H-polarized Plane Wave Diffraction by an Acute-angled Dielectric Wedge: A Time Domain Solution

M. Frongillo[1], **G. Gennarelli**[2], **and G. Riccio**[1]

[1]D.I.E.M. — University of Salerno, Via Giovanni Paolo II 132, Fisciano, SA 84084, Italy
[2]Institute for Electromagnetic Sensing of the Environment
National Research Council, Via Diocleziano 328, Naples 80124, Italy

Abstract— This work deals with the time domain diffraction phenomenon originated by a penetrable acute-angled dielectric wedge when illuminated by a plane wave at normal incidence. In particular, the H-polarization case is here considered. The diffraction coefficients are determined in closed form via an inverse Laplace transform by taking advantage of the formulation of the frequency domain coefficients in terms of the transition function of the Uniform Theory of Diffraction. Then the transient diffracted field originated by an arbitrary function plane wave can be evaluated via a convolution integral.

1. INTRODUCTION

Electromagnetic wave diffraction by a penetrable wedge is an exciting canonical problem with potential applications. Moreover, time domain (TD) diffraction problems are receiving great attention from both the research community and industry interested to ultra wide band (UWB) communication and radar systems. Because of the large bandwidth of the UWB signals, the analysis of the wave propagation mechanisms in the TD framework is preferable to the frequency domain (FD) data processing. Furthermore, the analysis of transient scattering phenomena is of importance for predicting the effects of electromagnetic pulses (EMPs) on civil and military structures.

Numerical discretization techniques represent useful tools to support the research activity, but they become rapidly intractable when considering excitation pulses with high frequency content. Analytical TD solutions are not available in literature for penetrable wedges apart from those reported in [1, 2], where the TD counterparts of the frequency domain Uniform Asymptotic Physical Optics (FD-UAPO) solutions [3, 4] for the diffraction coefficients related to right- and obtuse-angled dielectric wedges were obtained.

FD-UAPO solutions for evaluating the diffracted field in the dielectric region and the surrounding free-space were recently derived in the case of a lossless acute-angled wedge illuminated by H-polarized plane waves at normal incidence [5]. They were expressed in closed form and given in terms of the Geometrical Optics (GO) response of the structure and the transition function of the Uniform Theory of Diffraction (UTD) [6].

2. GEOMETRY OF THE PROBLEM

The considered problem and the related geometric parameters are shown in Fig. 1. The wedge-shaped dielectric region and its surfaces are denoted by Ω_d, S_0 and S_n, respectively. It has internal apex angle $\alpha < \pi/2$ and is filled by a lossless non-magnetic ($\mu_r = 1$) material having relative permittivity ε_r. The symbol Ω_0 indicates the free-space region surrounding the wedge. The incidence direction is assumed perpendicular to the edge and defined by the angle ϕ', which is here considered in the range from $\pi/2$ to $\pi - \alpha$, so that only S_0 is illuminated by an incident H-polarized plane wave having the magnetic field along the z-axis. The observation point is $P(\rho, \phi)$.

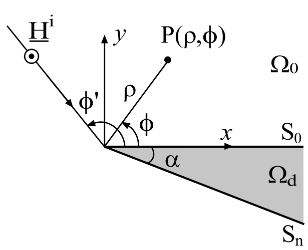


Figure 1: Geometry of the problem.

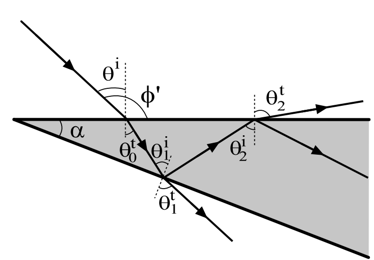


Figure 2: GO propagation mechanisms.

If $\theta^i = \phi' - \pi/2$ is the incidence angle, the wave penetrates into Ω_d according to the transmission angle $\theta_0^t = \sin^{-1}(\sin\theta^i/\sqrt{\varepsilon_r})$, and undergoes multiple reflections (see Fig. 2) until the reflection angle $\theta_p^r = \theta_p^i = p\alpha + \theta_0^t$ is greater than $\pi/2 - \alpha$. The number of total internal reflections is given by $M = \mathrm{Int}[(\pi/2 - \theta_0^t)/\alpha]$, where Int $[\cdot]$ denotes the integer part of the argument.

Waves transmitted through S_0 and S_n define shadow boundaries in Ω_0 and exist until $\theta_p^i < \theta_c = \sin^{-1}(1/\sqrt{\varepsilon_r})$, with $p = 1, \ldots, M^* = \mathrm{Int}[(\theta_c - \theta_0^t)/\alpha] \leq M$.

