Scattering Analysis of Reflectarray Antennas Illuminated by a Point Source for Near Field Focus Applications

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Abstract— The scattering and diffraction mechanisms of reflecting and transmitting arrays illuminated by the radiation of a point source for a general near-field focus problem are summarized. In particular, the phenomena are interpreted by the ray decompositions of Floquet waves and result in the solutions, analogous to the ordinary UTD-type formulations for the phased array antennas with a proper parameter mapping. In this approach, the radiations of periodically located current sources are first decomposed into Floquet modes, in which every mode is equivalent to the radiation of continuous current sources with additional linear phase impressions. Fast convergence in the radiation computation can be achieved because relatively few modes will propagate to the target zone with the rest being evanescent. The ray decomposition is performed by asymptotically evaluating the radiation integral of each Floquet mode and interprets the total fields in terms of rays directly radiated from the aperture and diffracted from aperture truncations.

1. INTRODUCTION

Reflectarray and transmitting array antennas are planar structures popularly used to replace the reflector and lens antennas, respectively with an advantage of low profile for easy installment. In comparison with the competing candidate of planar phased array antennas, they are also superior in simplifying the beam forming networks by using the radiation of a feeding antenna to illuminate the arrays. In the past, the majority of the researches in the literature focus on design and implementation methodologies for their applications in the far-field radiations. The radiation mechanisms have not been well developed. In addition, recent interests in the applications of near-field communications have further driven the need to analyze and explore the radiation mechanisms with handful interpretation methodologies, in which ray decompositions based on uniform geometrical theory of diffractions (UTD) has been well recognized to be very effective. The UTD-type ray decomposition has been successfully employed to analyze the radiation mechanisms of phased array antennas. In this approach, the radiations of periodically located current sources are first decomposed into Floquet modes, in which every mode is equivalent to the radiation of continuous current sources with additional linear phase impressions. Fast convergence in the radiation computation can be achieved because relatively few modes will propagate to the target zone with the rest being evanescent. The ray decomposition is performed by asymptotically evaluating the radiation integral of each Floquet mode and interprets the total fields in terms of rays directly radiated from the aperture and diffracted from aperture truncations. More recently we have developed a general solution for the radiation of finite array antennas excited to focus the radiation in the near zone, which accommodate the needs of analysis for the general applications of phased array antennas in both the far- and near-zones. This paper is organized in the following format. Section 2. describes the formulation for the radiation from an one-dimensional (1-D) semi-infinite to (2-D) NSA array. The characteristics are investigated in Subsections 2.1–2.4. In particular, the study focuses on the truncation diffraction mechanism as a compensation to the shortage of Floquet mode phenomena in [1–5], which examine only the radiation mechanisms of infinite array of antennas. Numerical examples are presented in Section 3 for demonstration and validation. Finally a short discussion is presented in Section 4 as a conclusion.

2. 2-D FINITE ARRAY RADIATION PROBLEM

2.1. Composition of 2-D Radiation from a 1-D Array

The semi-infinite, linear array of line sources is illustrated in Figure 1, whose elements are indexed by $n = -\infty \sim N$ and located at $\bar{r}'_n = (nd_x, 0)$ on the x-axis with a period d_x. The line source is located at $\bar{\rho}_f = (x_f, z_f)$ and radiates fields by

$$u_f(\bar{\rho}) = \frac{1}{4j} I_0 H_0^{(2)}\left(k|\bar{\rho} - \bar{\rho}_f|\right) \cong \frac{e^{-j\pi/4} I_0}{4} \sqrt{\frac{2}{\pi k|\bar{\rho} - \bar{\rho}_f|}} e^{-jk|\bar{\rho} - \bar{\rho}_f|} \tag{1}$$

which will excite the reflecting or transmitting elements in Figures 1(a) and (b), respectively. In (1), $k = 2\pi/\lambda$ is the wave number with λ being the wavelength in free space, and I_0 is used as the reference amplitude of line source. The fields scattered from the array can be found from the radiation of equivalent current, $I(\bar{\rho}')$, on the closed surface enclosing the array structure as illustrated in Figure 1. To the degree of accuracy in the Kirchhoff approximation, the induced current is assumed to be found from the infinite array structure illuminated by the same incident field. Thus it can be approximately expressed as

