A Microwave Radiation Interferometry Method Based on Adaptive Super-sparse Sampling

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Abstract— The Interferometry Synthetic Aperture Imaging Radiometers (SAIRs) is to sample visibility function based on the Nyquist theory of space interferometry measurement, which do not need the mechanical scanning and can directly image. Due to the complex structure of the imaging system and low imaging resolution, the SAIRs practical application is limited seriously. According to the characteristic of the image is sparse or can be sparse representation in transform domain, the Compressed Sensing (CS) can project the high-dimensional signal to low-dimensional space, so the quantity of the projection measurement data is far less than that by the Nyquist sampling method. Also the microwave radiation interferometry conducted in the frequency domain, which has the characteristics of low frequency information less and high frequency information richer, and the distribution of them is centralized; at the same time, the microwave radiation image itself have the specialty of the gradient sparsity and the local smoothness, it can be sparse representation in differential domain. On the basis of the priori information about the observation and the sparse domain, we establish the incoherent optimization model between the observation matrix and the sparse matrix according to the principle of the two matrixes satisfying the irrelevant in the CS. Using the incoherent optimization model, we can adaptively obtain the spatial measurement with different probability, to realize super sparse interferometry. The adaptive super-sparse sampling method can overcome the disadvantage of equal probability of the Fourier random sampling methods. In order to reconstruct the microwave radiation image, we establish the imaging model based on total variation regularization constraint, and use the alternating iterative algorithm to realize the reconstruction. The simulation and experiment results show that it is fast to reconstruct microwave radiation image with the adaptive super-sparse sampling method, and it can greatly improve the quality of the microwave radiation image in the case of the same sampling rate.

1. INTRODUCTION

Synthetic Aperture Imaging Radiometers (SAIRs) [1] was suggested in the 1980s as an alternative to real aperture radiometry for the observation at low microwave frequencies with high spatial resolution. Instead of the traditional radiometer to directly measure in the spatial domain, SAIRs measurement does not need mechanical scanning, and can imaging by inverted the interference measurements. However, the practical application of microwave radiometric imaging system will be limited by the complexity and low resolution of the structure. The key to solve these disadvantages is reducing the data quantity of the measurement.

According to the principle of microwave radiation and synthetic aperture interferometry [2], the visibility function V and the modified brightness temperature T is related by a Fourier transform for an ideal interferometer:

$$V(u,v) \propto \iint_{\xi^2+\eta^2<1} \frac{T(\xi,\eta)}{\sqrt{1-\xi^2-\eta^2}} F_k(\xi,\eta) F_j^*(\xi,\eta) R\left(-\frac{u\xi+v\eta}{f_0}\right) e^{-j2\pi(u\xi+v\eta)} d\xi d\eta \tag{1}$$

where $F_{k,j}$ is the normalized voltage antenna pattern, f_0 is the center frequency, R is the eliminated stripe function, $(\xi,\, \eta)$ represents the direction cosine, and $(u,\, v)$ is the baseline coordinates of the wavelength normalized. The space frequency can be discretization on the basis of (1), and then the microwave radiation interferometry model is represented by the following form:

$$V = F_\wedge T + e \tag{2}$$

Let $T \in R^{N\times N}$ is 2-D image, and $V \in R^{M\times 1}$ is the measurement vector. Where $F_\wedge \in R^{M\times N}$ is the sparse interferometry operator and e is the noise of receiver. In the ideal measurement condition, $F_\wedge$ is the random sampling of Fourier frequency component. However in most cases, the number of salient features hidden in brightness temperature T is much fewer than its resolution according to the Compressed Sensing (CS) theory [3–5], so $M \ll N$. That means that the brightness temperature distribution T is sparse or compressible under a suitable basis. Let $\Psi \in C^{N\times N}$ be an orthonormal basis, and then T can be represented by the following form:

$$T = \Psi x \tag{3}$$

The vector x contains k nonzero and T is K-sparse under Ψ. Due to the fact that the T possess the sparsity in wavelet domain and horizontal and vertical difference domain, so Ψ can be represent the wavelet basis, the vertical difference basis, or the horizontal difference basis. Then the (2) can be written as:

$$V = F_{\wedge}\Psi x + e \tag{4}$$

Let $F \in R^{N\times N}$ represent the 2-D discrete Fourier transform matrix and $F_{\wedge}$ is the part Fourier transform matrix which select M rows from F. In most situations, the selection is simply random; it is rarely consider the structure information of measurement.

In order to reduce the data quantity, the variable density compressed image sampling method has been proposed [6, 7]. This approach is rely on the mutual coherence between sparse basis and sensing basis, the smaller the mutual coherence the less the required number of measurements for exact recovery. However this method is just for the theoretical analysis, it is suitable for the 1-D signal. Michael Lustig [8] pointed that the energy of magnetic resonance imaging (MRI) signals is usually concentrated at low spatial frequencies, thus they propose to select Fourier basis vectors according to a variable density sampling profile selecting more low frequencies than high frequencies. However in [8], the variable density sampling approach is only empirical.

