Design and Analysis of Planar Phased MIMO Antenna for Radar Applications

Nour El-Din Ismail, Sherif Hanafy Mahmoud, Ahmed Hamed, and Alaa Hafez
Faculty of Engineering, Alexandria University, Alexandria, Egypt

Abstract— Phased-multiple-input multiple-output (phased-MIMO) radar using one-dimensional transmit arrays has been thoroughly investigated in the literature. In this paper, we consider two-dimensional phased-MIMO radar array which is called planar-phased-MIMO radar. This new technique aggregates the advantages of the linear-phased-MIMO radar without sacrificing either the main advantage of the planar-phased-array radar, which is the coherent processing gain, or the main advantage of the planar MIMO array radar, which is the diversity processing gain. The essence of the proposed technique is to partition the planar transmit array into a number of planar subarrays that are allowed to overlap. Then, each subarray is used to coherently transmit a waveform which is orthogonal to the waveforms transmitted by other subarrays. Coherent processing gain can be achieved by designing a weight matrix for each subarray to form a beam towards a certain direction in space. Moreover, the subarrays are combined jointly to form a planar-MIMO radar resulting in higher angular resolution capabilities. Substantial improvements is offered by the proposed planar-phased-MIMO radar technique with respect to the linear-phased-MIMO, planar-MIMO and planar-phased-array radar techniques. The achieved improvements are demonstrated analytically and by simulations through analyzing the corresponding beampatterns, the resultant peak side lobe level, mean side lobe level, and directivity. Both analytical and simulation results validate the effectiveness of the proposed planar-phased-MIMO radar.

1. INTRODUCTION

Phased-array technique have been widely employed in different radars to provide electronic beam steering of radiated or received electromagnetic signals operating at the same frequency [1–3]. Controlling the phase shifts across elements, the beam can be steered to the desired direction. Detecting/tracking weak target echoes and suppressing sidelobe interferences from other directions can be obtained due to its high directional gain. The desire for new more advanced antenna array technologies has been derived by the requirements of many emerging applications [4–8]. Multiple input multiple output (MIMO) radar systems are next-generation radar systems with multiple transmit and receive apertures, equipped with the capability of transmitting arbitrary and differing signals at each transmit aperture. The emerging MIMO radar literature can be broken into two broad areas. The first is characterized by spatially distributed assets that are not phase-coherent on transmitting or receiving, that takes advantage of spatial diversity to gain multiple views of a target and achieves improving in stability of statistical hypothesis tests for target detection. These approaches are often characterized as statistical MIMO. Fishler et al. [9] introduces the statistical MIMO radar concept which provides great improvements over other types of array radars. Haimovich et al. [10] reviews some recent work on MIMO radar with widely separated antennas and it is shown that with noncoherent processing, a target's RCS spatial variations can be exploited to obtain a diversity gain for target detection. Bliss et al. [11], describes the theory behind the improved surveillance radar performance and illustrates this with measurements from experimental MIMO radars. In contrast, there is a MIMO radar literature built around an assumption of colocated assets like one might find with an antenna array or phased-array radar.

Much interesting work has been done in this arena by other researches [12–14] introduce performance advantages of colocated MIMO radars.

A. Hassanien et al. [15] proposes a new technique for MIMO radar with colocated antennas which called linear phased-MIMO radar. This technique enjoys the advantages of the MIMO radar without sacrificing the main advantage of the phased-array radar which is the coherent processing gain at the transmitting side. This paper studies the design of a new technique for partition the planar transmit array into a number of planar subarrays that are allowed to overlap. Each subarray is used to coherently transmit a waveform which is orthogonal to the waveforms transmitted by other subarrays. Then, compares this design with previous techniques through analyzing the corresponding beampatterns, and directivity. Significant improvements offered by the planar-phased-MIMO radar technique.

2. PLANNER PHASED-MIMO MODEL

In this section, the transmit array is divided into multiple $(K \times L)$ subarrays which can be disjoint or overlapped, as shown in Figure 1. Each transmit sub-array can be composed of any number of elements ranging from 1×1 to $M \times N$. However, we will partition the sub-array with different values of $K \times L$ with different numbers of transmitting elements in each sub-array following the rule $M_{KL} = M_K \times N_L$ where $M_K = (M - K + 1)$, $N_L = (N - L + 1)$ and M_{KL} is the number of elements in each sub-array. A beam can be formed by each sub-array towards a certain direction. The beamforming weight vector can be properly designed to maximize the coherent processing gain [16]. At the same time, different waveforms are transmitted by different sub-arrays. Suppose the (k, l)th sub array consists of $M_{KL} < MN$ transmit elements, the equivalent baseband signal model of the (k, l)th sub array can be modeled as,

