi_2)$ at the position (r_2, L) is characterized by

$$E_2(r_2,\psi_2)=\left(-\frac{i}{\lambda B}\right)\exp(ikL)\iint_s E_1(r_1,\psi_1)\exp\left\{\frac{ik}{2B}\left[Ar_1^2+Dr_2^2-2r_1r_2\cos(\psi_1-\psi_2)\right]\right\}r_1dr_1d\psi_1. \tag{2}$$

The electric field of reconstruction beam satisfy rotational symmetry. If the radius of the beam illuminating on focusing lens is b, by taking the integration of r and ψ, we have

$$E_2(r_2) = \frac{i}{\lambda\cdot B}\exp(i\cdot k\cdot L)\int_0^b E_1(r_1)\cdot 2\pi J_0\left(\frac{k\cdot r_1\cdot r_2}{B}\right)\cdot\exp\left[\frac{i\cdot k}{2B}\left[A\cdot(r_1)^2+D\cdot(r_2)^2\right]\right]r_1dr_1, \tag{3}$$

where k is the wave number which is related to λ by $k = 2\pi/\lambda$. The transfer matrix elements A, B, C, D following

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix} = \begin{pmatrix} 1-\frac{z+l}{f} & z+l \\ \frac{-1}{f} & 1 \end{pmatrix}. \tag{4}$$

On substituting from Eqs. (1) and (4) into (3), we obtain

$$\begin{aligned} E(r,z) \ =\ & \frac{-i\cdot k}{(z+l)}A_0\exp(i\cdot k\cdot z)\exp\left(\frac{ik}{2(z+l)}r^2\right)\cdot\int_0^b J_0(k_r\cdot r_1)\cdot J_0\left(\frac{k\cdot r_1\cdot r}{z+l}\right) \\ & \cdot\exp\left[\frac{i\cdot k\cdot(r_1)^2}{2(z+l)}-\frac{i\cdot k}{2f}\cdot(r_1)^2\right]r_1dr_1. \end{aligned} \tag{5}$$

And the intensity distribution of the reconstruction beams reads

$$I(r,z)=\left(\frac{k\cdot A_0}{z+l}\right)^2\cdot\left[\left|\int_0^b J_0\left(k_r\cdot r_1\right)\cdot J_0\left(\frac{k\cdot r_1\cdot r}{(z+l}\right)\cdot\exp\left[\frac{i\cdot k\cdot(r_1)^2}{2(z+l)}-\frac{i\cdot k}{2f}\cdot(r_1)^2\right]r_1dr_1\right|\right]^2. \quad (6)$$

For correcting the transverse spatial frequency of the reconstructed beam restored to the original, a second lens is introduced to straighten the overlapping curved wave-fronts. The position z where the rectifying lens placed must be chosen in $z = f + f'$, and the foci of rectifying lens and the focusing lens satisfy $f = f'$. The rectifying transfer matrix elements A, B, C, D following:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}=\begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{-1}{f'} & 1 \end{pmatrix}\begin{pmatrix} 1 & f+f' \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix}=\begin{pmatrix} \frac{-f'}{f} & \frac{f\cdot f'+f^2-f\cdot z}{f'} \\ \frac{f'}{f^2}-\frac{1}{f'} & \frac{-f}{f'} \end{pmatrix}. \quad (7)$$

Let $f = f'$, we obtain

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}=\begin{pmatrix} -1 & 2f-z \\ 0 & -1 \end{pmatrix}. \quad (8)$$

Substituting from Eqs. (1) and (8) into (3), the electric field of the rectifying beam can be derived

$$E\left(r,z\right)=\frac{ik\cdot A_0}{2f-z}\exp\left(i\cdot k\cdot z\right)\cdot\exp\left(-\frac{i\cdot k}{4f-2z}\cdot r^2\right)\int_0^b J_0(k_r\cdot r_1)\cdot J_0\left(\frac{k\cdot r_1\cdot r}{2f-z}\right)\cdot\exp\left[\frac{i\cdot k}{4f-2z}r_1^2\right]r_1dr_1. \quad (9)$$

The intensity distribution reads

$$I\left(r,z\right)=E(r,z)\cdot E(r,z)^*=\left(\frac{k\cdot A_0}{2f-z}\right)^2\cdot\left[\left|\int_0^b J_0\left(k_r\cdot r_1\right)\cdot J_0\left(\frac{k\cdot r_1\cdot r}{2f-z}\right)\cdot\exp\left[\frac{i\cdot k\cdot(r_1)^2}{4f-2z}\right]r_1dr_1\right|\right]^2, \quad (10)$$

where the asterisk * represents the complex conjugation.

3. NUMERICAL CALCULATION RESULTS AND ANALYSES

The 3D intensity distribution and the intensity cross sections in different propagation distances are simulated, shown in Fig. 1 and Fig. 2 respectively.