On the basis of previous studies, and combined with the specialty of the distribution that low frequency and high frequency is centralized, we propose an adaptive super-sparse sampling (ASSp) method. It is based on the variable density sampling principle of 1-D signal, and transforms the 2-D image into two 1-D signal processing to realize the 2-D image sampling. According to this method, we give a optimal sampling profile based on the importance of the frequency for the whole image, that means that the sampling probability for the important information is larger than that for inconsequential information, and the data quantity for the reconstruction is much less than uniformly random sampling (URSp). Our method also gives a mathematical model which for the theoretical analysis.

2. THE ADAPTIVE SUPER-SPARSE SAMPLING

We know that if one want to sample more important frequency spectrums at the condition that the total sampling ratio is very low, the sampling probability of the vital spectrum needs to be high. The ASSp method will provide an optimal sampling profile according to the sampling probability of each spectrum, so as to realize adaptive sampling. In the following sections, we will focus on the introduction of ASSp.

Let $A = F\Psi \in R^{N\times N}$, and the 1-D signal $T \in R^N$ is S sparse on the basis Ψ. According to the (2), we need to select the sampling indices $\wedge = \{l_1, l_2, \ldots, l_M\}$ randomly and independently according to a discrete probability measure P defined on $\{1, 2, \ldots, N\}$. The incoherent optimization model can be defined as:

$$\mu(P) = \frac{1}{N^{1/2}} \max_{1\le i,j\le N} \frac{|\langle F_i, \psi_j\rangle|}{P^{1/2}(i)} \tag{5}$$

where M needs to satisfy the following condition:

$$M \ge CN\mu^2(P)s\log^2(6N/\varepsilon) \tag{6}$$

In the (5), $\mu(P)$ is a function about P and stands for the mutual coherence between the measurement basis F and the sparse basis Ψ. The value depends on the probability measure P and satisfies $\mu(P) \ge N^{-1/2}$. Due to the fact that this model is just suitable for 1-D signal, we should transform the 2-D image into the form of 1-D signal in the setting of 2-D image sampling. Fortunately there is the following mathematical formula can achieve this transformation:

$$F(T) = WTW \tag{7}$$

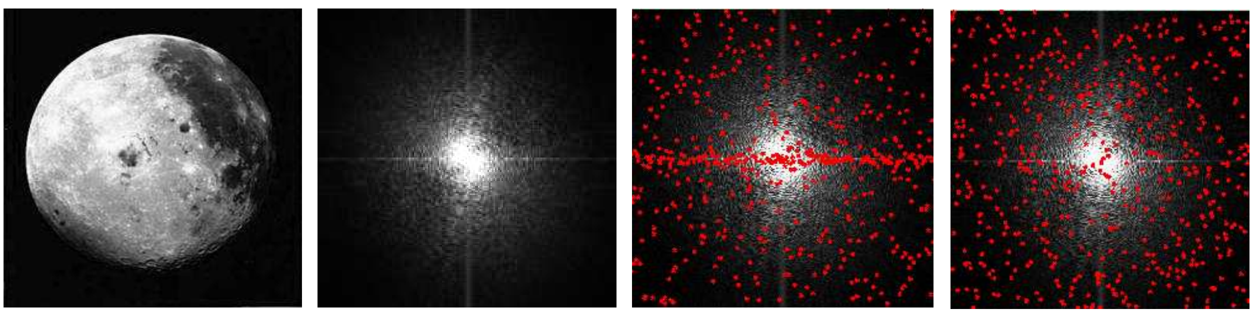
where F is a 2-D discrete Fourier transform matrix, T is the original image and W is 1-D discrete Fourier transform matrix. So based on the mathematical formula, it is realizable to transform the 2-D Fourier transform into two 1-D Fourier transform, which are the horizontal direction transform and the vertical direction transform. Then the image is sparse in these two directions respectively. For the wavelet basis, it is the same for these two direction sparsity. While for the difference basis, it is not in that case. Let Ψ_A be the vertical difference basis, and Ψ_B is the horizontal difference basis, the relationship of them is $\Psi_A = \Psi'_B$.

So for the 2-D image, the whole sampling can be seen as two sampling procedure, and it is need two functions about P based on (5), which one is $\mu(P_1)$, and the other is $\mu(P_2)$. Let $\mu(P_1)$ denote the vertical direction function about the vertical discrete probability measure P_1, and $\mu(P_2)$ denote the horizontal direction function about the horizontal discrete probability measure P_2. They can be denoted by the following form:

$$\mu(P_1) = \frac{1}{N^{1/2}} \max_{1\leq i,j\leq N} \frac{|\langle W_i, \psi_{Aj}\rangle|}{P_2^{1/2}(i)} \tag{8}$$

$$\mu(P_2) = \frac{1}{N^{1/2}} \max_{1\leq i,j\leq N} \frac{|\langle W_i, \psi_{Bj}\rangle|}{P_2^{1/2}(i)} \tag{9}$$

In practice, the choice of the optimized sampling profile P_1 and P_2 is difficult and should be adapted with the number of measurements M in order to obtain the best reconstruction qualities. In this paper, we try to solve this problem by convex optimization algorithms according to [7]. However, some frequencies are repeated sampled in the procedure. So in the final sampling profile, we need to modify the information and guarantee that every frequency is sampled once.



