New Scaling of Electron Thermionic Emission from Single-layer Graphene

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Abstract— In this work, a simple model is developed to explore the electron thermionic emission from a single-layer graphene. This study is based on our prior efforts in showing that electron emission mechanisms from single-layer graphene is quite different from the traditional models, such as graphene field emission, and graphene laser-induced over-barrier emission, etc.. Now, we establish a new scaling for thermionic emission from graphene, quite different from the conventional Richardson's law responsible for bulk materials. Our analytical formula obtained in this work is demonstrated to agree well with numerical results.

1. INTRODUCTION

Electron thermionic emission describes the process of thermal electron evaporation from the surface of a hot cathode material, which is first discovered by Owen Willans Richardson in 1901 [1]. Due to this discovery, he was awarded the Nobel Prize in 1928. The mathematical description of thermionic emission is given by Richardson-Dushman equation,

$$J = AT^2 \exp\left[-\frac{\Phi}{k_B T}\right]. \tag{1}$$

where, T is the cathode temperature, Φ is the work function of metal, k_B is the Boltzmann constant, and the prefactor $A = 1.2 \times 10^6\,\mathrm{Am^{-2}K^{-2}}$ is the Richardson constant. This equation has very important implications in the design of vacuum electronics device and the study of interface materials, as well as energy conversion. Specifically, the recent studies [2, 3] make a very promising future of the domestic application of energy converter based on thermionic emission. However, all the current applications above are based on the traditional Richardson's law, which is derived for bulk materials.

Recently, the studies of two-dimensional materials, like graphene [4], have attracted lots of attention. Graphene has many unique properties, such as ultrahigh mobility, linear dispersion relation, etc.. Fundamentally, the single-layer graphene differs from conventional bulk materials in its liner band structure. Thus this essential difference arises an interesting question: Is the conventional Richardson's law still valid for the thermionic emission from a single-layer graphene? In this work, we are going to answer this question.

2. MODEL

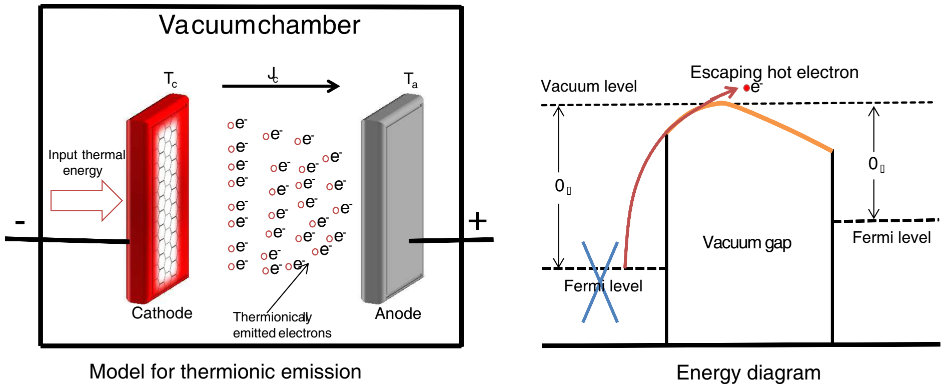
In this work, we develop a model, as shown in Fig. 1. Assume that the cathode has been heated up to a uniform temperature T_c, the electrons will be evaporated from the cathode and then condensed on the anode (which is at temperature T_a) by passing over the potential barrier as shown in Fig. 1(b).

