FEM Analysis of Conical Type Coaxial Open-ended Probe for Dielectric Measurement

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Abstract— This paper deals with a numerical modelling technique based on finite elements method (FEM) in microwave frequency for computing the dielectric constant and loss factor of a homogeneous dielectric material by using conical-type coaxial probe. This sort of probe can be easily inserted into a wide range of biological tissue types and semi-rigid materials like rubber, some plastics, and organic materials (ex: dairy, butter, etc. for measuring moisture content). Where his feature is considered to be very important in biological and industrial applications. The measuring principle is based on detecting conductance and capacitance change with respect to the dielectric mass movement in the fringe electrical field. A three-dimensional finite-element formulation is employed in the dielectric material region and a small neighbouring region of the probe structure on which it is mounted. The electrical input admittance as well as the reflection coefficient are found from the finite-element analysis. From this, the conductance and capacitance related to the conical probe and fringing field are calculated and consequently the dielectric constant and loss factor are determined. In contrast to many other modelling techniques used for coaxial probe which are approximate and hence limited, the finite-element model is more exact and is applicable to complicated geometries. To demonstrate the accuracy of the numerical model, a parallel experimental study was carried out in the laboratory for the same geometric dimensions. The two results are compared and show excellent agreement, and also demonstrating that finite-element modelling is a good approach for optimized conical coaxial probe design.

1. INTRODUCTION

Measurement of dielectric properties of materials at microwave frequencies has been the subject of many literatures [1–4]. The dielectric constant is an unique and essential property of dielectric materials to characterize the interaction between an electric field and matter. Accurate measurements of these properties can provide scientists and engineers with essential and valuable information to properly incorporate the materials into their intended applications. For example, this information is useful in the area of aerospace, automotive, agriculture, food engineering, bioengineering, medical treatments, and electronics applications, etc.. There are many techniques that have been developed to material characterization like the Transmission/Reflection Line, Open-ended coaxial probe, Free space and resonator method (cavity method) [3–5]. The open-ended coaxial probe has been widely researched over the years as a way to effectively and quickly characterize material properties in a wide frequency spectrum (200 MHz to near 50 GHz). Many models have been built to analyze open ended coaxial probes terminated by semi-infinite homogeneous materials [6–8]. There are four typical models (Capacitive model, Antenna model, Virtual line model, Rational function model,etc.). However, there is an undiscovered area in this field, the study of conical type open ended coaxial line which is a coaxial line ending in a conical shape geometry. The advantage of this kind of probe is the possibility of placing the probe into the solid materials which are difficult for dielectric measurement. In this research, The characteristic impedance of probe are calculated in the different frequencies for various materials by using finite element method, and then antenna model is chosen to relate the conical type coaxial probe characteristic impedance to the dielectric constant and loss factor of the material under test. The sensitivity and accuracy of the models in measuring lossy dielectric or biological material in compare to experimental results are investigated.

2. BASIC EQUATIONS

The problem of electromagnetic field analysis on a macroscopic level is fully described by Maxwell equations subject to certain boundary condition. The conical probe is assumed to be uniform along its longitudinal z axis; therefore maxwell curl equations for time harmonic fields are:

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \quad (1)$$
$$\nabla \times \mathbf{H} = -j\omega\epsilon\mathbf{E} \quad (2)$$

By taking the curl of (1) and (2), we have the following curl-curl equation for E:

$$\nabla \times \left(\frac{1}{\mu}\nabla \times \mathbf{E}\right) - k_0^2\epsilon\mathbf{E} = 0 \quad \text{in } \Omega \tag{3}$$

where k_0 is the free-space wave number and Ω is the probe cross section. An Neumann and Dirichlet boundary condition must be applied by the field vectors in all boundary and interface between two materials. The Galerkin method is chosen for finding numerical solutions to differential equations where the solution residue is minimized giving rise to the well-known weak formulation of problems. The basic idea of this method is to approximate the solution to a differential equation by very simple functions. The residual associated with the PDE Equation (3) is given by:

