A Novel Knowledge-aided Approach for Training Data Selection

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Abstract— This paper proposes a novel knowledge-aided approach for selecting training data in space-time adaptive processing (STAP) whose performance suffers from a severe degradation in heterogeneous interference environment. The proposed approach exploits distances between interference covariance matrices of training data and tested data as the measurements of interference statistical similarities, which helps us gain a deeper insight into the statistics from the point of geometry. Three distances including Euclidean distance, Riemannian distance and a physical distance are combined to distinguish various heterogeneous phenomenons. A prior knowledge is employed in estimating the interference covariance matrices of both training data and tested data. Simulation results illustrate the effectiveness of the proposed approach.

1. INTRODUCTION

Space-time adaptive processing (STAP) is an important technique for suppressing strong clutter in airborne radar systems which designs an adaptive filter by learning the statistical characteristics of the interference environment [1]. Traditional STAP implementation employs training data near the range under test to estimate interference covariance matrix and generate the optimal weight vector under the assumption that they are independent and identically distributed with the tested data [2]. However, realistic interference environments usually appear heterogeneous, thereby violating the assumption, leading to covariance estimation errors, adaptive weight vector mismatch and consequently, degradation in STAP performance [3]. One useful approach to mitigate the effect of heterogeneity is screening the training data before covariance estimation.

Lots of screening methods have been proposed, such as power-selected training (PST) [4], the combination of phase and power-selected training [5] and the nonhomogeneity detector (NHD) [6]. PST selects the most powerful range cells to generate a deep clutter null for avoiding clutter residues, and phase and power-selected training attempts to remove those chosen powerful range cells which may potentially include target-like signals. Both of these two techniques make the assumption that the interference heterogeneity only lies in the power. NHD, however, takes the interference statistical properties into consideration instead of power, hence is a better criterion in theory. But the performance of NHD method depends upon a reliable selection metric [7]. Performance loss may occur when the metric is estimated using training data due to its heterogeneity.

Recently, distances between interference covariance matrices of the training data and tested data are put forward as one of the means to measure the statistical similarities within them, which provides a method of judging the interference heterogeneity from the perspective of geometry [8]. The interference covariance matrix of tested data is usually estimated using sample covariance matrix whose performance may degrade significantly when sample covariance matrix contains heterogeneous range cells. A knowledge-aided interference covariance matrix estimation based on clutter model is employed in this paper to estimate the statistical characteristic of tested data, which mitigates the heterogeneity from the root since it exploits no information of training data. In addition, a prior knowledge of noise spectral density is used to construct a positive-definite interference covariance matrix of training data which is necessary in calculating distances. Three different distances containing Euclidean distance, Riemannian distance and a physical distance are combined to separate all kinds of interference heterogeneities mentioned above. Simulation results illustrate the effectiveness.

2. STAP SIGNAL MODEL

Consider a uniform linear array radar system with N antenna elements. The radar transmits M pulses in a coherent processing interval (CPI), and there are K fast-time samples for each returned pulse. STAP is inherently a two-dimensional filtering process in space and slow-time plane which adaptively designs a weight vector $\mathbf{w}_k \in C^{MN\times 1}$ for the space and slow-time data snapshot $\mathbf{x}_k \in C^{MN\times 1}$ in the kth range cell. According to the usual linearly-constrained minimum-variance (LCMV) principle, the optimal weight vector for the kth range cell can be expressed as follows [1]

$$\mathbf{w}_{opt} = \mu \mathbf{R}_k^{-1}\mathbf{v} \tag{1}$$

where $\mu = 1/(\mathbf{v}^H \mathbf{R}_k^{-1} \mathbf{v})$ is a constant, $\mathbf{R}_k$ is the interference covariance matrix in the kth range cell and $\mathbf{v}$ is the space-time steering vector for a moving target with a given radial velocity and the direction of arrival.

However, the interference covariance matrix is usually unknown in reality, thereby should be estimated from measurement observations or other prior knowledge. The interference includes clutter, noise and jammer. Conventionally, the interference covariance is estimated from multiple space-time data snapshots of range cells nearby to the range under test (RUT) [1], defined as sample covariance matrix

$$\hat{\mathbf{R}} = \frac{1}{N_s} \sum_{i=1}^{N_s} \mathbf{x}_i \mathbf{x}_i^H \tag{2}$$

where $\mathbf{x}_i \in C^{MN \times 1}$ is the ith training data, N_s is the number of training data, and the superscript H denotes conjugate transpose.

