A Method of Two-dimensional MIMO Planar Array Design Based on Sub-array Segmentation for Through-wall Imaging

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Abstract— In three-dimensional (3-D) through-wall imaging (TWI) applications, imaging performance is directly affected by the configuration of the arrays. In this paper, the concept of equivalent array is introduced to design the 2-D MIMO array. A method named sub-array segmentation is used to circumvent the de-convolution process between the equivalent array and MIMO array. To get the optimum array among the results after the de-convolution, a concept of comprehensive size is used to evaluate the physical size of the MIMO array by using the information of sub-array segmentation, which is inspired by the split transmit virtual aperture (STVA) that has the shortest physical size given the virtual aperture. Finally, a 2-D UWB-MIMO array is designed for TWI as an example to validate the proposed method.

1. INTRODUCTION

Recent years, radar imaging technology has made a great progress with the introduction of various new concepts. To meet the needs of anti-terrorism in the cities, street fighting, as well as the personnel rescue buried in the ruins, ultra-wideband TWI technology has been widely studied. To solve the problem of the shadow effect and space ambiguity phenomena in 2-D imaging, getting the 3-D information of target scene by 2-D spatial distribution of antennas becomes an urgent need.

In order to overcome the disadvantages of time costing of synthetic aperture imaging system and the prohibitive cost of real aperture imaging in 3-D TWI, MIMO imaging [1], which uses the virtual array concept to get a larger virtual imaging aperture while using much lesser actual number of array elements, are widely used. Equivalent array, such as virtual aperture [2–4], which makes the bistatic pattern of MIMO array equivalent to the monostatic pattern, provides an important tool for the analysis and design of MIMO array.

In [5] Lockwood proposed a framework for designing the 2-D sparse array by selecting different element spacing within the transmit/receive elements, Smith obtained Several typical configurations of 2-D array under far-field condition using Fast Fourier transform methods in [6], X. D. Zhuge in [7] extended the method of separable aperture functions for designing an uniform linear array to an uniform rectilinear array. As the bistatic-to-monostatic equivalence condition is valid only at a small range of angle, in most cases, the size of the designed MIMO array is expected to be as small as possible to keep the similar illumination geometry with the equivalent array. Refer to the STVA given by Lu Biying in [2], a method based on the sub-array segmentation is introduced to solve the problem of de-convolution, and a best choice for the smallest physical size of the MIMO array is made among the results after the de-convolution according to the criterion of comprehensive size. An example of 2-D planar array is made to validate the proposed method in the end.

2. VIRTUAL APERTURE

Virtual aperture [2–4] is based on the equivalence between the two-way bistatic array and the two-way monostatic array, using the principle of Phase Center Approximation (PCA). Consider the 2-D MIMO array composed of M transmit and N receive elements located on the xz plane, in which the location of mth transmit element is $\mathbf{r}_{t,m} = (x_{t,m}, 0, z_{t,m})$, $m = 0, 1, 2, \ldots, M-1$, and the location of nth receive element is $\mathbf{r}_{r,n} = (x_{r,n}, 0, z_{r,n})$, $n = 0, 1, 2, \ldots, N-1$. The virtual aperture element $\mathbf{r}_{v,i}$, come from $\mathbf{r}_{t,m}$ and $\mathbf{r}_{r,n}$, may be expressed as

$$\mathbf{r}_{v,i} = (x_{v,i}, 0, z_{v,i}) = ((x_{t,m} + x_{r,n})/2, 0, (z_{t,m} + z_{r,n})/2) \tag{1}$$

where $i = (m+1)(n+1) - 1$, $m = 0, 1, 2, \ldots, M-1$, $n = 0, 1, 2, \ldots, N-1$.

3. 2-D MIMO ARRAY DESIGN

As known that the equivalent virtual array can be regarded as the space convolution between the transmit and receive elements of MIMO array, got as

$$D_e(r) = D_t(r) * D_r(r) \tag{2}$$

where $D_e(r)$ is the distribution function of the equivalent array, $D_t(r)$ and $D_r(r)$ are the distribution functions of the transmit and receive array, "$*$" represents the space convolution operation.

