Study on Accuracy and Efficiency of the Numerical Algorithm for Electromagnetic Scattering from Targets and Rough Surface

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Abstract— Some studies about the accuracy and efficiency of the Propagation-Inside-Layer-Expansion (PILE) combined with the (Generalized) Forward-Backward Method ((G)FBM) hybrid scheme for electromagnetic scattering from targets and rough surface are carried out. The Relative Residual Error (RRE), that is set as the important evaluation criterion of algorithm accuracy is evaluated. The computational complexity and CPU time, acting as two key criterions for judging the efficiency are investigated. Numerical results demonstrate that, for multi-targets and rough surface cases, only needing a few iteration, the PILE and FBM hybrid scheme can reach enough accuracy, and also improve the efficiency, compared with some other classical numerical methods.

1. INTRODUCTION

Recently, scattering of electromagnetic wave in target and rough surface has been becoming an interesting topic as its important applications in electromagnetics, remote sensing, antennas theory and design, marine science, and electronic countermeasures, etc. Many analytical and numerical algorithms have been presented for this domain [1–6], such as the SPM, the KA, the MOM, the FMM, the FEM, the FDTD, etc., within which, lately, a fast and efficient numerical scheme named as the Propagation-Inside-Layer-Expansion (PILE) has been presented by N. Déchamps et al. [4]. In this paper, the accuracy and efficiency of the Propagation-Inside-Layer-Expansion (PILE) combined with the (Generalized) Forward-Backward Method ((G)FBM) hybrid scheme for electromagnetic scattering from targets and rough surface are investigated. The theory, formulations, and numerical results are given and corresponding discussions are addressed in each section.

2. BRIEF THEORY AND FORMULATIONS

When the PILE combined with (G)FBM hybrid scheme [5, 6] is applied to study the electromagnetic scattering from targets and rough surface, two important factors is needed to investigate: the accuracy and the efficiency. To validate the accuracy of EPILE+(G)FBM, the Relative Residual Error are necessarily investigated. The Relative Residual Error of using the EPILE+FBM and comparing with the MOM(LU/CGM) [5] is defined as the norm of the following form

$$\mathrm{RRE}=\frac{\sum_{-90}^{90}\left|\sigma_{(\mathrm{EPILE+(G)FBM})}-\sigma_{(\mathrm{MOM(LU/CGM)}}\right|^2}{\sum_{-90}^{90}\left|\sigma_{\mathrm{MOM(LU/CGM)}}\right|^2} \tag{1}$$

where, σ is the scattering coefficient. To confirm the efficiency of the EPILE+(G)FBM, the computational complexity $o(\ )$ and CPU time are necessarily investigated. Take the targets both above and below rough surface case for example (be suitable for the EPILE+FBM hybrid scheme), the complexities per iteration of the terms $\mathbf{Y}_{1,2}^{(p)}$, $\mathbf{Y}_r^{(p)}$ [5, 6] are given below,

$$\mathbf{M}_r\cdot\mathbf{Y}_r^{(p-1)}=\underbrace{(\bar{\Re}^1)^{-1}\cdot\bar{\Re}^2\cdot\overbrace{\underbrace{(\bar{\Re}^3)^{-1}\cdot\overbrace{\bar{\Re}^4\cdot\mathbf{Y}_r^{(p-1)}}^{o(2N_1N_3+2N_2N_3)\ (a)}}_{\substack{o(M_{iter}12(N_1+N_2)^2)\\ or\ o(4(N_1/3+N_2/3)^3)\ \ (b)}}}^{o(2N_1N_3+2N_2N_3)\ (c)}}_{o(4(N_3)^2)\ (d)}, \tag{2}$$

$$\mathbf{M}_{1,2}\cdot\mathbf{Y}_{1,2}^{(p-1)} = \underbrace{(\bar{\Re}^3)^{-1}\cdot\overbrace{\bar{\Re}^4\cdot\underbrace{(\bar{\Re}^1)^{-1}\cdot\overbrace{\bar{\Re}^2\cdot\mathbf{Y}_{(t1,t2)}^{(p-1)}}^{o((2N_1N_3)\times(+2N_2N_3))\ (e)}}_{o((2N_3)^2)\ (f)}}^{o(2N_1N_3+2N_2N_3)\ (g)}}_{o(M_{iter}12(N_1+N_2)^2)\ or\ o(4(N_1/3+N_2/3)^3)\ (h)} \quad (3)$$

where, $\mathbf{M}_r$, $\mathbf{M}_{1,2}$ is the corresponding characteristic matrix on rough surface, and on targets above/below, N_1, N_2, N_3 denotes the number of sampling points for targets above/below, and rough surface, $\bar{\Re}^1$, $\bar{\Re}^2$, $\bar{\Re}^3$, $\bar{\Re}^4$ corresponds to the local interactions on the rough surface, the targets→rough surface coupling, the local interactions on the rough surface, and the rough surface→targets coupling, respectively. The CPU time refers to the computer computational time for each scattering solving. Often, considering the possible instability, the averaged CPU time that is an average value of plenty of CPU time for each computation is a more useful information for evaluating the efficiency of algorithm. Obviously, if the average CPU time is less, the efficiency of algorithm is more efficient (i.e., computationally faster), otherwise, its efficiency is lower.