3. RESULTS

Wallace's electron theory of graphene provides the basis for all unique and excellent properties of graphene. The unique linear-band structure of graphene leads to massless dirac fermion electron, which obeys a 2-D dirac equation. Using this model, the density of electron states per cell in the graphene is obtained,

$$\rho(E)dE = \frac{2}{(2\pi)}^2 \iint d^2k = \frac{2E}{\pi(\hbar v_f)^2}dE, \tag{2}$$

where $\hbar$ is the reduced Planck constant and v_f (10^6 m/s) is the velocity of massless Dirac fermions in the graphene. For the thermionic emission, the current density of the emitted electrons along



(a) (b)

Figure 1: Schematic diagram of the (a) electron emission process of a graphene-based thermionic energy converter (TIC), where T_c and T_a are the cathode temperature and anode temperature, respectively; J_c is current ow from cathode to anode. (b) energy level for thermionic energy converter (TIC) shown in (a). Φ_c and Φ_a are the work function of cathode and of anode, respectively. The orange line represents the potential energy in the vacuum. The deep red line traces the thermionic emission process of an electron into vacuum.

the direction perpendicular to graphene plane is calcuated by

$$J(E_F, T) = \int_{\Phi}^{\infty} eN(E_x)dE_x, \tag{3}$$

with

$$N(E, E_x)dEdE_x \equiv \frac{2f_{F-D}(E)}{2\pi}^2 \iiint_{E,E_x} v_x d^2k dk_x = \frac{1}{\pi\hbar^3 v_f^2} E f_{FD}(E)dEdE_x. \tag{4}$$

where, $f_{F-D}(E)$ is the Fermi-Dirac distribution. To solve Eq. (4) analytically, we assume that $f_{F-D}(E)$ can be replaced with Maxwell-Boltzmann distribution $f_{M-B}(E) = \exp[-(E - E_F)/k_B T]$.

In doing so, we obtain an analytical solution to Eq. (3).

$$J(E_F, T) = \frac{ek_B^2 T^2}{\pi\hbar^3 v_f^2}(k_B T + \Phi) \exp\left[-\frac{\Phi - E_F}{k_B T}\right] \tag{5}$$

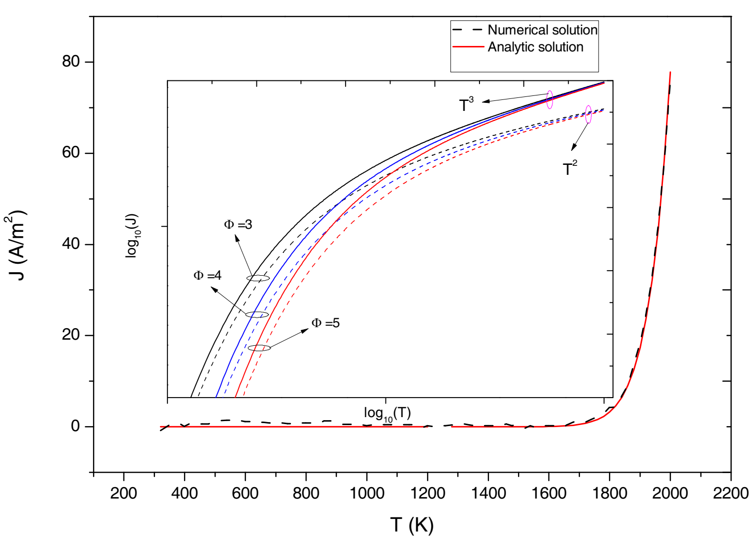


Figure 2: Comparison of our analytical results [in red line] with the numerical solution of Eq. (3) [in black dashed line] as a function of temperature. The Inset demonstrates comparison between our model (solid line) and Richardson's law (dashed line) at different work function 3 eV, 4 eV and 5 eV.

We found that Eq. (5) is completely different from the conventional Richardson's law given by Eq. (1), due to the linear band structure of graphene. And the numerical calculation of Eq. (3) is carried out to verify the analytical solution, as shown in the Fig. 2. In addition, the comparison between our results and Richardson's law is also presented in the Fig. 2. From this figure, it is found that, at high temperature limit ($k_B T \gg \Phi$), Eq. (5) also gives a new scaling of T^3, which is different from the T^2 scaling based on the Richarson's law.

4. CONCLUSION

In the work, we study the thermionic emission from a single-layer graphene, and a new scaling of thermionic emission is established. It is found that the newly proposed scaling is quite different from the conventional Richardson's law. This finding may be very useful in the design of energy conversion devices and in the study of transport near interface.

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