TM Wave Mode Analysis of Circular Dielectric Resonator with Anisotropic Permittivity

Hepi Ludiyati[1,2], **Andriyan Bayu Suksmono**[1], **and Achmad Munir**[1]

[1]Radio Telecommunication and Microwave Laboratory
School of Electrical Engineering and Informatics, Institut Teknologi Bandung, Indonesia
[2]Department of Electrical, Politeknik Negeri Bandung, Indonesia

Abstract— This paper presents the theoretical analysis of transverse magnetic (TM) wave mode for circular dielectric resonator with anisotropic permittivity. The analysis which is emphasized on its resonant frequency is required to investigate specific properties of resonator in TM wave mode which are useful for microwave application. By introducing the anisotropic permittivity, the resonator is expected to have the property with unique characteristic compared to the conventional one. To determine the resonant frequency of resonator for TM wave mode, Maxwell's equations with proper boundary condition are applied for circular waveguide that encapsulates an anisotropic circular dielectric resonator. The anisotropic permittivity is established by assuming the different values of relative permittivity in each axis of cylindrical coordinate, i.e., ϵ_ρ, ϵ_ϕ and ϵ_z. The analysis result shows that the circular dielectric resonator with anisotropic permittivity in z axis, i.e., propagation direction, has significantly changed the resonant frequencies of TM wave mode which can be applicable for resonance mode selection.

1. INTRODUCTION

Over 4 decades ago, after being first time introduced by Ritchmyer [1], dielectric resonators have been widely exploited for the implementation in numerous microwave applications including oscillators, filters and antennas [2–4]. One of the important milestones in the dielectric ceramic industry occurred in the mid nineteen seventies when the first temperature-stable, low-loss barium tetratitanate ceramics were developed [5]. Whilst the investigation of their properties in term of artificial dielectrics to explore the unique characteristics for microwave devices, e.g., anisotropic permittivities, has been conducted for a long time. In fact, more than a half century ago, one of the concepts has been expressed for the application of artificial dielectric when a dielectric lens was made by replacing the refractive material using a mixture of metal disks in a matrix [6]. From the concept, some applications based on artificial dielectrics have been proposed with the intention for a plane wave or TEM wave incidence, such as wave absorber, polarizer and phase shifter [7]. Moreover, by use of the artificial dielectric, a rectangular dielectric resonator has been successfully fabricated for the first time [8].

In principle, for various applications the performance of a dielectric resonator depends on its characteristics. This also applies for the dielectric resonator with anisotropic permittivity. In some specific application, it is sometimes required to implement dielectric resonators which have high relative permittivity as well as capability in resonance mode selectivity [9]. The kind of features, i.e., high relative permittivity, is rarely obtainable through simple chemical treatment such as natural dielectric. However, the capability of resonance mode selection needs another treatment which is required to yield dielectric resonators with anisotropic permittivity. One method to obtain the anisotropic permittivity which have been implemented experimentally is by configuring the orientation of constituent materials or particles [8]. In spite of implementation for a rectangular dielectric resonator, the method is also supposed to be applicable for a circular dielectric resonator. In order to have better understanding in the implementation for circular dielectric resonator, i.e., TM wave mode, in this paper, the theoretical analysis of resonance mode for circular dielectric resonator with anisotropic permittivity is investigated with emphasizing on its resonant frequency for the first 3 TM wave modes. Here, the resonant frequency is determined by applying Maxwells equations for a circular waveguide that encapsulates the circular dielectric resonator with different values of relative permittivity in each axis of cylindrical coordinate.

2. CALCULATION OF RESONANT FREQUENCY FOR TM WAVE MODE

In the analysis of resonator characteristics, one of the most essential properties is the resonant frequency which is determined by the value of relative permittivity. The proposed method to calculate the resonant frequency of circular dielectric resonator from its relative permittivity is by use of a short-open termination [10]. Here, to obtain the resonant frequency of circular dielectric resonator with anisotropic permittivity for the TM wave mode, the resonator is assumed to be homogeneous and follows the standard analysis for an isotropic resonator.