3. NUMERICAL RESULTS AND DISCUSSIONS

In formulas (2) and (3), operations (a), (c), (e), and (g) are matrix-vector multiplications: their computational complexities are $o(2N_1N_3 + 2N_2N_3)$. Operations (d), (f) are the FBM iterative inversions, their complexities are $o(4(N_3)^2)$. Operations (b) and (h) are the MOM (CGM or LU scheme), whose complexities are $o(M_{iter}12(N_1 + N_2)^2)$ or $o((4N_1/3 + 4N_2/3)^3)$, where M_{iter} is the number of iterations in the CGM scheme. Therefore, the total computational complexity is $o(p(4N_1N_3 + 4N_2N_3 + M_{iter}12(N_1 + N_2)^2 + 4(N_3)^2))$ or $o(p(4N_1N_3 + 4N_2N_3 + (4N_1/3 + 4N_2/3)^3 + 4(N_3)^2))$ for both the calculation, where p is the number of iterations in the EPILE scheme, for $N_3 \gg N_1$ and $N_3 \gg N_2$, i.e., the number of samples for the rough surface is much more than those of the targets, or the size of the rough surface is far larger than that of the targets, the complexity is about $o(p(4N_3 + 4(N_3)^2)) \approx o(p(4(N_3)^2))$ (for N_3 is large enough), p is generally less than 10, so this method is much faster than the direct LU inversion, of order $o((4N_3/3)^3)$ and the CGM, of order $o(M_{iter}12(N_3)^2)$. for examples, when $N_3 = 100$, the complexity of EPILE+FBM is 4×10^4, of LU is 2.25×10^6, of CGM is $> 1.2\times10^6$; when $N_3 = 1000$, the complexity of EPILE+FBM is 4×10^6, of LU is 2.25×10^9, of CGM is $> 1.2\times10^9$, obviously, when the N_3 is larger, the complexity gap between the EPILE+FBM and the MOM(LU/CGM) is more evident.

Table 1: Comparison of different order EPILE combined with different iteration number of FBM in RRE and averaged CPU time for 100 rough surface realizations (HH polarization).

EPILE() +FBM()	(2)+(2)	(2)+(3)	(2)+(4)	(2)+(5)	(2)+(6)
RRE	8.2×10^{-3}	7.103×10^{-4}	6.645×10^{-5}	1.615×10^{-5}	1.504×10^{-5}
CPU time	≈ 35 S	≈ 40 S	≈ 45 S	≈ 50 S	≈ 55 S
EPILE() +FBM()	(3)+(2)	(3)+(3)	(3)+(4)	(3)+(5)	(3)+(6)
RRE	7.850×10^{-3}	6.814×10^{-4}	6.231×10^{-5}	1.453×10^{-5}	1.098×10^{-5}
CPU time	≈ 40 S	≈ 46 S	≈ 50 S	≈ 55 S	≈ 60 S

The 2, 3 order EPILE combined with 2, 3, 4, 5, 6 iteration number of FBM for both HH and VV polarizations of 100 rough surface realizations have also been calculated. The corresponding RRE and averaged CPU time are listed in Tables 1, 2, respectively. Comparing the data in row indicates that, for a given EPILE iteration number, the Relative Residual Error decreases as the FBM iteration number increases, and all the RRE trends to be less than 10^{-3} order of magnitudes even through the averaged CPU time increases slightly, the FBM increases one iteration step, the averaged CPU time increases about 5 seconds (S). Comparing the data in column suggests that, for a given FBM order, the RRE also decreases and trends to be less than 10^{-3} order of magnitudes as the EPILE order increases, the EPILE increases one order, the averaged CPU time also increases about 5 seconds, while the averaged CPU time of MOM is about 5 minutes, hence, it can be

Table 2: Comparison of different order EPILE combined with different iteration number of FBM in RRE and averaged CPU time for 100 rough surface realizations (VV polarization).

EPILE() +FBM()	(2)+(2)	(2)+(3)	(2)+(4)	(2)+(5)	(2)+(6)
RRE	1.386×10^{-4}	2.001×10^{-5}	1.162×10^{-5}	1.130×10^{-5}	1.115×10^{-5}
CPU time	≈ 32 S	≈ 37 S	≈ 42 S	≈ 47 S	≈ 52 S
EPILE() +FBM()	(3)+(2)	(3)+(3)	(3)+(4)	(3)+(5)	(3)+(6)
RRE	1.324×10^{-4}	1.813×10^{-5}	1.135×10^{-5}	1.086×10^{-5}	1.018×10^{-5}
CPU time	≈ 37 S	≈ 42 S	≈ 47 S	≈ 52 S	≈ 58 S

summarized that, the EPILF+FBM is efficient in a whole, increasing the order of either EPILE or FBM, the RRE decreases and is in 10^{-3} or less order of magnitudes, in fact under which, an essentially exact result have be obtained, and a slightly increased CPU time is the only cost.

4. CONCLUSIONS

In this paper, the accuracy and efficiency of the Propagation-Inside-Layer-Expansion (PILE) combined with the (Generalized) Forward-Backward Method ((G)FBM) hybrid scheme for electromagnetic scattering from targets and rough surface are studied. The numerical results demonstrate that, the EPILE+FBM hybrid scheme can provide accurate results under the premise of enough efficiency. The accuracy can be further improved by increasing the order of EPILE or the iteration step of FBM, although a rather slightly increased time is needed.

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