$$S_{k,l}(t) = \sqrt{\frac{MN}{KL}}\psi_{k,l}(t)\tilde{w}^*_{k,l} \quad k = 1, 2, \ldots, K; \quad l = 1, 2, \ldots, L \tag{1}$$

where $K \times L$ is the number of sub-arrays, $\tilde{W}_{k,l}$ is the $M \times N$ unit-norm complex matrix which consists of M_{KL} beamforming weights corresponding to the active antennas of the (k, l)th sub-array, that is, the number of nonzero in $\tilde{W}_{k,l}$ equals to M_{KL} and the number of zeros equals to $(M \times N) - M_{KL}$. Note that $\sqrt{MN/KL}$ is used to obtain an identical transmission power constraint for subsequent comparison, which means the transmit energy within one pulse repetition interval (PRI) is given by

$$E_{k,l} = \int_{Tp} S^H_{k,l}(t)S_{k,l}(t)dt = \frac{MN}{KL} \tag{2}$$

This means that the total transmitted energy for the phased-MIMO radar within one radar pulse is equal to MN [17]. The signal reflected by a hypothetical target located at direction in the far-field can be then modeled as

$$\begin{aligned} S_{r(t,\theta,\emptyset)} &= \sqrt{\frac{MN}{KL}}\sigma_s\left(\theta,\emptyset\right)\sum\nolimits_{k=1}^{K}\sum\nolimits_{l=1}^{L}\tilde{W}^*_{k,l}\tilde{a}_{k,l}\left(\theta,\emptyset\right)\psi_{k,l}(t) \\ &= \sqrt{\frac{MN}{KL}}\sigma_s\left(\theta,\emptyset\right)\sum\nolimits_{k=1}^{K}\sum\nolimits_{l=1}^{L}W^*_{k,l}a_{k,l}\left(\theta,\emptyset\right)e^{-j\tau_{k,l}(\theta,\emptyset)}\psi_{k,l}(t) \end{aligned} \tag{3}$$

where σ_s is the reflection coefficient of the hypothetical target, $W_{k,l}$ and $a_{k,l}(\theta, \emptyset)$ are the $M_K \times N_L$ beamforming matrix and transmit steering matrix, respectively, which contain only the elements corresponding to the active antennas of the (k, l)th sub-array, $\tau_{k,l}(\theta, \emptyset)$ is the time required for the

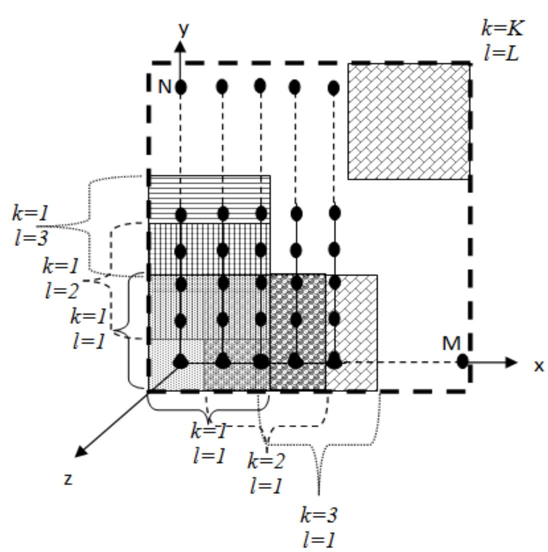

Figure 1: Illustration of the planar phased-MIMO array.

wave to travel across the spatial displacement between the first element of the transmit array and the first element of the (k,l)th sub-array, and stands for the Hadamard (element-wise) product. In (3), we assume that the first element of $a_{k,l}(\theta,\emptyset)$ is taken as a reference element. Let us introduce the $K \times L$ transmit coherent processing matrix.

$$C(\theta,\emptyset) = \begin{bmatrix} \sum_{m_k=1}^{M_K}\sum_{n_l=1}^{N_L} W_{1,1}^{*} a_{1,1}(\theta,\emptyset) & \dots & \sum_{m_k=1}^{M_K}\sum_{n_l=1}^{N_L} W_{1,L}^{*} a_{1,L}(\theta,\emptyset) \\ \vdots & \ddots & \vdots \\ \sum_{m_k=1}^{M_K}\sum_{n_l=1}^{N_L} W_{K,1}^{*} a_{K,1}(\theta,\emptyset) & \dots & \sum_{m_k=1}^{M_K}\sum_{n_l=1}^{N_L} W_{K,L}^{*} a_{K,L}(\theta,\emptyset) \end{bmatrix} \tag{4}$$

where $m_k = 1,\dots,M_K$ and $n_l = 1,\dots,N_L$ are the orders of elements in each subarray in x-direction and y-direction respectively.