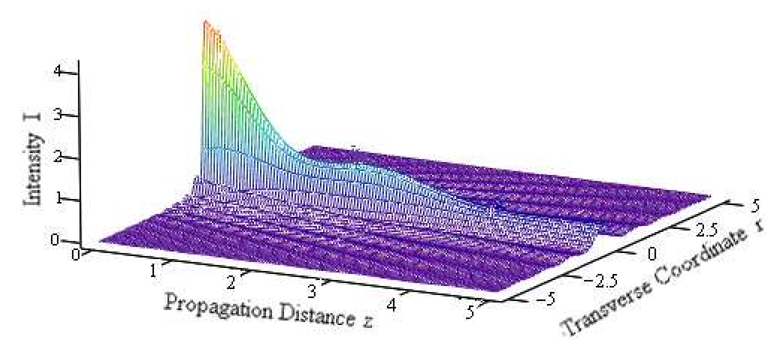
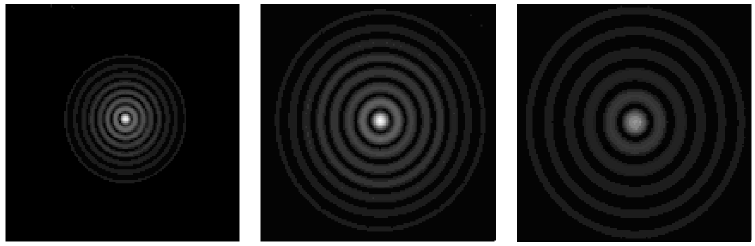


Figure 1: The 3D intensity distribution of reconstruction beam.

For the sake of convenience, the focal length f is normalized to $f = 3$. Fig. 1 shows that the intensity of the reconstruction beams attenuates quickly, the central spot size of the beams increasing with increasing propagation distance. The phenomenon can be seen easily in the Fig. 2. In Fig. 2, we note that the intense central spot size becomes larger. Comparing (a) with (c), the intensity of the beam reduces quickly. These results demonstrate the reconstruction beam is aberrational. It limits the application, so the reconstruction beam needs to be rectified.

Using the Eq. (10), the intensity distribution of rectifying beams is simulated, where the parameters $f = 3$, $f' = 3$. The results are compiled in Fig. 3 and Fig. 4.



(a) (b) (c)

Figure 2: The reconstruction beam patterns at different points on z-axis. (a) $z = 1$; (b) $z = 3$; (c) $z = 5$.

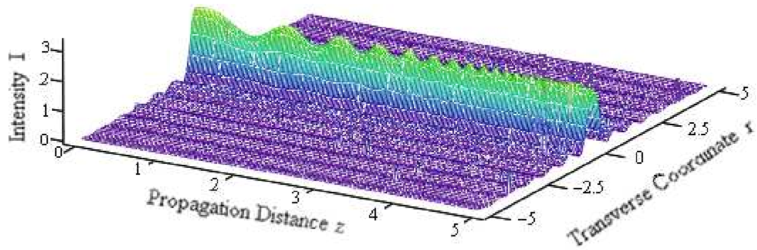
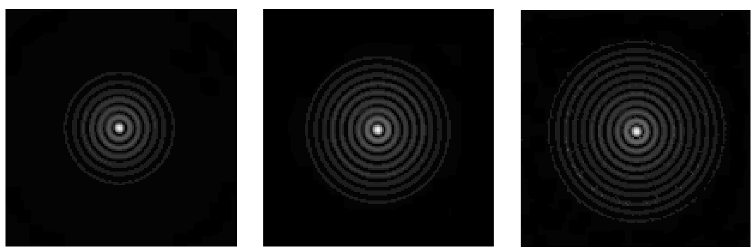


Figure 3: The 3D intensity distribution of rectified beam.



(a) (b) (c)

Figure 4: The rectified beam patterns at different points on z-axis behind the rectifying lens. (a) $z = 0$; (b) $z = 2$; (c) $z = 4$.

Figure 3 gives the 3D intensity distribution of rectified beam from which we can see that the intensity of the beam is almost invariable with increasing propagation distance. According to the Fig. 4, the intense central spot size, and the intensity of the rectified beam do not change any more with the propagation distance. Only the number of the bright rings increases. The rectified beam has the non-diffracting properties.

4. CONCLUSIONS

By using the generalized Huygens-Fresnel diffraction integral, the propagation properties of self-reconstruction beam are studied. It is shown that the non-diffracting beams can reconstruct its intensity profile after focusing. But the reconstruction beam occurs aberration opon propagation. A second lens is introduced to rectify the beam aberration and to get the original non-diffracting beam. Furthermore, we can generate different parameter non-diffracting beams by altering the propagation ratio of the foci of rectifying lens and focusing lens. The results obtained in this paper would be useful for application in micro-manipulation.

ACKNOWLEDGMENT

This research is supported by the National Natural Science Foundation of China (Grant No. 61178015) and the Natural Science Foundation Project of Fujian Province (Grant No. 2012J01278). We would like to thank Prof. Ke Wang for valuable discussion.

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