Under ideal homogenous interference environment, if $2MN$ independent and identically distributed (IID) training data are available, the performance of this technique denoted as SMI can approach the ideal, known-covariance case to within 3 dB [2]. Unfortunately, interference environments of real data appear heterogeneous or non-IID, which deviates from the assumptions, thereby leading to covariance matrix estimation errors, adaptive filter mismatch, and consequently, degradation in performance of SMI. To mitigate the effect of heterogeneity, training data should be screened before covariance matrix estimation to select range cells which share similar statistical characteristics with the range under test.

3. TRAINING DATA SELECTION METRIC

The goal of training data selection metric is to measure the interference statistical similarity between training data and tested data. The best criterion is possibly the interference probability density function which completely represents the statistical characteristics. However, the probability density function is hardly to obtain accurately in practical applications. Instead, interference covariance matrix, as an important mathematical characteristic, gains a lot of attention and is widely used. In the angle of geometry, the similarity between two covariance matrices can be measured by the distance between them.

The most common distance is Euclidean distance which calculates the Frobenius norm of difference between the interference covariance matrix of training data denoted as $\mathbf{R}_1$ and the interference covariance matrix of tested data denoted as $\mathbf{R}_0$

$$d_1^2 = \|\mathbf{R}_1 - \mathbf{R}_0\|_F^2 \tag{3}$$

Clearly, Euclidean distance only cares about the power difference between two covariance matrices, and doesn't make use of the covariance structure.

Another kind of distance is Riemannian distance which is defined as

$$d_2^2 = \left\| \log \left(\mathbf{R}_0^{-1/2} \mathbf{R}_1 \mathbf{R}_0^{-1/2} \right) \right\|_F^2 \tag{4}$$

where $\mathbf{R}_0$ and $\mathbf{R}_1$ are constrained to positive-definite matrices. Riemannian distance is the foundation of information geometry and is theoretically more effective than Euclidean distance to measure the statistical similarity since covariance structure information is considered.

In addition, a physical distance is defined in [8] by

$$d_3^2 = \left\| \mathbf{R}_0^{-1/2} \mathbf{R}_1 \mathbf{R}_0^{-1/2} - \mathbf{I} \right\|_F^2 \tag{5}$$

where $\mathbf{I}$ is the identity matrix. Under ideal conditions, this physical distance is a first order Taylor approximation of Riemannian distance. Its effectiveness has been verified in [8] using measured data.

4. KNOWLEDGE-AIDED COVARIANCE MATRIX ESTIMATION

The key point in the calculation of aforementioned distances is the interference covariance matrix estimations of both training data and tested data since they are unknown. Traditionally, the interference covariance matrix of tested data is estimated by sample covariance matrix according to Equation (2). However, it suffers from errors when training data is heterogeneous. An alternative way to the estimation is based on the clutter model [1] whose structure is given by (jammer is not considered for simplicity)

$$\hat{\mathbf{R}} = \mathbf{R}_c + \mathbf{R}_n = \sum_{p=1}^{N_c} |\alpha_p|^2 \mathbf{v}_p \mathbf{v}_p^H \circ \mathbf{T}_p + \sigma_n^2 \mathbf{I} \tag{6}$$

where $\mathbf{R}_c$ and $\mathbf{R}_n$ are the covariance matrix of clutter and noise, respectively, N_c is the number of independent clutter patches in tested data, α_p and $\mathbf{v}_p$ are the complex amplitude and space-time steering vector, respectively, for the pth ground clutter patch, $\circ$ is the Hadamard matrix product, $\mathbf{T}_p$ is a covariance matrix taper (CMT) that allows for incorporation of subspace leakage effects such as internal clutter motion (ICM) and channel error, σ_n^2 is the average noise power per measurement. Covariance matrix constructed in this way only takes statistics of the range under test into consideration. As a result, it can mitigate fundamentally the effect of heterogeneity brought by training data.