The $K \times L$ waveform diversity matrix is

$$d(\theta,\emptyset) = \begin{bmatrix} e^{-j\tau_{1,1}(\theta,\emptyset)} & \dots & e^{-j\tau_{1,L}(\theta,\emptyset)} \\ \vdots & \ddots & \vdots \\ e^{-j\tau_{K,1}(\theta,\emptyset)} & \dots & e^{-j\tau_{K,L}(\theta,\emptyset)} \end{bmatrix} \tag{5}$$

Then, the reflected signal (3) can be rewritten as,

$$S_{r(t,\theta,\emptyset)} = \sqrt{\frac{MN}{KL}}\sigma(\theta,\emptyset)\left(C(\theta,\emptyset)\odot d(\theta,\emptyset)\right)\odot\psi_{k,l}(t) \tag{6}$$

where $\psi_{k,l}(t) = \begin{bmatrix} \psi_{1,1}(t) & \dots & \psi_{1,L}(t) \\ \vdots & \ddots & \vdots \\ \psi_{K,1}(t) & \dots & \psi_{K,L}(t) \end{bmatrix}$ is the $K \times L$ matrix of waveforms. Assuming that the target of interest is observed in the background of D interfering targets with reflection coefficient $\{\sigma_i\}_{i=1}^{D}$ and locations $\{\theta_i,\emptyset_i\}_{i=1}^{D}$, the $M_r \times N_r$ received complex matrix of array observations can be written as,

$$X(t) = S_r(t,\theta_s,\emptyset_s)\,b(\theta_s,\emptyset_s) + \sum_{i=1}^{D} S_r(t,\theta_i,\emptyset_i)\,b(\theta_i,\emptyset_i) + n(t) \tag{7}$$

where $S_r(t,\theta_s,\emptyset_s)$ and $S_r(t,\theta_i,\emptyset_i)$ are defined as in (6) and $b(\theta,\emptyset)$ is the receive steering matrix associated at angles $(\theta,\emptyset)$. By matched-filtering $X(t)$ to each of the waveforms $\{\psi_{k,l}\}_{k,l=1,1}^{K,L}$ we can form the $KM_r \times LN_r$ virtual data matrix,

$$y = \begin{bmatrix} X_{1,1} & \dots & X_{1,L} \\ \vdots & \ddots & \vdots \\ X_{K,1} & \dots & X_{K,L} \end{bmatrix} = \sqrt{\frac{MN}{KL}}\sigma_s u(\theta_s,\emptyset_s) + \sum_{i=1}^{D}\sqrt{\frac{MN}{KL}}\sigma_i u(\theta_i,\emptyset_i) + \tilde{n} \tag{8}$$

where the $KM_r \times LN_r$ matrix

$$u(\theta,\emptyset) = \left(c(\theta,\emptyset)\odot d(\theta,\emptyset)\right)\otimes b(\theta,\emptyset) \tag{9}$$

is the virtual steering matrix associated with direction $(\theta,\emptyset)$ and $\tilde{n}$ is the $KM_r \times LN_r$ noise term (noise received by $M_r \times N_r$ receivers containing KL matched filters at each receiver) whose covariance is given by $\tilde{R}_n = \sigma_n^2 I_{KM_rLN_r}$ where σ_n^2 is the noise power.

2.1. Planar Phased-MIMO Beamforming

In the case of non-adaptive beamforming, the corresponding conventional beamforming weight matrix is given for the (k,l)th transmitting subarray as [18]:

$$w_{k,l} = \frac{a_{k,l}(\theta_s,\emptyset_s)}{\left\|a_{k,l}(\theta_s,\emptyset_s)\right\|_F} = \frac{a_{k,l}(\theta_s,\emptyset s)}{\sqrt{M-k+1}\sqrt{N-l+1}}, \quad k=1,\dots,K \quad l=1,\dots,L \tag{10}$$

where $\|a_{k,l}(\theta_s,\emptyset_s)\|_F$ is the Frobenius of matrix $a_{k,l}(\theta_s,\emptyset_s)$ and equals to $\sqrt{\sum_{i=1}^{M-k+1}\sum_{j=1}^{N-l+1}\left(a_{i,j}(\theta_s,\emptyset_s)\right)^2}$.