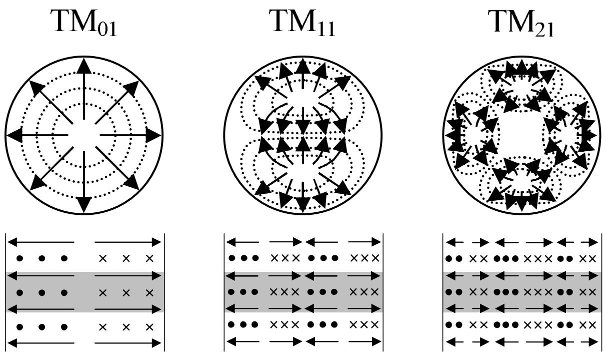

Figure 1: Field distribution of first 3 TM wave modes.

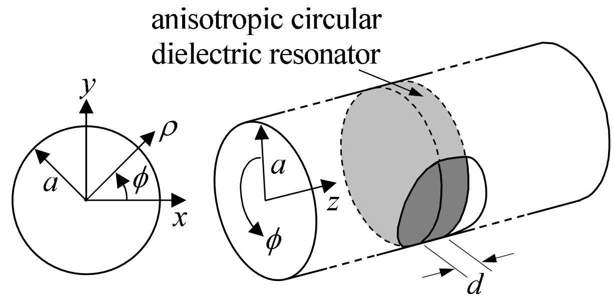

Figure 2: Illustration of resonant frequency calculation for anisotropic circular dielectric resonator.

As shown in Figure 1, field distribution of the first 3 TM wave modes, i.e., TM_{01}, TM_{11}, TM_{21}, are illustrated where the TM_{01} wave mode has a simple one since it has no variation of magnetic field in angular direction. While the depiction of resonant frequency calculation for anisotropic circular dielectric resonator with the radius of a and the thickness of d is indicated in Figure 2 in which the cylindrical coordinate system is the most appropriate one for its analysis in view of cylindrical geometry. The permittivity in term of tensor and the permeability of circular dielectric resonator are expressed in (1).

$$[\epsilon] = \epsilon_0 \begin{bmatrix} \epsilon_\rho & 0 & 0 \\ 0 & \epsilon_\phi & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}, \qquad [\mu] = \mu_0 \tag{1}$$

where ϵ_ρ, ϵ_ϕ and ϵ_z are relative permittivity in the ρ, ϕ and z axes, respectively. Whilst, ϵ_0 and μ_0 are permittivity and permeability in free space, respectively. The field with time variation $e^{(j\omega t)}$ suppressed in the resonator is assumed to propagate in the $+z$ direction as $e^{-(j\beta z)}$. By substituting (1) into the Maxwell's equations with proper boundary condition and solving them with respect to E_z for the TM wave mode, the following equation can be obtained.

$$\frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial E_z}{\partial \rho} + \left(\frac{k_\rho\sqrt{\epsilon_\phi}}{k_\phi\sqrt{\epsilon_\rho}}\right)^2 \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial^2 \phi} + \frac{\epsilon_z}{\epsilon_\rho}k_\rho^2 E_z = 0 \tag{2}$$

where $k_\rho^2 = \omega^2\mu_0\epsilon_0\epsilon_\rho - \beta^2$ and $k_\phi^2 = \omega^2\mu_0\epsilon_0\epsilon_\phi - \beta^2$.

The general solution of (2) known as scalar Helmholtz equation for the electric field in cylindrical coordinate system is given by (3).

$$E_z = R(\rho)\Phi(\phi) \tag{3}$$

By substituting (3) into (2), the relation of $R(\rho)$ and $\Phi(\phi)$ can be obtained.

$$\frac{1}{\rho R}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial R}{\partial \rho}\right) + \frac{1}{\rho^2\phi}\left(\frac{k_\rho\sqrt{\epsilon_\phi}}{k_\phi\sqrt{\epsilon_\rho}}\right)^2 \frac{\partial^2 \Phi}{\partial^2 \phi} = -\frac{\epsilon_z}{\epsilon_\rho}k_\rho^2 \tag{4}$$

The solution of (4) can be taken from the method of variable splitting. Therefore, (4) can be split into 2 equations as function of ρ and ϕ which have the general solution as expressed in (5) and (6), respectively.