(a) (b) (c) (d)

Figure 1: The spectrum analysis of the moon image. (a) The original image. (b) The frequency spectrum. (c) The sampling of ASSp. (d) The sampling of URSp.

For the ASSp, it can sample more important low frequencies than URSp effectively. An example of a moon image is shown in Figure 1(a). Figure 1(b) is the frequency spectrum of the moon. In order to exactly illustrate the comparison of these two sampling approaches, Figure 1 gives the sampling location in frequency domain at the sampling data is 512, and Figure 1(c) is the sampling of ASSp, Figure 1(d) is the sampling of URSp. We use the red point to denote the sampling location. Figure 1(c) shows that the ASSp can really sample more low information which located in the central position than URSp, and almost all the low frequencies could be got with the increase of sampling data. So in the ASSp, the sampling probability for low frequency is larger than that for high information, and our sampling method realize the adaptive sampling.

For the image reconstruction, considering the situations of microwave image itself is not only can be sparse in a wavelet base but also has the specialties of gradient sparsity and local smoothness, the use of difference base and wavelet base in regularization to exploit image sparsity and preserve edges. This paper refers the algorithm of RecPF in the [9], and adopts the Alternating Direction Algorithm (ADM) to achieve the correct result. By utilizing the separable structure of the variables, we gain the minimum value of the unknown parameters alternately at the each iteration, and then obtain the optimal reconstructed image after numerous iterations.

3. EXPERIMENTAL RESULTS

To test our proposed ASSp algorithm and compared it with URSp, a 256×256 moon temperature image is adopted, which is shown in the Figure 1(a). All experiments were performed under Windows XP and MATLAB R2008a running on a Acer laptop with an Intel Core i3 CPU at 2.4 GHz and 2 GB of memory. In the experiment, we simply set the maximum number of iterations is 100, and the sampling rates (SR) are 10%, 20%, 40%, 50%, 60% and 80%. The specific parameters refer the [9]. All the results are the mean value of the code running 5 times.

The peak signal to noise ratio (PSNR) and the running time t are used to judge the quality of the reconstructed image. In our test, we adopt the Db1 wavelet basis and obtain the optimized sampling profile according to (8) and (9). The reconstructed images of two different sampling methods have been shown in the Figure 2.

(a) (b) (c) (d)

Figure 2: The reconstructed image of the two sampling method. (a) URSp SR = 20%. (b) ASSp SR = 20%. (d) URSp SR = 60%. (e) ASSp SR = 60%.

From the Figure 2, we can see from the visual perspective that the ASSp method could have better results than URSp at the same SR, whether it is 20% or 60%. That indicates the proposed ASSp method for the microwave radiation interferometry is effective. The superiority compared with URSp is obvious, and the reconstruction result become better and better with the SR increasing. In order to further illustrate the effectiveness, we analyze the comparison from the reconstructed parameter, and the reconstruction result of different radio is given in Table 1.

Table 1: Comparison between the two sampling methods.

SR(%)	PSNR/dB		t/s		SR(%)	PSNR/dB		t/s	
	URSp	ASSp	URSp	ASSp		URSp	ASSp	URSp	ASSp
10	26.65	28.74	2.4	22.8	50	28.94	47.31	2.3	14.7
20	27.20	36.23	2.4	19.3	60	37.44	50.85	2.3	14.0
40	28.28	42.98	2.3	15.4	80	42.97	53.30	1.4	13.1

As illustration, Table 1 shows that compared with the URSp, the ASSp can greatly improve the quality of reconstructed image. When the SR is very low, the superiority of ASSp is inconspicuous. However with the increase of SR, the advantage of ASSp become noticeable, the PSNR improves 13.41 dB when the SR is 60%. Simulation results illustrate that the ASSp is robust and effective for the microwave interferometry. While in the Table 1, we can see that the spending time of ASSp is much more than URSp. Therefore it is the focus of the next research and we will pay attention to the optimization problem in the future.

4. CONCLUSION

In this paper, we proposed a new sampling method on the basis of URSp. Compared with the traditional variable density sampling algorithm, the ASSp not only consider the prior information of the microwave image, but also introduce the theory of 1-D signal sampling into 2-D image. By computing the coherence between the sparsity and sensing bases, we can get the optimal sampling profile. The simulation results show that our sampling method can greatly reduce the sampling data and have good effect on the microwave image reconstruction.

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