3. TD-UAPO DIFFRACTED FIELD

According to [1, 2, 5], two separate problems relevant to Ω_d and Ω_0 are considered for the evaluation of the TD diffracted field

$$\underline{e}^d(P,t) = \frac{1}{\sqrt{\rho}} \int_{t_0}^{t-\rho/c} d\left(t - \frac{\rho}{c} - \tau\right) \underline{e}^i(Q,\tau)\,\mathrm{d}\tau \qquad \text{if} \qquad t - \frac{\rho}{c} > t_0 \tag{1}$$

where in $\underline{e}^i$ is the incident field (forcing function), $t = t_0$ is the time at the diffraction point Q when the forcing function is turned on, d is TD diffraction coefficient and c is the speed of light in the considered observation region. In the hypothesis that the material parameters are assumed independent on the frequency, the following expressions result for the TD-UAPO diffraction coefficient.

3.1. Internal Region Ω_d

$$d_{\Omega_d} = (d_{S_0})_{\Omega_d} + (d_{S_n})_{\Omega_d} \tag{2}$$

$$\begin{aligned}(d_{S_0})_{\Omega_d} = \frac{-1}{2\sqrt{2\pi}} T_0 \Bigg\{ & \frac{\sin\phi - \cos\theta_0^t}{\cos\phi + \cos(\theta_0^t + \pi/2)} g\left(2\rho\cos^2\left(\frac{(2\pi-\phi) + (\theta_0^t + \pi/2)}{2}\right), t\right) \\ & + \sum_{\substack{m=1 \\ m\ even}}^{M} \left(\prod_{p=1}^{m-1} R_p\right) \frac{(1+R_m)\sin\phi + (1-R_m)\cos\theta_m^i}{\cos\phi + \cos(\theta_m^i + \pi/2)} g \\ & \left(2\rho\cos^2\left(\frac{(2\pi-\phi) + (\theta_m^i + \pi/2)}{2}\right), t\right)\Bigg\}\end{aligned} \tag{3}$$

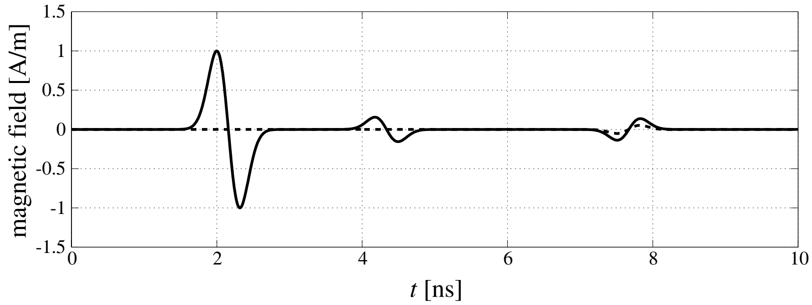
$$\begin{aligned}(d_{S_n})_{\Omega_d} = \frac{-1}{2\sqrt{2\pi}} T_0 \Bigg\{ & \sum_{\substack{m=1 \\ m\ odd}}^{M} \left(\prod_{p=1}^{m-1} R_p\right) \frac{(1-R_m)\cos\theta_m^i - (1+R_m)\sin(\phi+\alpha)}{\cos(\phi+\alpha) + \cos(\theta_m^i + \pi/2)} \\ & \times g\left(2\rho\cos^2\left(\frac{\phi - (2\pi-\alpha) + (\theta_m^i + \pi/2)}{2}\right), t\right)\Bigg\}\end{aligned} \tag{4}$$

3.2. External Region Ω_0

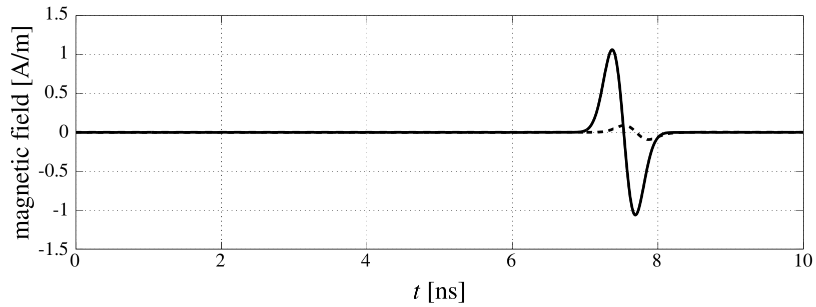
$$d_{\Omega_0} = (d_{S_0})_{\Omega_0} + (d_{S_n})_{\Omega_0} \tag{5}$$

$$\begin{aligned}(d_{S_0})_{\Omega_0} = \frac{-1}{2\sqrt{2\pi}} \Bigg\{ & \frac{(1-R_0)\sin\phi' - (1+R_0)\sin\phi}{\cos\phi + \cos\phi'} g\left(2\rho\cos^2\left(\frac{\phi \pm \phi'}{2}\right), t\right) \\ & - T_0 \sum_{\substack{m=1 \\ m\ even}}^{M^*} T_m \left(\prod_{p=1}^{m-1} R_p\right) \frac{\sin\phi + \cos\phi_m^t}{\cos\phi + \cos(\theta_m^t + \pi/2)} g\left(2\rho\cos^2\left(\frac{\phi \pm (\theta_m^t + \pi/2)}{2}\right), t\right)\Bigg\}\end{aligned} \tag{6}$$

