Electormagnetic Field-focusing EBG Lens

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Abstract— The electromagnetic field focusing capabilities of non-continuous periodic Mikaelian lens is considered. Theoretical continuous Mikaelian lens has the electromagnetic field focusing properties and concentrates incident wave field in one point on the border. Also this lens has sophisticated structure and cannot be implemented using current technologies. The alternative way to implement similar structure is by using periodic layers of constant permittivity which are separated by the periodic layers of air. The idea is to replace continuous lens with the another one, which structure could be easily implemented in practice. Continuous lens has permittivity reducing from 2.56 on the axis of symmetry to 1.0 at the borders. The new lens is a periodic structure, that does not have the same redundancy of permittivity as the continuous one. Such lens has layers containing both the non-conductor and the air, and the ratio of thickness of air to thickness of material increases in the direction from axis of symmetry towards the borders. Thickness ratio of air to thickness of material in each layer is calculated according to the permittivity in the continuous lens on the same distance from the axis of symmetry. Material and air create necessary permittivity in total. It is shown, that the ratio of wavelength to the one period length should be higher than specific value. FDTD method is used for the numerical simulation. The thickness of one layer expressed in the number of grid points is important numerical parameter. When this number is too small, structure of lens close to the axis of symmetry cannot be resolved, and this leads to loss of focusing capabilities. To avoid this, the number of grid points per layer should be at least equal to 10, but the higher this number is the better result would be achieved. The problem with increasing the number is that grid size should also be increased, and the amount of needed memory will also significantly rise. Number of computational experiments is performed and electromagnetic-field focusing capability of layered lens is obtained.

1. INTRODUCTION

A. L. Mikaelian found a solution of synthethis problem of gradient lens with index of refraction which depends on one cartesian coordinate [1]. Gradient Mikaelian lens focuses field of electromagnetic wave in focal point on its surface. There are different ways to implement them in reality, for example ion implantation. This way is used in optics. Another way is to approximate this gradient lens, and approximated one will have almost the same focusing features. EBG Mikaelian lens was proposed in [2]. In this work the focusing features of its special case are studied.

2. GRADIENT LENS

Three-dimensional structure with two metal surfaces is equal two-dimensional, when wave vector is parallel to metal surfaces and orthogonal to periods, and field vector E is orthogonal to surfaces. Figure 1 shows three-dimensional structure where two parallel bricks are metal surfaces and 6 parallel bricks are dielectric periods.

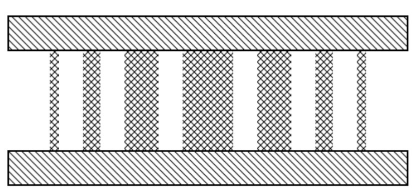

Figure 1: Three-dimensional structure.

A. L. Mikaelian has shown a solution for such gradient lenses. Structure, which has reflection index changing across only one axis proportional to the minus first power of hyperbolic cosine, is a lens for electromagnetic waves. Let reflection index change across the O_y axis, and planar wave will propagate across the O_x axis. Lens width by O_x axis is **L**. This lens has an axis of symmetry and the O_x axis is set to exactly match it, the axis of symmetry is parallel to the wave vector and orthogonal to the O_y axis. Axis of symmetry divides structure into two symmetrical parts. O_y axis is set to exactly match left border of the lens, i.e., the first border wave reaches during propagation.

Reflection index changes according to [1]:

$$n(y) = \frac{n_0}{ch\left(\frac{\pi|y|}{2L}\right)} \tag{1}$$

Note that permittivity is expressed like this using reflection index:

$$\varepsilon(y) = n^2(y) \tag{2}$$

Permittivity and reflection index reduce from the maximum value on the O_x axis (axis of symmetry) to the minimun value on upper and lower borders. **R** is the thickness of both upper and lower symmetrical parts by O_y axis. Let the maximum value of reflection index be $n_0 = 1.6$, the minimum value equals 1.0, which is the same to the value of the vacuum, surrounding the structure. The maximum value of permittivity is $\varepsilon_0 = 2.56$. The thickness **R** becomes determined after setting the width **L**, because **R** is retrieved from the next formula:

$$n(R) = \frac{n_0}{ch\left(\frac{\pi R}{2L}\right)} = 1.0 \tag{3}$$

3. LAYERED APPROXIMATION

Gradient lens is difficult to create because of continuous permittivity and refraction index change. But permittivity could be divided into parts, i.e., gradient change could be replaced with discrete change. New layered lens has discrete levels of permittivity, and permitivity also reduces from the axis of symmetry to upper and lower border, but not in a continuous way. Figure 2 shows difference in permittivity distribution between this layered lens and a gradient one.

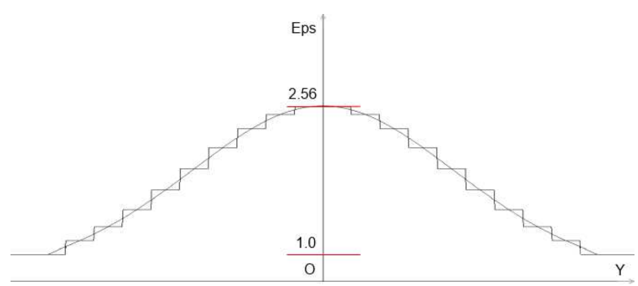

Figure 2: Permittivity distribution in gradient and layered lenses.