$$R(\rho) = A_1 J_n\left(\frac{\sqrt{\epsilon_z}}{\sqrt{\epsilon_\rho}}k_\rho \rho\right) + A_2 N_n\left(\frac{\sqrt{\epsilon_z}}{\sqrt{\epsilon_\rho}}k_\rho \rho\right) \tag{5}$$

$$\Phi(\phi) = A_3 \sin\left(\frac{k_\phi\sqrt{\epsilon_\rho}}{k_\rho\sqrt{\epsilon_\phi}}n\phi\right) + A_4 \cos\left(\frac{k_\phi\sqrt{\epsilon_\rho}}{k_\rho\sqrt{\epsilon_\phi}}n\phi\right) \tag{6}$$

where J_n and N_n are Bessel and Neumann functions, respectively. After substituting (5) and (6) into (3) and applying proper boundary condition in which the $R(\rho)$ is defined for $\rho < a$ and the value of $\Phi(\phi)$ is suitable for the defined value of $R(\rho)$, thence the solution of E_z in term of time domain which propagates in the $+z$ direction is revealed in (7).

$$E(z,t) = E_{z0} J_n \left(\frac{\sqrt{\epsilon_z}}{\sqrt{\epsilon_\rho}} k_\rho \rho \right) \cos \left(\frac{k_\phi \sqrt{\epsilon_\rho}}{k_\rho \sqrt{\epsilon_\phi}} n\phi \right) e^{j(\omega t - \beta z)} \tag{7}$$

From (7), the related fields in other axes can be calculated from Maxwell's equations. Furthermore, the evanescent fields, i.e., fields outside the resonator, can be obtained by replacing ϵ_ρ, ϵ_ϕ and ϵ_z in (1) with 1 and β with $-j\alpha$. Then, by using the method of short-open termination which is applied at the middle of resonator ($d/2$), the input impedances are calculated to attain the following relation (8).

$$Z_g \tan \left(\beta \frac{d}{2} - \frac{s\pi}{2} \right) = \frac{\alpha}{\omega \epsilon_0} \tag{8}$$

where s and Z_g are the integer number (1, 3, 5, ...) and the wave impedance of resonator for appropriate TM wave mode, respectively.

3. NUMERICAL RESULT AND ANALYSIS

In order to demonstrate the potentiality of anisotropic permittivity, the resonant frequencies of circular dielectric resonator property with for TM_{01}, TM_{11}, TM_{21} wave modes are calculated and compared to the natural circular dielectric resonator. In case of natural circular dielectric resonator, ϵ_ρ, ϵ_ϕ and ϵ_z are replaced by ϵ_r; whilst k_ρ and k_ϕ are replaced by k_c where $k_c^2 = \omega^2 \mu_0 \epsilon_0 \epsilon_r - \beta^2$ and ϵ_r is the relative permittivity. The relative permittivity of circular dielectric resonator is taken as 5, 5 and 10 for ϵ_ρ, ϵ_ϕ and ϵ_z respectively; while for the natural one it is 5 being isotropic (ϵ_r). The resonator thickness (d) is set from 0.1 mm to 5 mm and the radius (a) is 16.27 mm. The cut-off frequency of hollow circular waveguide itself is 7.058 GHz, 11.246 GHz, and 15.069 GHz for TM_{01}, TM_{11}, and TM_{21} wave modes, respectively. The calculation results of resonant frequency as a function of resonator thickness (d) for TM_{01}, TM_{11}, TM_{21} wave modes are plotted in Figure 3. From the results, it can be seen that the resonant frequencies of circular dielectric resonator with anisotropic permittivity, i.e., ϵ_z higher than ϵ_ρ and ϵ_ϕ, are lower than of natural circular dielectric resonator for TM_{01}, TM_{11}, and TM_{21} wave modes. This means that the anisotropic permittivity in z direction has significant influences in lowering resonant frequency of circular dielectric resonator for TM wave modes.