$$\begin{aligned}(d_{S_n})_{\Omega_0} = -\frac{1}{2\sqrt{2\pi}} T_0 \Bigg\{ & \sum_{\substack{m=1 \\ m\ odd}}^{M*} T_m \left(\prod_{p=1}^{m-1} R_p\right) \frac{\sin(\phi+\alpha) - \cos\theta_m^t}{\cos(\phi+\alpha) + \cos(\theta_m^t + \pi/2)} \\ & \times g\left(2\rho\cos^2\left(\frac{(\phi+\alpha) \pm (\theta_m^t + \pi/2)}{2}\right), t\right)\Bigg\}\end{aligned} \tag{7}$$



(a)



(b)

Figure 3: Magnetic field amplitude: (a) $P(2\,\mathrm{m}, 10°)$; (b) $P(2\,\mathrm{m}, 310°)$.

Note that the sign $+$ $(-)$ must be used in (6) when $0 < \phi < \pi$ $(\pi < \phi < 2\pi - \alpha)$, and in (7) when $0 < \phi < \pi - \alpha$ $(\pi - \alpha < \phi < 2\pi - \alpha)$. If $\theta_p^t = \sin^{-1}(\sqrt{\varepsilon_r} \sin\theta_p^i)$, the reflection and transmission coefficients used in (3)–(4) and (6)–(7) are so defined:

$$T_0 = \frac{2\cos\theta^i}{\sqrt{\varepsilon_r}\cos\theta^i + \cos\theta_0^t}; \quad R_0 = \frac{\sqrt{\varepsilon_r}\cos\theta^i - \cos\theta_0^t}{\sqrt{\varepsilon_r}\cos\theta^i + \cos\theta_0^t} \tag{8}$$

$$T_p = \frac{2\sqrt{\varepsilon_r}\cos\theta_p^i}{\cos\theta_p^i + \sqrt{\varepsilon_r}\cos\theta_p^t}; \quad R_p = \frac{\cos\theta_p^i - \sqrt{\varepsilon_r}\cos\theta_p^t}{\cos\theta_p^i + \sqrt{\varepsilon_r}\cos\theta_p^t} \tag{9}$$

Moreover,

$$g(X, t) = \frac{X}{\sqrt{\pi c t}\,(t + X/c)} \tag{10}$$

4. NUMERICAL TESTS

The reported examples refer to a lossless wedge characterized by $\varepsilon_r = 2$ and $\alpha = 15°$ in the case of H-polarized plane wave impinging at $\phi' = 110°$. Fig. 3(a) shows the field contributions arriving at $P(2\,\mathrm{m}, 10°)$. In order of arrival, it is possible to identify the Geometrical Optics (GO) contributions (incident, specular reflection, and transmission-reflection-transmission waveforms in solid line), and the diffraction one (dashed line). Note that P is very close to the shadow boundary (10.73 degrees) for the transmission-reflection-transmission waveform. The field contributions arriving at $P(2\,\mathrm{m}, 310°)$ are plotted in Fig. 3(b). Only the transmission-transmission waveform contributes to the GO field.

REFERENCES

1. Gennarelli, G. and G. Riccio, "Time domain diffraction by a right-angled penetrable wedge," *IEEE Trans. Antennas Propagat.*, Vol. 60, 2829–2833, 2012.
2. Gennarelli, G. and G. Riccio, "Obtuse-angled penetrable wedges: A time domain solution for the diffraction coefficients," *Journal of Electromagnetic Waves and Applications*, Vol. 27, No. 16, 2020–2028, 2013.
3. Gennarelli, G. and G. Riccio, "A uniform asymptotic solution for diffraction by a right-angled dielectric wedge," *IEEE Trans. Antennas Propagat.*, Vol. 59, 898–903, 2011.

4. Gennarelli, G. and G. Riccio, "Plane-wave diffraction by an obtuse-angled dielectric wedge," *J. Opt. Soc. Am. A*, Vol. 28, 627–632, 2011.
5. Frongillo, M., G. Gennarelli, and G. Riccio, "Diffraction by an acute-angled dielectric wedge: The H-polarization case," *PIERS Proceedings*, 846–849, Stockholm, Sweden, Aug. 12–15, 2013.
6. Kouyoumjian, R. G. and P. H. Pathak, "A uniform geometrical theory of diffraction for an edge in a perfectly conducting surface," *Proc. of IEEE*, Vol. 62, 1448–1461, 1974.