Resonant Properties of Subwavelength Voids in Anisotropic Metamaterials

A. V. Vozianova[1], **P. B. Ginzburg**[2], **and A. N. Poddubny**[1,3]

[1]ITMO University, St. Petersburg, Russia
[2]Department of Physics, Kings College London, Strand, London WC2R 2LS, United Kingdom
[3]Ioffe Physical Technical Institute of the Russian Academy of Sciences, St. Petersburg, Russia

Abstract— Resonant properties of subwavelength voids in strongly anisotropic (hyperbolic) metamaterials were investigated by means of spherical harmonics expansion. These harmonics, being eigen resonances in isotropic environment, are nontrivially coupled by strong anisotropy and corresponding boundary conditions. Eigen resonances of the void are linked to the local field corrections for spontaneous emission rates inside metamaterial environment.

1. INTRODUCTION

Spontaneous emission processes are efficiently manipulated by nano-structured environment, which could almost on demand control both rates and coherence of quantum emitters [1]. Local density of states, being the key characteristic of electromagnetic space, could be dramatically enhanced by so-called hyperbolic metamaterials — strongly anisotropic artificial composites [2, 3]. However, radiating sources, situated inside a material, strongly interact with their surrounding by inducing additional local polarizations of the matter. One of the well-established approaches, enabling to address this issue, is the Clausius-Mosotti theory, which introduces a virtual cavity around an emitter [4]. This virtual void, being non-resonant for transparent dielectric environment, has nontrivial eigen resonances in the case of hyperbolic metamaterial. Investigation of the nature of these resonances and their impact on the local field correction for emission processes will be studied here.

2. QUASISTATIC MODEL FOR SPHERICAL VOID INSIDE ANISOTROPIC UNIAXIAL MEDIUM

We consider a subwavelength spherical void with radius R and dielectric constant ε_{in} situated inside anisotropic uniaxial medium with the dielectric tensor $\varepsilon_{out} = \mathrm{diag}(\varepsilon_{\parallel}, \varepsilon_{\parallel}, \varepsilon_{\perp})$.

The problem of eigen modes of the system was solved in the quasistatic limit by adopting Laplace equation for electric potential u:

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0, & r \leq R \\ \varepsilon_{\parallel}\frac{\partial^2 u}{\partial x^2} + \varepsilon_{\parallel}\frac{\partial^2 u}{\partial y^2} + \varepsilon_{\perp}\frac{\partial^2 u}{\partial z^2} = 0, & r > R \end{cases} \tag{1}$$

The appropriate boundary conditions for normal field components are following:

$$D_n^{in}\big|_{R-0} = D_n^{out}\big|_{R+0}. \tag{2}$$

In the case of isotropic embedding medium the resonances (material must have negative permittivity in order to support a one) have well-defined sequence — dipole, quadrupole, and higher-poles, defined by spherical harmonic decomposition. However, in the case of anisotropic medium all spherical harmonics are non-trivially coupled via boundary conditions.

The well-known solution of the first equation in (1) may be written in spherical coordinates [5]:

$$u_{in} = \sum_{n=1}^{\infty} A_{nm} P_n^m(\cos\theta)\, R^{n+1}(\cos m\phi + \sin m\phi) \tag{3}$$

But the potential u_{out} outside the sphere can be obtained by using scale transformation theory [5] in a more complex form

$$u_{out} = \sum_{n=1}^{\infty} F_{nm} P_n^m\left(\frac{\cos\theta}{q}\right)\left(\frac{\sqrt{\varepsilon_{\perp}}}{Rq}\right)^{n+1}(\cos m\phi + \sin m\phi) \tag{4}$$

where denominator $q = \sqrt{k \sin^2\theta + \cos^2\theta}$, coefficient $k = \frac{\varepsilon_\perp}{\varepsilon_\parallel}$, P_n^m — associated Legendre polynomial, A_{nm}, F_{nm} — indeterminate coefficients. In this work we consider the case of azimuthally uniform oscillations to coordinate ($m = 0$).

We substituted solutions (3) and (4) in the boundary condition (2), and further we multiplied the resulting expression by $P_n(\cos\theta)$ and integrated over spatial angle we obtain the linear system of equations for the indeterminate coefficients:

$$\begin{aligned}\varepsilon_{in} R^{k-1} A_k \;=\; & -\varepsilon_\parallel \sum_{n=1}^{\infty} \frac{F_n(n+1)}{R^{n+2}} \int_0^\pi P_n\left(\frac{\cos\theta}{q}\right)\left(\frac{\sqrt{\varepsilon_\perp}}{q}\right)^{n+1} P_n(\cos\theta)\sin\theta d\theta \\ & + \left(\varepsilon_\parallel - \varepsilon_\perp\right) \sum_{n=1}^{\infty} \frac{F_n(n+1)}{R^{n+2}} \int_0^\pi P_n\left(\frac{\cos\theta}{q}\right)\left(\frac{\sqrt{\varepsilon_\perp}}{q}\right)^{n+1} P_n(\cos\theta)\cos^2\theta\sin\theta d\theta \\ & - \left(\varepsilon_\parallel - \varepsilon_\perp\right) \sum_{n=1}^{\infty} \frac{F_n(n+1)}{R^{n+2}} \int_0^\pi P_n\left(\frac{\cos\theta}{q}\right)\left(\frac{\sqrt{\varepsilon_\perp}}{q}\right)^{n+1} P_n(\cos\theta)\cos\theta\sin^2\theta d\theta \quad (5)\end{aligned}$$

In this system Legendre polynomial with different arguments are not orthogonal as the result, causes the coupling between harmonics.