$$\partial F(\mathbf{E}) = 0 \tag{4}$$

where

$$R(\mathbf{E}) = \frac{1}{2}\iint_{\Omega}\left(\frac{1}{\mu}(\nabla \times \mathbf{E})\cdot(\nabla \times \mathbf{E})^* - k_0^2\epsilon\mathbf{E}\cdot\mathbf{E}^*\right)d\Omega \tag{5}$$

suppose that E can be approximated by the expansion

$$E(x, y, z) = \sum_{j=1}^{n} N_j^e(x, y, z)E_j^e \tag{6}$$

where $N_j^e(x, y)$ denotes the basis function defined over the entire domain, M is the number of elements in the domain, and n is the number of edge in the element, and E_j^e is constant coefficients to be determined. By substituting Equation (6) in (5), we can obtain the generalized matrix form eigenvalue problem. More detail about this part can be find in [9] and [10]. This method is used to calculate dielectric field in the aperture of the conical probe and then the reflection coefficient and characteristic impedance of probe at measurement port (which is shown in Fig. 1). Consequently, antenna model is chosen to relate the conical type coaxial probe characteristic impedance to the dielectric constant and loss factor of the material under test. The 2-dimensional electromagnetic structure of conical-type open-ended coaxial probe is shown in Fig. 2 is considered. And the global constant related to structure and martial property are presented in Table 1.

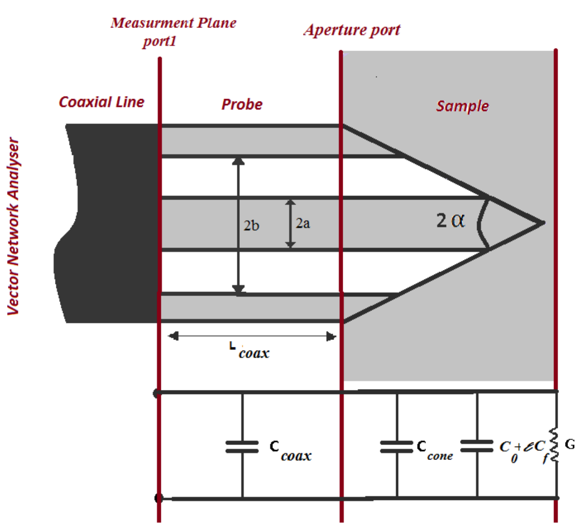


Figure 1: Conical-type coaxial probe geometry and equivalent circuit.

Figure 2: Handmade probes and network analyzer.

In antenna model, the probe is modelled by tow capacitance C_f and C_{cone}, while the material under test (MUT) is modelled by a capacitance $\epsilon^* C_m$ and a conductance G connected in parallel to capacitance for modelling the radiation in MUT. The admittance of this equivalent circuit is given by:

$$\frac{Y}{Y_0} = j\omega C_f Z_0 + j\omega C_{\text{cone}} Z_0 + j\omega C_m Z_0(\omega, \epsilon_0) + Z_0 G(\omega, \epsilon_0) \tag{7}$$

Table 1: Global constant for electromagnetic model.

Name	Expression	Description
a	0.455 [mm]	Coax inner radius
b	1.49 [mm]	Coax outer radius
L_{coax}	250 [mm]	Length of conical coax probe
f	300 [MHz]–3 [GHz]	Frequency
α	30	Cone Angle

For coaxial line whose dimensions are small compared to a wavelength, C_f and C_{cone} are frequency independent, while C_m and G are dependent on frequency. For coaxial probe immersed in a lossy medium it has been shown in [7] that

$$Y\left(\omega,\epsilon^*\right)=\sqrt{\epsilon^*}Y\left(\sqrt{\epsilon^*}\omega,\epsilon_0\right) \tag{8}$$

Hence Equation (7) may be written

$$\frac{Y}{Y_0}=j\omega C_f Z_0+j\omega C_{\text{cone}}Z_0+j\omega\epsilon^* C_m Z_0+Z_0 G\epsilon^{*5/2} \tag{9}$$

Two unknown parameters, C_f and C_0, are estimated accurately by using two material with known properties (air and water). Analytical computation of C_{cone} part is complicated due to the different size of two conductors and their irregular locations. However, dividing the aperture into many thin layer and each layer can be approximated constant radius provides an elegant way to calculate it accurately. Results for different frequencies are shown in Table 2.