Furthermore, if the relative permittivity of circular dielectric resonator is taken as 10, 5 and 5 for ϵ_ρ, ϵ_ϕ and ϵ_z respectively, in which ϵ_ρ is higher than others; and the natural one is 5 same as in the previous calculation, the resonant frequencies for first 3 TM wave modes have slightly lower than the natural one for the thickness of resonator thinner than 2 mm. The calculation result is

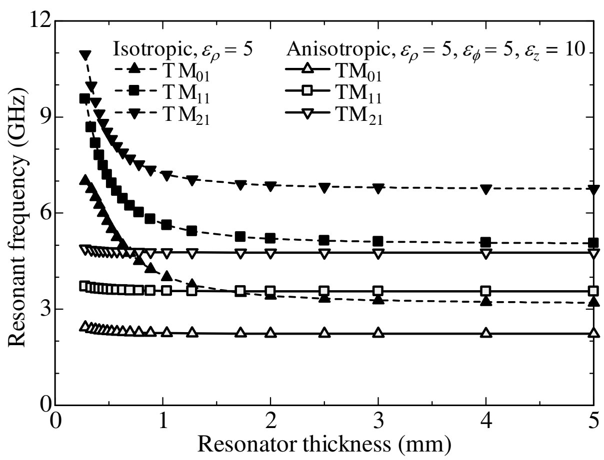

Figure 3: Resonant frequencies of first 3 TM wave modes ($\epsilon_r = 5$ for isotropic, while $\epsilon_\rho = 5$, $\epsilon_\phi = 5$ and $\epsilon_z = 10$ for anisotropic).

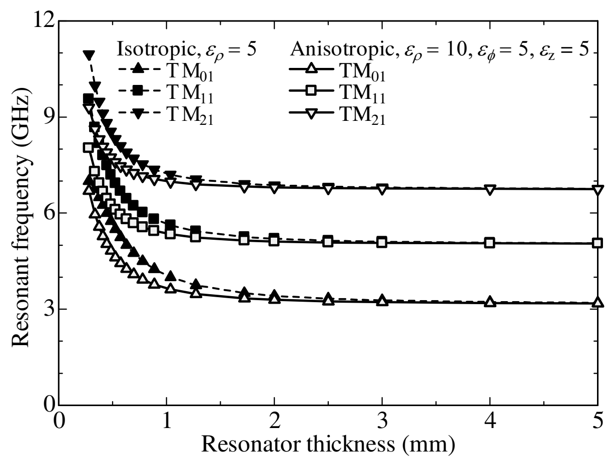

Figure 4: Resonant frequencies of first 3 TM wave modes ($\epsilon_r = 5$ for isotropic, while $\epsilon_\rho = 10$, $\epsilon_\phi = 5$ and $\epsilon_z = 5$ for anisotropic).

shown in Figure 4. From the result, it is noticeable that the anisotropic permittivity in ρ direction will have no effect if the thickness of resonator is more than 0.85 times of the radius. The results in Figures 3 and 4 are satisfied with (7) in which ϵ_z as the nominator plays an important role to the field. Although there is also k_ρ in (7) which covers ϵ_ρ, however its impact is lesser that ϵ_z. From (7), it can also be understood that ϵ_ϕ will have no effect in lowering the resonant frequency for TM wave modes.

4. CONCLUSION

The TM wave mode of circular dielectric resonator with anisotropic permittivity has been analyzed theoretically and compared to the natural one. The analysis result which was emphasized on its resonant frequency for the first 3 TM wave modes has indicated significant improvement for the resonant frequencies of resonator. This is beneficial in microwave application for a resonance mode selection and spurious property enhancement. It has been demonstrated that the anisotropic permittivity in z axis, i.e. propagation direction, has significantly changed the resonant frequencies of circular dielectric resonator. The similar impact was also shown for other anisotropic permittivity in ρ axis for the resonator thickness less than 0.15 times of radius. However, the anisotropic permittivity in z direction has had stronger impact in lowering the resonant frequency than in ρ direction. From the result, it can be concluded that the resonant frequencies of circular dielectric resonator for TM wave modes can be enhanced effectively by changing the relative permittivity in appropriate direction. In near future, FDTD-based computation method using cylindrical coordinate system is still underdevelopment for analyzing the property of circular dielectric resonator with anisotropic permittivity where the comparison results will be reported later.

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