$$I(\bar{\rho}') = u_f(\bar{\rho}_i) Q(\bar{\rho}') e^{j\phi(\bar{\rho}')} \tag{2}$$

where $\bar{\rho}_i$ and $\bar{\rho}'$ are the position vectors on the closed surface. The function, Q, is related to the reflection and transmission coefficients for the cases of Figure 1, with ϕ being the phase. In the current investigation, one is interested in the scattering field in the $+z$ space, thus only the current on S_t is considered with the others omitted for simplification. The scattering field can be expressed as

$$u_s(\bar{\rho}) = \frac{1}{4j} \int_{x_a}^{x_b} u_f(\bar{\rho}_i) Q(\bar{\rho}') e^{j\phi(\bar{\rho}')} H_0^{(2)}(k|\bar{\rho} - \bar{\rho}'|) dx' \tag{3}$$

where $x_a \to -\infty$ and $x_b = Nd_x$ in the current development.

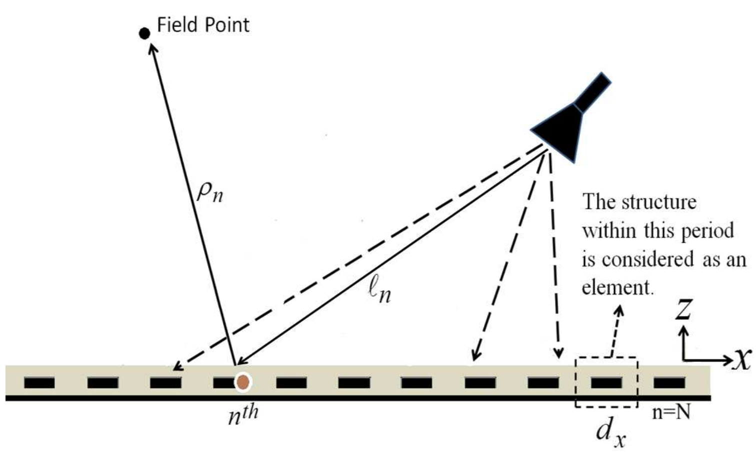

Figure 1: Illustration of 2-D reflectarray radiation problems, where the fields are focused in the near zone.

2.2. Composition of Uniform Asymptotic Formulations

The Floquet waves are obtained by using the following Poisson sum formula:

$$\sum_{n=-\infty}^{N} f(n) = -\frac{f(N^+)}{2} + \sum_{p=-\infty}^{\infty} \int_{-\infty}^{N^+} f(v) e^{-j2\pi pv} dv \tag{4}$$

The evaluation of (3) can be performed asymptotically by employing the spectrum representation of Hankel functions to decompose the radiating fields into components of diffraction mechanisms. Within the UTD framework, the asymptotic evaluation of (3) can be formulated into the following format:

$$A^p_{net}(\bar{r}) = A^p_{dir}(\bar{r}) \cdot U(\mathrm{Re}(x_e - x_s)) + A^p_{end}(\bar{r}) F(ka) \tag{5}$$

where $U(\cdot)$ is the Heaviside step function and $F(\cdot)$ is a transition function to assure the uniform field distribution. In (5), $A^p_{dir}(\bar{r})$ is the asymptotic solution of radiation when the array is extended to a infinity while $A^p_{end}(\bar{r})$ accounts for the effects of truncation. Both components arise from a radiation point, x_s, on the array aperture, and a diffraction point, x_e, on the edge. This formulation remains valid as the radiation point is close to the diffraction point, i.e., $x_s \to x_e$. However, as mentioned in [?], it exists two radiation points in a general NSA problem. The solution in (5) becomes singular as these two radiation points coincide near the diffraction point at the edge. The characteristics of these terms are addressed in the following subsections.