Due to the nature of convolution, the equivalent array can be seen as a combination of receive array copies, when the number of transmit elements is given. Thus, in some cases, the de-convolution of the equivalent array can be simplified to the segmentation of the equivalent array with the same structure of sub-arrays.

For a 2-D planar array, assume that the 2-D uniform plane array composed with $N \times M$ equivalent virtual elements is considered. The intervals between the elements along the x-label and the z-label are respectively d_x and d_z. The distribution function of the equivalent virtual array can be expressed as

$$D_e(r) = \left\{ (r, w_e(r)) \middle| \begin{array}{l} r(x) = x_m,\ m = 1, 2 \ldots, M \\ r(z) = z_n,\ n = 1, 2 \ldots, N \end{array},\ w_e(r) = 1 \right\} \tag{3}$$

The center position of the equivalent array is

$$O(x_0, z_0) = ((x_1 + x_M)/2, (z_1 + z_N)/2) \tag{4}$$

Given the number of transmit elements T, which is a divisor of $N \times M$, then the number of receive elements can be determined by $R = N \times M/T$. Factor R, and let N_R represents the number of factorizations. For the ath factorization $R = R_{ax} \times R_{az}$, factor R_{ax} and R_{az} respectively, the number of factorizations N_{ax} and N_{az} can be got. If R_{ax} is not a divisor of M or R_{az} is not a divisor of N, N_{ax} or N_{az} should be set to null. If R_{ax} equals to M or R_{az} equals to N, N_{ax} or N_{az} should be set to unity. Note that the pth factorization of R_{ax} is $R_{ax} = R_{apr} \times R_{aps}$, and the qth of R_{az} is $R_{ax} = R_{apr} \times R_{aps}$. The number of sub-array segmentations for the equivalent array can be formulated as

$$N_E = \sum_{a=1}^{N_R} N_{ax} \times N_{az} \tag{5}$$

Take any kind of the sub-array segmentation from N_E, a structure of sub-array can be obtained. Thus, a copy of the receive array can be expressed as follows

$$\begin{aligned} D_{r0}(r) \ = \ & \left\{ (r, w_{r0}(r)) \middle| \begin{array}{l} r(x) = x_m,\ m = (R_x - 1) \times M/R_{aps} + 1, 2 \ldots, R_{aps},\ R_x = 1, \ldots, R_{apr}; \\ r(z) = z_n,\ n = (R_z - 1) \times N/R_{aqs} + 1, 2 \ldots, R_{aqs},\ R_z = 1, \ldots, R_{aqr}; \end{array} \right., \\ & w_{r0}(r) = 1 \Big\} \end{aligned} \tag{6}$$

The center position of the receive array copy is

$$\begin{aligned} & O(x'_0, z'_0) = ((x_1 + x_{m0})/2, (z_1 + z_{n0})/2), \\ & m0 = (R_{apr} - 1) \times M/R_{aps} + R_{aps},\ n0 = (R_{aqr} - 1) \times N/R_{aqs} + R_{aqs} \end{aligned} \tag{7}$$

Then, the distribution function of the receive array can be expressed as

$$\begin{aligned} D_r(r) = & \left\{ (r, w_r(r)) \middle| \begin{array}{l} r(x) = 2(x_m + x_0 - x'_0),\ m = (R_x - 1) \times M/R_{aps} + 1, \ldots, R_{aps},\ R_x = 1, \ldots, R_{apr} \\ r(z) = 2(z_n + z_0 - z'_0),\ n = (R_z - 1) \times N/R_{aqs} + 1, \ldots, R_{aqs},\ R_z = 1, \ldots, R_{aqr} \end{array} \right., \\ & w_r(r) = 1 \Big\} \end{aligned} \tag{8}$$