For the receiving array, the conventional beamforming weight matrix is given by:

$$w_d = [c(\theta_s, \emptyset_s) \odot d(\theta_s, \emptyset_s)] \otimes b(\theta_s, \emptyset_s) \tag{11}$$

Let $G(\theta, \emptyset)$ be the normalized beampattern

$$G(\theta, \emptyset) = \frac{\left| \sum_1^{KM_r} \sum_1^{LN_r} \left(w_d^* \odot u(\theta, \emptyset) \right) \right|^2}{\left| \sum_1^{KM_r} \sum_1^{LN_r} \left(w_d^* \odot u(\theta_s, \emptyset_s) \right) \right|^2} = \frac{\left| \sum_1^{KM_r} \sum_1^{LN_r} \left(u^*(\theta_s, \emptyset_s) \odot u(\theta, \emptyset) \right) \right|^2}{\| u(\theta_s, \emptyset_s) \|_F^4} \tag{12}$$

Considering the special case of a uniform planar array (UPA), we have $\sum_1^{M_K} \sum_1^{N_L} a_{1,1}^*(\theta_s, \emptyset_s) \odot a_{1,1}(\theta, \emptyset) = \sum_1^{M_K} \sum_1^{N_L} a_{1,2}^*(\theta_s, \emptyset_s) \odot a_{1,2}(\theta, \emptyset) = \ldots = \sum_1^{M_K} \sum_1^{N_L} a_{2,1}^*(\theta_s, \emptyset_s) \odot a_{2,1}(\theta, \emptyset) = \ldots = \sum_1^{M_K} \sum_1^{N_L} a_{K,L}^*(\theta_s, \emptyset_s) \odot a_{K,L}(\theta, \emptyset)$.

Using (12), the beampattern of the phased-MIMO radar for UPA with partitioning to KL transmit sub-arrays can be written as

$$G_{k,L}(\theta, \emptyset) = \frac{\left| \left(\sum_1^{M_K} \sum_1^{N_L} \left(a_{k,l}^*(\theta_s, \emptyset_s) \odot a_{k,l}(\theta, \emptyset) \right) \right) \left(\sum_1^{KM_r} \sum_1^{LN_r} \left[(d(\theta_s, \emptyset_s) \otimes b(\theta_s, \emptyset_s))^* \odot (d(\theta, \emptyset) \otimes b(\theta, \emptyset)) \right] \right) \right|^2}{\left\| a_{k,l}^*(\theta_s, \emptyset_s) \right\|_F^4 \| d(\theta_s, \emptyset_s) \otimes b(\theta_s, \emptyset_s) \|_F^4} \tag{13}$$

After some algebra and using the facts that $\|a_{K,l}(\theta_s, \emptyset_s)\|_F^2 = (M-K+1)(N-L+1)$, $\|d(\theta_s, \emptyset_s)\|_F^2 = KL$, and $\|b(\theta_s, \emptyset_s)\|_F^2 = M_r N_r$, the beampattern (13) can be rewritten as

$$G_{k,L}(\theta, \emptyset) = C_{k,L}(\theta, \emptyset) D_{k,L}(\theta, \emptyset) \cdot R(\theta, \emptyset) \tag{14}$$

where $C_{k,L}(\theta, \emptyset) \triangleq \frac{|\sum_1^{M_K} \sum_1^{N_L} a_{k,l}^*(\theta_s, \emptyset_s) \odot a_{k,l}(\theta, \emptyset)|2}{(M-K+1)^2(N-L+1)^2}$ is the transmit beampattern, $D_{k,L}(\theta, \emptyset) \triangleq \frac{|\sum_1^{K} \sum_1^{L} d_{k,l}^*(\theta_s, \emptyset_s) \odot d_{k,l}(\theta, \emptyset)|^2}{K^2 L^2}$ is the waveform diversity beampattern, and $R(\theta, \emptyset) \triangleq \frac{|\sum_1^{M_r} \sum_1^{N_r} b^*(\theta_s, \emptyset_s) \odot b(\theta, \emptyset)|^2}{M_r^2 N_r^2}$ is the receive beampattern. Therefore, the overall beampattern (14) of the planar phased-MIMO radar with transmitting UPA can be seen as the product of three individual beampatterns.