For a clutter patch with azimuth angle θ and depression angle φ measured from the reference of platform moving direction, its space-time steering vector can be expressed as

$$\mathbf{v} = \mathbf{v}_t \otimes \mathbf{v}_s \tag{7}$$

$$\mathbf{v}_t = [1, \exp(jw_d), \ldots, \exp(j(M-1)w_d)]^T, \quad \mathbf{v}_s = [1, \exp(jw_s), \ldots, \exp(j(N-1)w_s)]^T \tag{8}$$

$$w_d = \frac{4\pi v}{\lambda f_r} \cos(\theta + \alpha) \cos\varphi, \quad w_s = \frac{2\pi d}{\lambda} \cos\theta \cos\varphi \tag{9}$$

where M is the number of pulses transmitted in a CPI, N is the number of antenna elements, d is the distance between adjacent antenna elements, f_r and λ are pulse repetition frequency and wavelength of the transmitted signal, respectively, v is the velocity of airborne radar platform and α is the crab angle, $\otimes$ is Kronecker product, $\mathbf{v}_t$ and $\mathbf{v}_s$ are time and space steering vector whereas w_d and w_s are normalized Doppler frequency and spatial frequency, respectively. If parameters of radar system, antenna platform are known as a prior or can be measured conveniently, the space-time steering vector of each clutter patch can be obtained according to their locations. The amplitude of each clutter patch α_p can be estimated directly from a prior knowledge such as SAR images [9] or indirectly form measured data [10]. CMT $\mathbf{T}_p$ can be similarly determined indirectly from measured data [10] under some proper models or directly from a prior knowledge. For example, a widely accepted Gaussian model for ICM only requires the specification of wavelength λ, pulse repetition frequency f_r and variance of the clutter spectral spread [10]. The power of the noise σ_n^2 can be accurately estimated through calibration procedures in practice.

The interference covariance matrix estimation of training data is generally a function of training data, and a possible choice could be the sample covariance matrix, namely $\mathbf{R}_i = \mathbf{x}_i \mathbf{x}_i^H$. However, it can't meet the requirement of positive-definiteness which is necessary to calculate the Riemannian distance, hence a technique proposed in [11] is exploited here to generate a modified positive-definite sample covariance matrix

$$\hat{\mathbf{R}}_i = \mathbf{U}_i \mathbf{\Lambda}_i \mathbf{U}_i^H \tag{10}$$

where $\mathbf{\Lambda}_i = \mathrm{diag}([\lambda_i, \sigma_n^2, \ldots, \sigma_n^2])$, with $\lambda_i = \max(\sigma_n^2, \|x_i\|^2)$ and σ_n^2 being the average noise power, $\mathbf{U}_i$ is a unitary matrix consisting of the eigenvectors of $\mathbf{x}_i \mathbf{x}_i^H$ whose first eigenvector corresponds to the eigenvalue $\|x_i\|^2$.

5. SIMULATIONS

The simulation parameters are as follows. Radar is flying at an altitude of 885 m and the distance between the radar and the center range is 10150 m. The radar platform moves at the velocity of 140 m/s. The pulse-repetition frequency of the transmitted signal f_r is 2800 Hz and the wavelength λ is 0.2 m. The number of pulses in a CPI M is 10 and the number of antenna receive elements N is 12. The distance between adjacent receive elements d equals to half of the wavelength. There are 151 range cells in total and the target is placed in the 76th range cell. Without loss of generality, the noise power level is assumed to be 0 dB, and the target power is set to be 10 dB.

The interference statistical characteristics of each range cell are listed in Table 1. All kinds of heterogeneities consisting of power differences, target-like signals, clutter discretes and spectral dissimilarities are taken into consideration. And the spectral dissimilarities are modeled using a well-known CMT model which is regularly used to describe channel error

$$\mathbf{T} = \mathbf{I}_{M\times M} \otimes \left(\sin c\left(\frac{(m-n)\Delta}{\pi}\right)\right) \qquad m, n = 1, 2, \ldots, N \tag{11}$$

where Δ is assumed to be a uniformly distributed random value among 0 and 0.1 in range cells from 121 to 151 and 0 in other range cells. In addition, we assume that each target-like signal has the same azimuth angle and velocity and so as the clutter discretes.

Table 1: Interference statistical characteristics of each range cell.