Combine Eq. (3), Eq. (8) and the positional relationship of the equivalent virtual array given in Eq. (1), the distribution function of the transmit array is

$$D_t(r) = \left\{ (r, w_t(r)) \middle| \begin{array}{l} r(x) = [2(i-1) - (M/R_{ax} - 1)] \times M/R_{aps} d_x + x_0,\ i = 1, 2 \ldots, M/R_{ax} \\ r(z) = [2(j-1) - (N/R_{az} - 1)] \times N/R_{aqs} d_z + z_0,\ j = 1, 2 \ldots, N/R_{az} \end{array}, w_t(r) = 1 \right\} \tag{9}$$

It is easy to find that different segmentations lead to different results of the de-convolution MIMO arrays, when given the same equivalent array. Refer to the STVA, which has the shortest physical size when given the effective aperture, a concept of comprehensive size, which use the information of sub-array segmentation to evaluate the truly physical size of the MIMO array, is introduced as follows

$$S_e = S_{r0} \cup d_{tx} d_{tz} (N_{tx} - 1)(N_{tz} - 1)) \tag{10}$$

where S_e represents the comprehensive size or the final physical size of the MIMO array, S_{r0} represents the size of sub-array, N_{tx} and N_{tz} represent the number of sub arrays along the x-label and z-label direction, d_{tx} and d_{tz} represent the intervals between sub-arrays along the x-label and z-label direction, "$\cup$" means to take the union.

According to the projection slice theory [8], the pattern of the planar array at a certain direction is determined by the projection of array elements onto a rotated axis at that particular angle within the array plane. If the beam patterns along two orthogonal directions are only concerned, a 2-D planar array can be designed as a composition with two basic one-dimensional linear arrays designed with the performance requirements previously.

Thus, the 2-D MIMO array can be designed as the following steps:

(1) Design two basic one-dimensional linear arrays meeting with the performance requirements of the azimuth, elevation direction [2];

$$D_1(r) = \{(r, w_1(r))|r(x) = x_m,\ m = 1, 2 \ldots, M,\ w_1(r) = 1\} \tag{11}$$
$$D_2(r) = \{(r, w_2(r))|r(z) = z_n,\ n = 1, 2 \ldots, N,\ w_2(r) = 1\} \tag{12}$$

(2) Compose the two basic one-dimensional linear arrays designed above into a 2-D uniform planar array, as shown in Eq. (3);

(3) For the given number of transmit elements, segment the 2-D uniform planar array according to Eq. (5) to obtain a copy of receive array as shown in Eq. (6), and then select the smallest comprehensive size among the receive array copies after segmentation according to Eq. (10).

(4) Obtain the receive array from the copy of receive array based on Eq. (8), and get the transmit array as shown in Eq. (9).

4. AN EXAMPLE AND SIMULATION RESULTS

Array Requirements: Design a 2-D UWB-MIMO array, with resolutions of 0.3 m in both the azimuth and elevation direction at a distance of 5m. The side lobe levels should be below −18 dB. The number of transmit elements is 9, and the physical size of the MIMO array should be as small as possible.

To meet the UWB imaging requirements, a stepped frequency signal is emitted, the center frequency is 1.5 GHz, the bandwidth is 2 GHz, the frequency step is 4 MHz. In accordance with the design steps described above and paper [2], the length of one-dimensional uniform linear array and the number of elements should satisfy

$$\sigma = 0.886\lambda_c/4\left(\frac{L/2}{\sqrt{(L/2)^2 + R^2}}\right) \tag{13}$$
$$N = \text{int}_{up}(\max(1/RL, f_H/(B \cdot GL))) \tag{14}$$

where $\text{int}_{up}(\cdot)$ is the top integral function.