3. SIMULATION RESULTS

The simulation assumes that the transmitting and receiving antennas are spaced half a wavelength apart from each other at the receiving end. The additive noise is modeled as a complex Gaussian zero-mean spatially and temporally white random sequence that has identical variances in each array sensor. The target of interest is assumed to reflect a plane-wave that impinges on the array from a direction of elevation angle $\theta_s = 30°$ and azimuth angle $\phi_s = 120°$. This simulation examines the received beam pattern of the transmit/receive beamformer for the different cases when the transmit/receive antennas are planar phased-array, planar MIMO, linear phased-MIMO, and also planar phased-MIMO. Figures 2, 3, 4, and 5 show the three-dimensional received beampatterns for the four different techniques, respectively. At Figures 2 and 3, both planar phased-array and planar MIMO have the same beam patterns. This occurs due to the equality of the coherent gain of the planar phased-array and the diversity gain of the planar MIMO.

Figure 4 shows the linear phased-MIMO beam pattern while Figure 5 shows the planar phased-MIMO beam pattern. Using calculation capabilities of Matlab program, average side lobe level can be obtained for planar phased-array and planar MIMO as (−24.5 dB) while it reaches (−29.4 dB) for linear phased-MIMO beam pattern. Moreover, for planar phased-MIMO array it reaches its minimum value at (−37.7 dB).

Figure 6 shows the beam pattern for the four techniques, planar phased-array, planar MIMO, linear phased-MIMO, and planar phased-MIMO in elevation plane (theta-plane). It is worth nothing that peak side lobe level in elevation plane for planar phased-array and planar MIMO radar is (−24 dB) while it reaches (−26.5 dB) for linear phased-MIMO and as lower as (−51.5 dB) for planar phased-MIMO.

Figure 7 shows the beam pattern of the same four techniques in azimuth plane (Phi-plane). Both planar phased-array and planar MIMO beam patterns have the same peak side lobe level

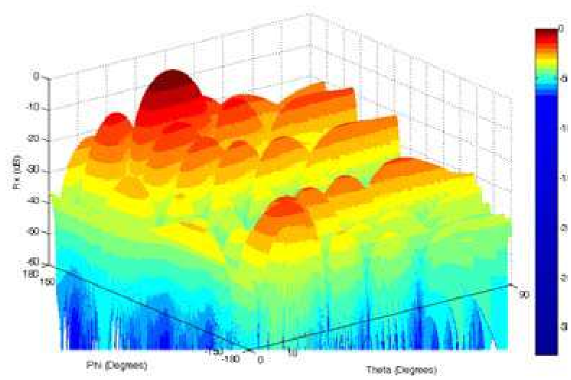

Figure 2: Received beam pattern for planar phased array radar.

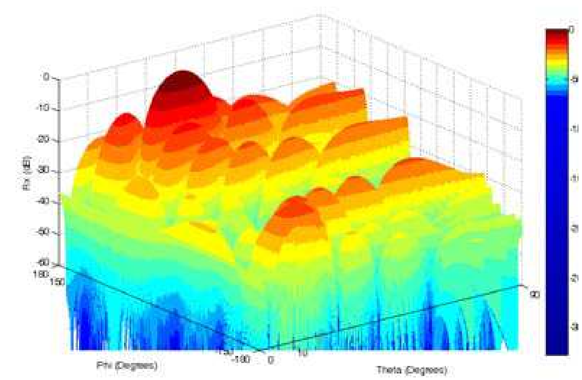

Figure 3: Received beam pattern for planar MIMO radar.

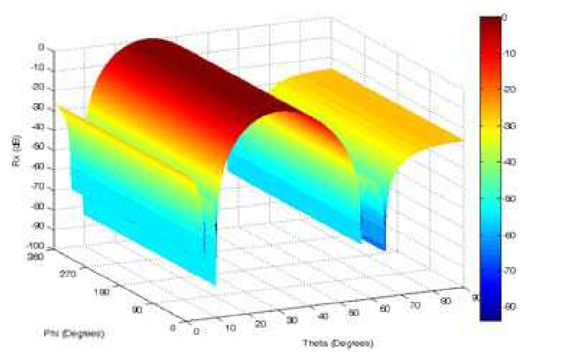

Figure 4: Received beam pattern for linear phased-MIMO radar.