2.3. The Asymptotic Solution of Direct Radiation from a Non-truncated Array, $A^p_{dir}(\bar{r})$

In (5), $A^p_{dir}(\bar{r})$ is identical to the solution of radiation for an infinite array, whose characteristics have been investigated. The formulation is summarized in the following by

$$A^p_{dir}(\vec{r}) = \frac{jI_oQ}{2dxk^s_z}\sqrt{\frac{-\rho_c}{\rho^s_v-\rho_c}}\frac{1}{\sqrt{\ell^s_v}}e^{-jk(\rho^s_v-\rho^s_{vo})}e^{-j\beta_p x_i} \tag{6}$$

where $x_s = v_s d_x$ with v_s being the contributing saddle point. Here the saddle point, $x_i = x_s$, satisfies the following relation by

$$\frac{k^s_x}{k} = \frac{k^i_x}{k} + \frac{p\lambda}{d_x}; \quad \left(k^s_x, k^i_x\right) = k\left(\frac{x-x_s}{\rho^s_v}, \frac{x_s-x_f}{l^s_v}\right) \tag{7}$$

2.4. The Asymptotic Solution of Edge Point Contribution from a Truncated Array, $A^p_{end}(\vec{r})$

The asymptotic edge point contribution in (5) is developed and summarized in the following formulation:

$$A^p_{end}(\vec{r}) \cong \frac{jI_oQ}{2dx\sqrt{\ell_e}\sqrt{\rho_e}}\left[\frac{-e^{-j\frac{\pi}{4}}}{\sqrt{2\pi k}[(k^e_{xo}-k^e_x)+\beta_p]}\right]e^{-jk(\rho_e-\rho_{eo})}e^{-j\beta_p x_e} \tag{8}$$

3. NUMERICAL EXAMPLES

One first considers the case of scattering characteristic for an 2-D semi-infinite reflectarray, and examines the fundamental mode ($p = 0$) and higher order modes ($p = -1$), whose results are shown in Figures 2(a), (b). The parameters as shown: feed point located at $(-20\lambda, 90\lambda)$, focused point at $(0, 40\lambda)$, number of array element $(-\infty \sim N_x)$, $N_x = 20$ and operation frequency at 10 GHz.

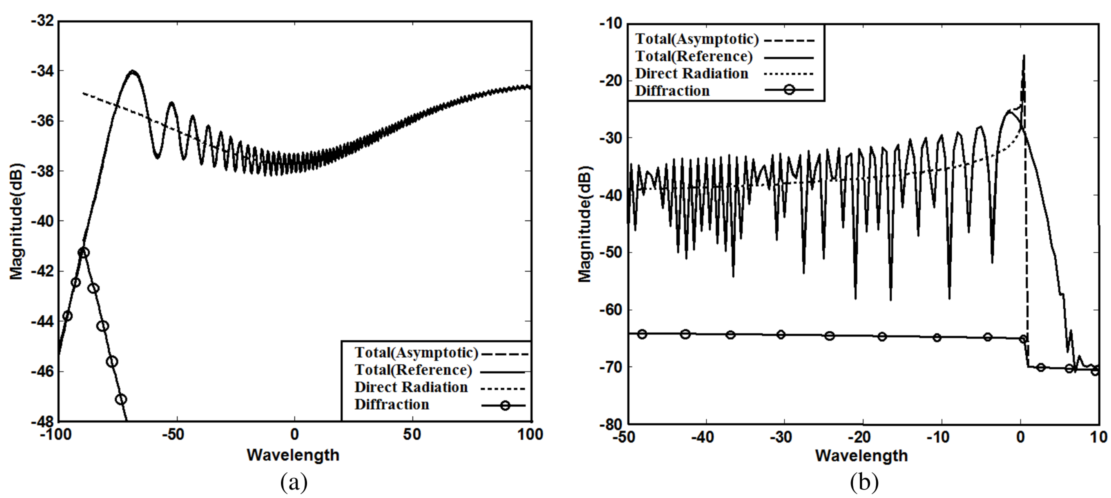

Figure 2: The scattering analysis of semi-reflectarray for dominant mode and higher order mode. (a) $p = 0$ mode. (b) $p = -1$ mode.

4. CONCLUSIONS

An uniform formulation of EM radiation from a semi-infinite, periodic reflectarray is presented in this paper. It decomposes the radiation mechanism with respect to the fundamental concept of UTD, and can be applied to interpret the wave propagation phenomena in a general radiation problem of focused field. In particular, the diffraction mechanisms from the array truncation are investigated with numerical examples demonstrations. This formulation remains valid as the radiation point approaches to the diffraction point on the truncation edge of array. As discussed in [1], there are two radiation points on the array aperture when the radiation focuses in the near-zone. The coincidence of these two radiation point occurs in the case that the field points are on the caustic curves in each Floquet mode. The presented formulation becomes singular as the diffraction point is near the coincident point of these two radiation points. These phenomena will be investigated in the future phase of this work.

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