Table 2: Capacitances of probe and fringe field (C_f and C_{cone}).

freq	G	C_{cone}	C_{coax}	C_0	C_f
300000000	$3.12E-04$	$1.28E-13$	$2.44E-11$	$3.79376E-14$	$1.2763E-13$
600000000	$4.29E-04$	$1.28E-13$	$2.44E-11$	$3.88725E-14$	$7.49492E-14$
900000000	$6.11E-04$	$1.28E-13$	$2.44E-11$	$3.95339E-14$	$6.85539E-14$
1200000000	$9.67E-04$	$1.28E-13$	$2.44E-11$	$4.07336E-14$	$8.74826E-14$
1500000000	0.001200643	$1.28E-13$	$2.44E-11$	$4.34477E-14$	$8.3988E-14$
1800000000	0.00130072	$1.28E-13$	$2.44E-11$	$4.69067E-14$	$6.81714E-14$
2100000000	0.001568596	$1.28E-13$	$2.44E-11$	$5.0166E-14$	$6.88041E-14$
$2.40E+09$	0.001915157	$1.28E-13$	$2.44E-11$	$5.52373E-14$	$7.18766E-14$
$2.70E+09$	0.002068937	$1.28E-13$	$2.44E-11$	$6.48864E-14$	$5.72198E-14$
$3.00E+09$	0.002208294	$1.28E-13$	$2.44E-11$	$7.79847E-14$	$3.9359E-14$

3. RESULTS AND DISCUSSION

The reflection coefficient factor, electromagnetic field at the probe aperture and permittivity of different materials were analyzed in different frequencies (300 MHz to 3 GHz). Fig. 3 shows the measured reflection coefficient of Air, Water, Methanol and Butanol at 20 degree with conical type open ended coaxial probe. Fig. 4 shows relative permittivity and conductivity for three different materials (Ethanol, Methanol, and Butanol) with conical type open ended coaxial probe. These permittivity are calculated from the reflection coefficient measured at port 1 in Fig. 1. The use of such model is a good approximation for large band of permittivity at radio and microwave frequencies.

The results show that open-ended conical coaxial probes can be successfully used especially at low frequencies with the advantage of increased accuracy and sensitivity. Moreover, it is less effected by unavoidable temperature variation when compared to standard at-plan open-circuit coaxial-probe. An additional advantage of the conical-type probe is that it does not have the problem of flat coaxial probe with solid surface.