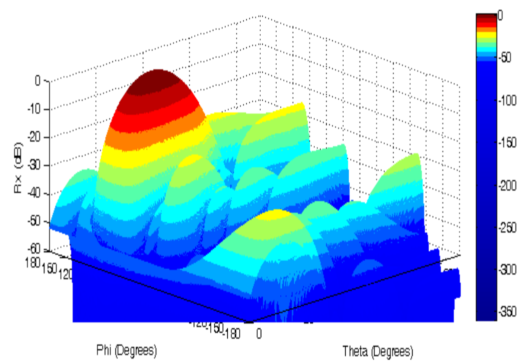

Figure 5: Received beampattern for planar phased-MIMO radar.

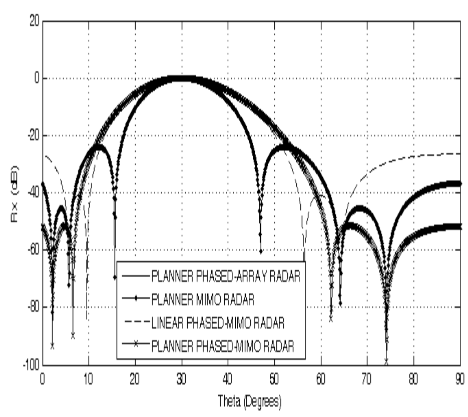

Figure 6: Received beampattern for the four techniques in elevation plane (theta-plane).

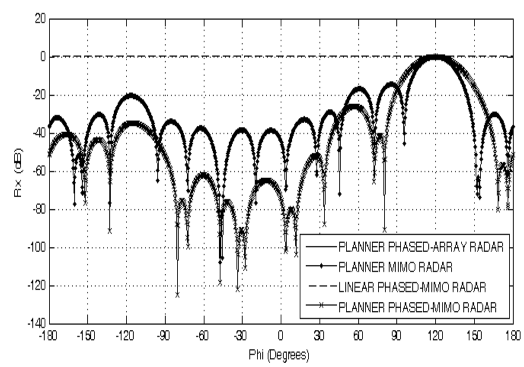

Figure 7: Received beampattern for the four techniques azimuth plane (Phi-plane).

at (-14.5 dB). Planner phased-MIMO beam pattern has a lower side lobe level at (-26 dB) while linear phased-MIMO beam pattern has a symmetrical uniform spatial power distribution at all azimuth angles. Figure 8 shows a spherical representation of linear phased-MIMO array while Figure 9 shows a spherical representation of planar phased-MIMO array. It's clear that while linear phased-MIMO array can steer the beam only along the elevation plane while maintaining symmetrical transmit/receive beam pattern in azimuth plane, planar phased-MIMO array can direct its beam at direction of the target of interest in both elevation and azimuth planes achieving higher directivity.

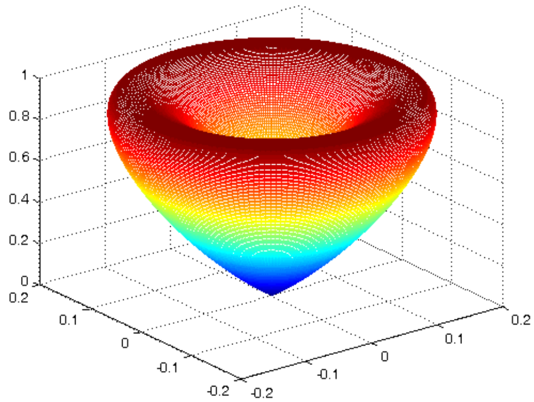

Figure 8: Linear phased-MIMO array directivity.

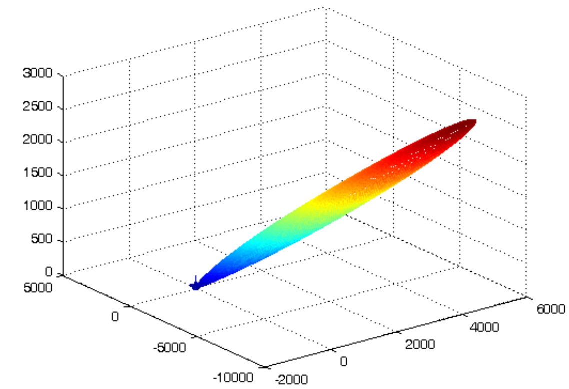

Figure 9: Planner phased-MIMO array directivity.

4. SUMMARY

The paper studies a new technique for partitioning the transmit array into a number of subarrays that are allowed to overlap in planar way. Then, each subarray is used to coherently transmit a waveform which is orthogonal to the waveforms transmitted by other subarrays. Significant improvements in average side lobe level, peak side lobe level and directivity are offered by the planar phased-MIMO radar technique over other techniques.

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