Range cell	**Interference statistical characteristics**
1–30, 61–151	$\sigma_c^2 = 30\,\mathrm{dB}$
31–60	$\sigma_c^2 = 35\,\mathrm{dB}$
50, 60, 70, 80, 90	Target-like signals, $\sigma_t^2 = 20\,\mathrm{dB}$
45, 55, 65, 75, 85	Target-like signals, $\sigma_t^2 = 35\,\mathrm{dB}$
40, 90, 100, 110, 120	Clutter discretes, $\sigma_d^2 = 40\,\mathrm{dB}$
35, 85, 95, 105, 115	Clutter discretes, $\sigma_d^2 = 45\,\mathrm{dB}$
121–151	CMT is considered

Three kinds of distances for each range cell except the 76th one are calculated using the sample covariance matrix and knowledge-aided covariance matrix estimation respectively. Figure 1 shows the results under the assumption that knowledge is accurate. The blue solid line denotes distances using knowledge-aid covariance matrix estimation while the red dashed one denotes distances using sample covariance matrix estimation. The result of Euclidean distance shows that both of the covariance matrix estimation work well, yet only power difference can be detected, weak target-like signals and spectral dissimilarities is hardly to be found. Moreover, it is impossible to distinguish between target-like signals and clutter discretes. The results of Riemannian distance and physical distance, however, show that knowledge-aided covariance estimation performs better than sample covariance estimation which fails to detect power differences, target-like signals and clutter discretes. Both of the knowledge-aided Riemannian distance and physical distance can well find the target-like signals even when their power is 10 dB lower than clutter and they can detect strong clutter discretes. Knowledge-aided Riemannian distance fails to detect weak clutter discretes, and it is interesting that the knowledge-aided Riemannian distances of clutter discretes are lower than ideal values while those of target-like signals are higher than ideal values. In addition, if target-like signals and clutter discretes exist simultaneously in one range cell, the statistical properties of target-like signals occupy the dominant position. Spectral dissimilarities are obvious comparing these three distances, and those range cells should be removed.

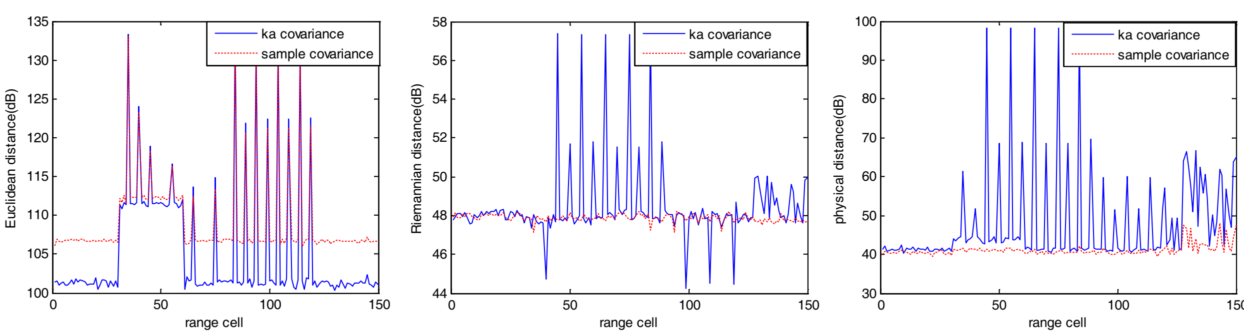

Figure 1: Three distances (Euclidean, Riemannian, physical) using sample covariance matrix and knowledge-aided covariance matrix under the assumption that knowledge is accurate.

However, knowledge of clutter parameters is not always accurate in practice, and power error is one of the most common situations. Figure 2 shows the condition that the clutter power is overestimated by 10 dB while Figure 3 displays the results that the clutter power is underestimated by 10 dB. Both of the figures testify the effectiveness of knowledge-aided distances although there is a 10 dB clutter power error. Actually, the error enhances the performance slightly in detecting clutter discretes for knowledge-aided Riemannian distance either in power overestimation or underestimation condition while it only improves the performance of knowledge-aided physical distance in power underestimation condition and reduces the performance moderately in power overestimation condition.

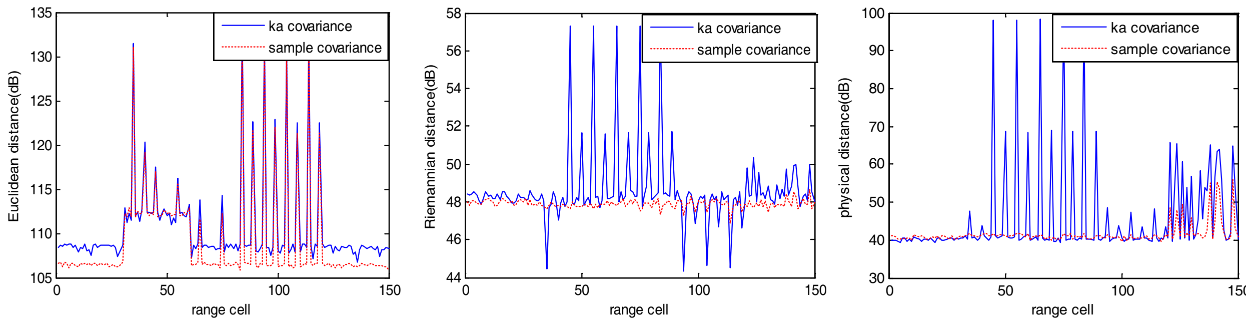

Figure 2: Three distances (Euclidean, Riemannian, physical) under the condition that the clutter power is 10 dB higher than actual value.