From Eq. (13) and Eq. (14), the length of one-dimensional uniform linear array $L = 1.5$ m and the number of elements $N = 12$ can be obtained. Thus, a two-dimensional uniform array of $12 \times 12 (1.5\,\text{m} \times 1.5\,\text{m})$ would be got from the two basic one-dimensional uniform linear arrays with the interval $d = 0.1364$ m between the elements. Given the number of transmit elements as $T = 9$, the square equivalent array could be segmented into 9 kinds of 16-sub-arrays with same structure as shown in Figure 1 according to the Eq. (5).

According to Eq. (10), the comprehensive apertures of the segmentations shown in Figure 1 can be calculated in Table 1. From Table 1, array 5 acquires the smallest comprehensive size and the corresponding smallest physical size of the MIMO array. The resulted MIMO array following the design steps is shown in Figure 2.

Table 1: Comprehensive sizes for the 9 different segmentations illustrated in Figure 1.

N	1	2	3	4	5	6	7	8	9
S_e	64	56	72	56	49	63	72	63	81

To validate the performance of the designed equivalent array and MIMO array, a simulation is taken and the resulted PSF is shown in Figure 3. It can be found that both the designed equivalent array and MIMO array well meet the design requirements and the PSFs keep almost identified, which has verified the accuracy of the method to use equivalent array to design the MIMO array and the effectiveness of the method to use the sub-array segmentation to circumvent the de-convolution process between the equivalent array and the MIMO array.

Adopt the designed 2-D MIMO array into the 3-D TWI application with a simulation. Assume that the MIMO array locates at 3 m far away from the wall, the thickness of the wall is 0.2 m and the dielectric constant is 4.2, two point targets distributed along the elevation direction locate at 2 m after the wall, the interval between the targets is 0.4 m. The illumination geometry and the result imaging are shown schematically in Figure 4. It can be found from the simulation results

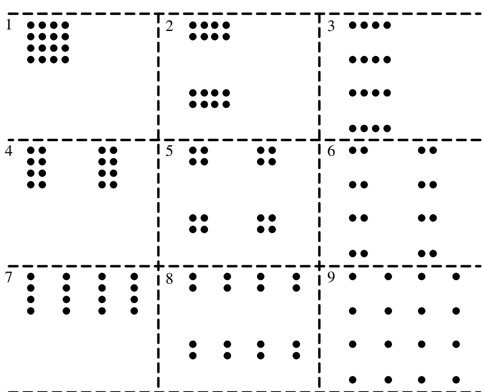


Figure 1: 9 kinds of structures for the 16-element sub-array.