3. RESULTS AND DISCUSSION

Uniaxial material, having well-defined crystallographic axis in its natural Cartesian coordinate system, was transformed into full 3×3 tensor in the spherical system, appropriated for solution of the spherical cavity.

$$\bar{\varepsilon}_{out} = \begin{bmatrix} \varepsilon_\parallel - \left(\varepsilon_\parallel - \varepsilon_\perp\right)\cos^2\theta & \left(\varepsilon_\parallel - \varepsilon_\perp\right)\cos\theta\sin\theta & 0 \\ 0 & \varepsilon_\parallel - \left(\varepsilon_\parallel - \varepsilon_\perp\right)\cos^2\theta & 0 \\ 0 & 0 & \varepsilon_\parallel \end{bmatrix} \quad (6)$$

The presence of off-diagonal terms, calculated with the help of scale transformation theory [5], breaks the natural rotation a symmetry of the problem. Consequently, eigen resonances of the void are described now by infinite series of spherical harmonics. The convergence of these series is strongly dependent on the strength of anisotropy. In the case of extreme anisotropy — hyperbolic metamaterial (ordinary and extraordinary permitivities have different sign) [2] the convergence is enabled solely by introduction of Joule losses. In order to address the impact of anisotropy on the radiation pattern, the boundary problem with the dipolar extrication was solved. While the classical dipole pattern appears at the classic isotropic case (Fig. 2), the strongly anisotropic regime manifests itself in multiple harmonic generations outside the void (Fig. 2–Fig. 4). The impact of these harmonics on the radiation will be addressed.

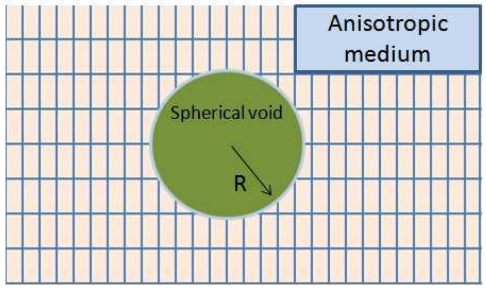

Figure 1: Geometry of problem.

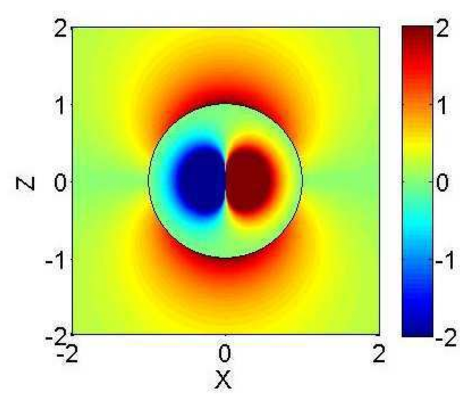

Figure 2: Radial components of electric field, excited by point vertically polarized dipole, situated in spherical void situated inside an isotropic medium ($\varepsilon_{in} = 1$, $\varepsilon_{out} = 3$).

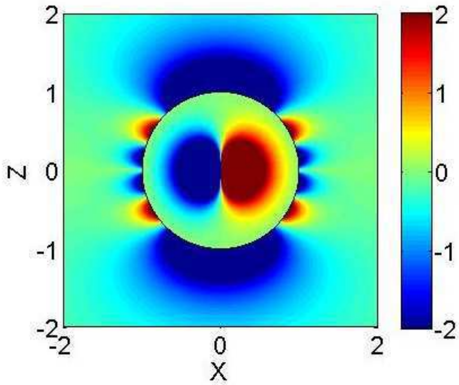

Figure 3: Radial components of electric field, excited by point vertically polarized dipole, situated in spherical void situated inside an anisotropic (elliptic) medium ($\varepsilon_{in} = 1$, $\varepsilon_{out} = \mathrm{diag}(2, 2, 3)$).

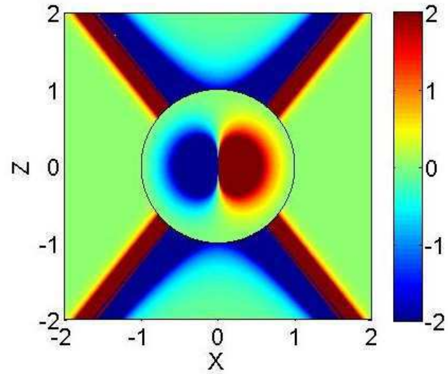

Figure 4: Radial components of electric field, excited by point vertically polarized dipole, situated in spherical void situated inside an anisotropic (hyperbolic) medium ($\varepsilon_{in} = 1$, $\varepsilon_{out} = \mathrm{diag}(2, 2, -3+0.1i)$).

In this paper we have used the scale transformation theory to consider the electric fields inside and outside subwavelength spherical void and have investigated the resonant properties of subwavelength voids in anisotropic (elliptic, hyperbolic) metamaterials by means of spherical harmonics expansion. These harmonics are nontrivially coupled by strong anisotropy and corresponding boundary conditions. Also we have solved the boundary problem with the dipolar extrication.

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REFERENCES

1. Poddubny, A. N., et al., "Spontaneous radiation of a finite-size dipole emitter in hyperbolic media," *Physical Review A*, Vol. 84, No. 2, 023807, 2011.
2. Poddubny, A. N., et al., "Hyperbolic metamaterials," *Nature Photonics*, Vol. 7, 948–957, 2013.
3. Barnett, S. M., et al., "Spontaneous emission in absorbing dielectric media," *Phys. Rev. Lett.*, Vol. 68, 2012.
4. Poddubny, A. N., et al., "Hyperbolic metamaterials," *Nature Photonics*, Vol. 7, 948–957, 2013.
5. Li, Y.-L. and J.-Y. Huang, "The scale-transformation of electromagnetic theory and its applications," *Chinese Physics*, Vol. 14, No. 4, 0646-10, 2005.