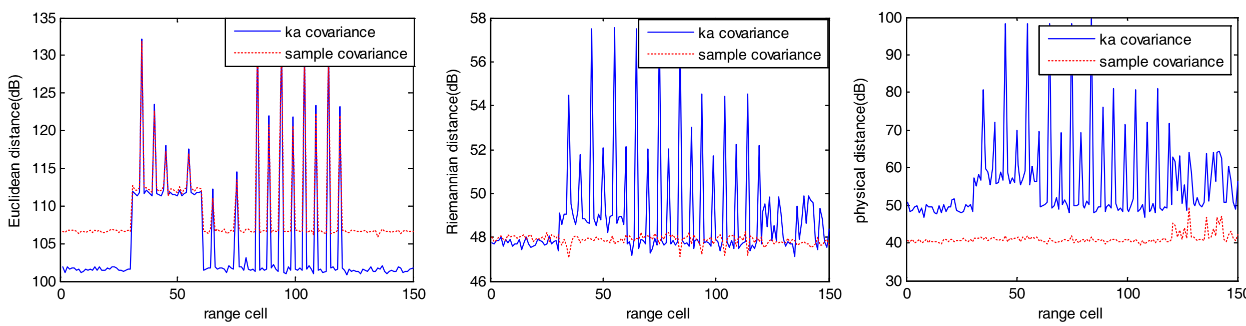

Figure 3: Three distances (Euclidean, Riemannian, physical) under the condition that the clutter power is 10 dB lower than actual value.

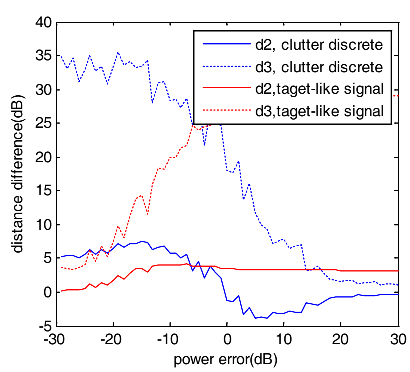

Figure 4: Riemannian and physical distance of clutter discrete and target-like signal compared with the ideal values under different clutter power estimations.

Next, based on the aforementioned results, the knowledge-aided Riemannian and physical distances of clutter discrete and target-like signal compared with ideal values under different clutter power estimations are discussed. Assume the power of clutter discrete and target-like signal are 45 dB and 20 dB, respectively. Figure 4 shows that the Riemannian distance of clutter discrete is higher than the ideal value at first and then lower than the ideal value with the change of clutter power estimation, yet that of target-like signal is always higher than ideal values and approximately invariant when the clutter power is overestimated. Besides, the difference between physical distance of clutter discrete and ideal value presents a tendency of descent while that of target-like signal presents an opposite ascendant tendency. Both of the properties can be used to distinguish clutter discrete and target-like signal, which is very useful since range cells containing target-like signals should be removed to avoid target elimination, nevertheless range cells containing clutter discretes sometimes should be exploited to drive a deep null.

6. CONCLUSIONS AND FUTURE WORK

In this paper, we measure the statistical similarities of data in different range cells from the perspective of geometry. Distances between interference covariance matrices of different range cells containing Euclidean distance, Riemannian distance and a physical distance are selected as the training data selection metrics. With regard to the interference covariance matrix estimation, two methods including sample covariance matrix estimation and a knowledge-aided covariance matrix estimation based on clutter model are compared. Simulation results show that different heterogeneous phenomenon consisting of power differences, target-like signals, clutter discretes and spectral dissimilarities can be detected and separated with the combination of three distances. With this guidance, training data can be well selected.

Significant future research is still required. Knowledge error except clutter power error should be considered and their impacts should be studied. In addition, our proposed approach should be testified using measured data rather than simulated data, and improvement of our approach should be investigated according to the effects analyzed from the measured data.

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