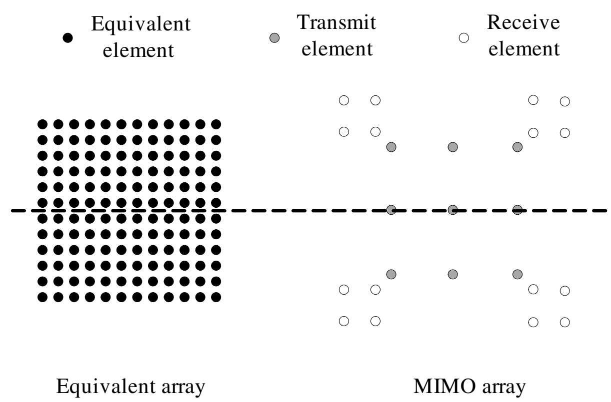
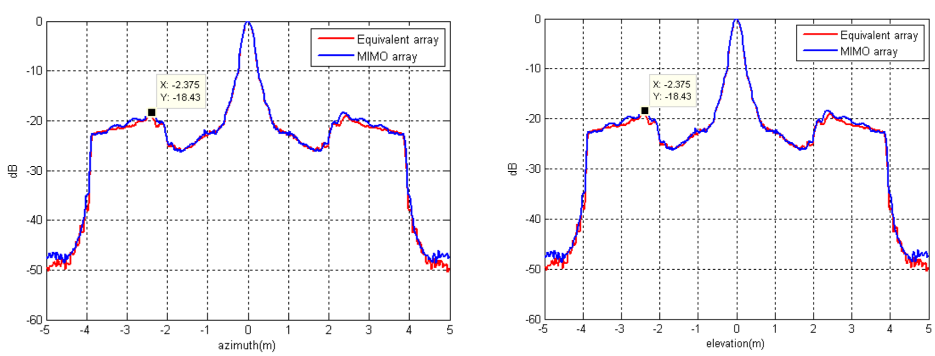
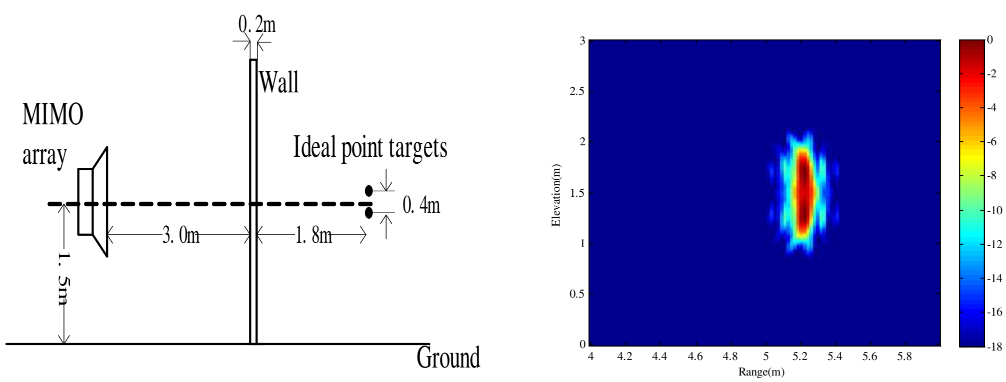


Figure 2: 2-D equivalent array and MIMO array

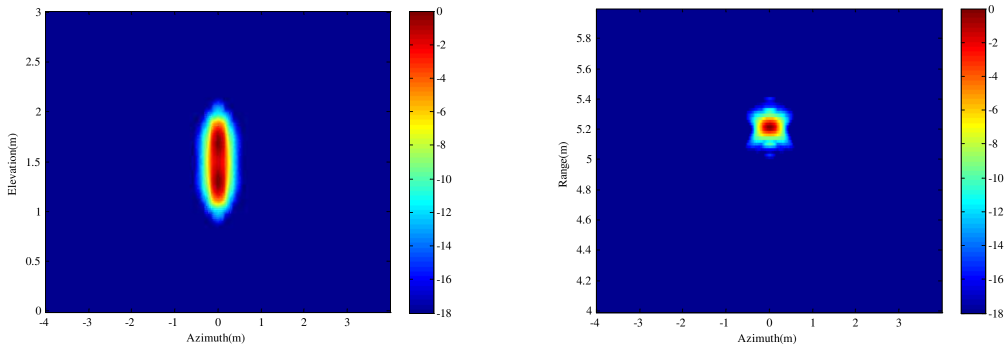


(a) (b)

Figure 3: The PSFs of the equivalent array and MIMO array in azimuth and elevation directions. (a) PSFs in azimuth direction. (b) PSFs in azimuth direction.



(a) (b)



(c) (d)

Figure 4: The illumination geometry and the result imaging for 3-D TWI. (a) The illumination geometry. (b) The slice of range-elevation plane. (c) The slice of azimuth-elevation plane. (d) The slice of azimuth-range direction.

that the designed 2-D MIMO array behave a perfect performance in the 3-D TWI.

5. CONCLUSIONS

In this paper, the concept of equivalent array is introduced to analyze and design the 2-D MIMO array. The principle of the shortest physical size for the one-dimensional STVA array is extended to the 2-D MIMO array with the concept of comprehensive size and the method of sub-array segmentation is used to circumvent the de-convolution process between the equivalent array and the MIMO array. At last, an example is taken to design a 2-D UWB-MIMO array, and the simulation gives a perfect result.

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