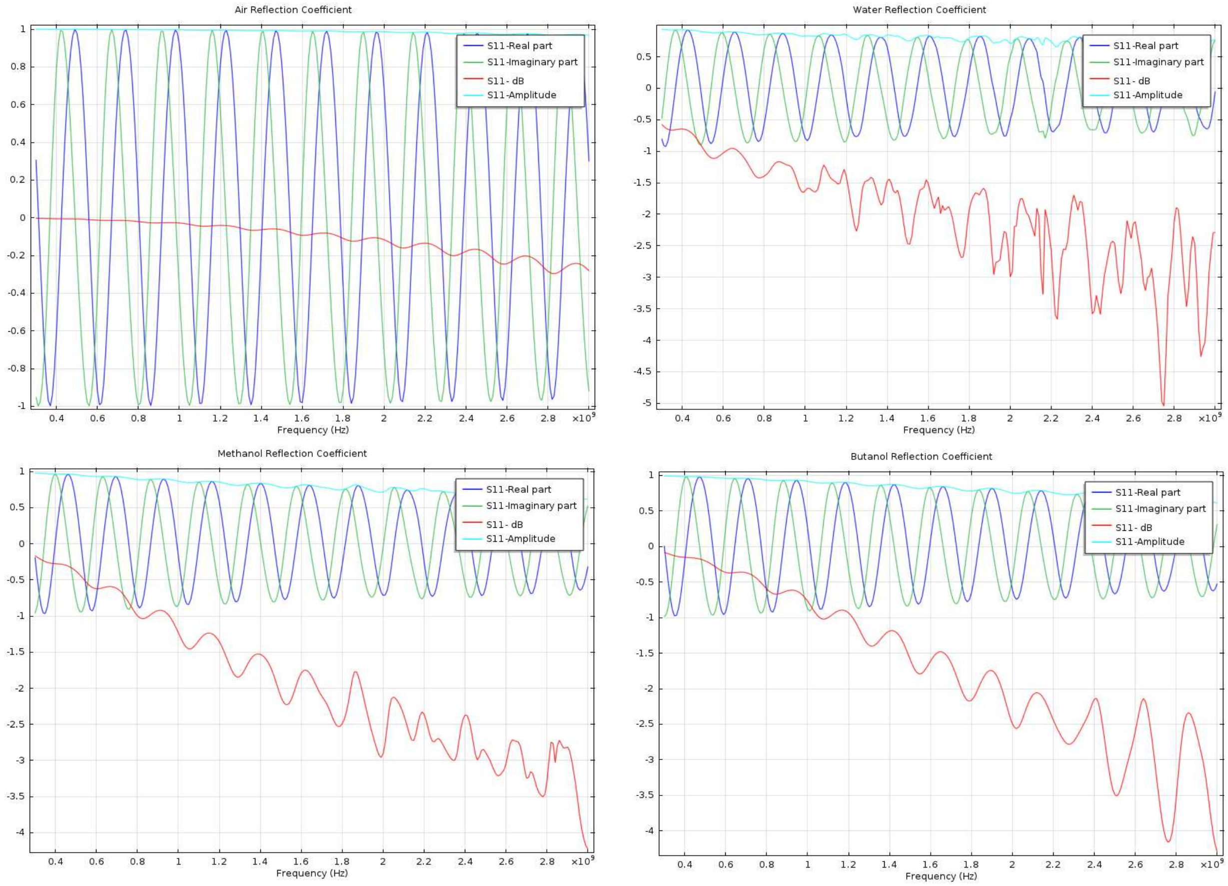
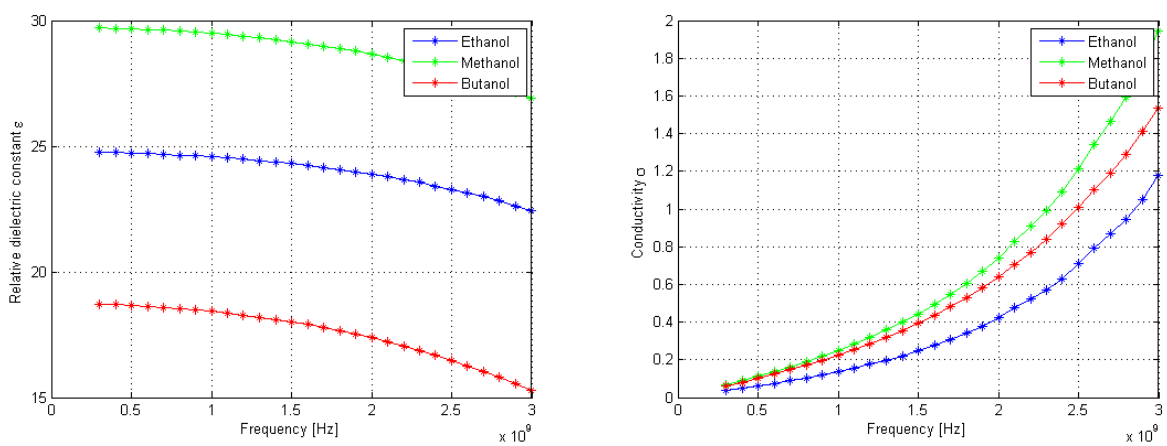


Figure 3: Reflection coefficient for Air, Water, Methanol and Butanol.



(a) (b)

Figure 4: (a) Relative permittivity. (b) Conductivity for open ended conical coaxial probe (cone angle 30 degree).

4. CONCLUSION

In this work, dielectric constant and loss factor of a homogeneous dielectric material by using conical-type coaxial probe are studied and reported. The advantages of the conical coaxial probe is not only in its easy insertion in material and biological application, but it can be successfully used especially at low frequencies with the advantage of increased accuracy and sensitivity.

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