In layered lens permittivity in each layer is set according to permittivity in gradient lens at the same point on O_y axis as the middle point of the layer. N is the total number of layers, i is the number of current layer. For reflection index:

$$n(y) = n_i, \quad l_i - \frac{R}{2N} \le y < l_i + \frac{R}{2N} \tag{4}$$

$$n_i = \frac{n_0}{ch\left(\frac{\pi l_i}{2L}\right)} \tag{5}$$

$$l_i = \left(i + \frac{1}{2}\right)\frac{R}{N} \tag{6}$$

$$0 \le i < N - 1 \tag{7}$$

Such a layered lens is easier to implement in reality than a gradient one, but let's use another approximation.

4. EBG APPROXIMATION

There is another way to approximate gradient lens [2]. Each layer (period) will be divided into two internal layers of material and vacuum. In each period internal layer of material will be closer to the axis of symmetry, besides, material will be the same everywhere and will have permittivity equal to the permittivity on the axis of symmetry. This approximation is called Electromagnetic Bandgap (EBG). Figure 3 shows difference between layered and EBG approximations. This lens is easier to create in reality.

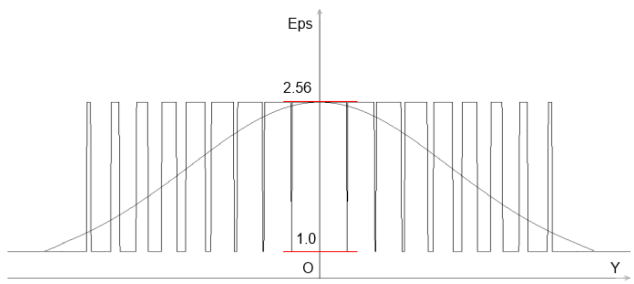

Figure 3: Permittivity distribution in gradient and EBG lenses.

The idea is that permittivity in each period in average will be equal to the permittivity in gradient lens at the same point on O_y axis as the middle point of the period. Average permittivity in period is

$$\varepsilon_{average_i} = \varepsilon_0 * \frac{l_{1_i}}{l_i} + \frac{l_{2_i}}{l_i} \tag{8}$$

$$l_{1_i} + l_{2_i} = l_i \tag{9}$$

l is the thickness of one period, l_1 is the thickness of material in period and l_2 is the thickness of vacuum. Period filling index is the ratio of material thickness to vacuum thickness [3]:

$$c_i = \frac{l_{1_i}}{l_{2_i}} = \frac{\varepsilon_{average_i} - 1}{\varepsilon_0 - 1} \tag{10}$$

Reflection index in EBG Mikaelian lens is calculated like this:

$$n(y) = \begin{cases} n_0, & l_i - \frac{R}{2N} \le y \le l_i - \frac{R}{2N} + c_i \\ 1, & l_i - \frac{R}{2N} + c_i < y < l_i + \frac{R}{2N} \end{cases} \tag{11}$$

$$\varepsilon_{average_i} = n_i^2 \tag{12}$$

Average refraction index in period n_i and coordinate of the middle point of the period l_i are retrieved from (5) and (6).

5. NUMERICAL MODELLING OF EBG MIKAELIAN LENSES

Numerical modelling was done using total field/scattering field method for wave excitation and perfectly mathced layer for scattered wave cancelation [4]. TMz mode was simulated.

Permittivity of material and vacuum create necessary permittivity in total. That's why this lens does not work as a lens to all the incident waves, the ratio of wavelength to the thickness of one period should be higher than some value, then wave sees structure in total and not as separated pieces of material. This ratio is the first significant parameter. Numerical experiments show that it should be equal at least 2, but the higher it is, the better results would be achieved.

The second significant parameter is the thickness of one period expressed in number of grid points. When this number is too small, periods close to the axis of symmetry lose all differences between them. They become almost fully-filled with material except one line of grid points, because there are not enough points to fulfill needed permittivity accuracy. Numerical experiments show that to avoid this problem number of grid points per period should be at least 10, but the higher it is, the better results would be achieved. The problem with increasing both significant parameters is that large amounts of memory are required.

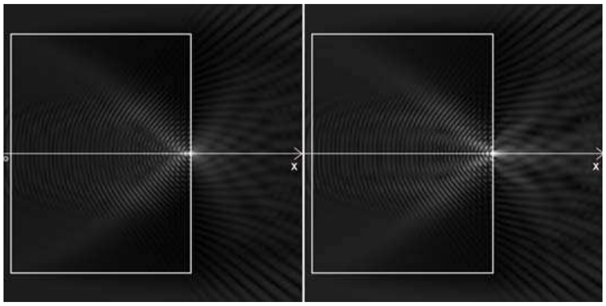

Figure 4: $|E_z|$ distribution for gradient and EBG lenses.

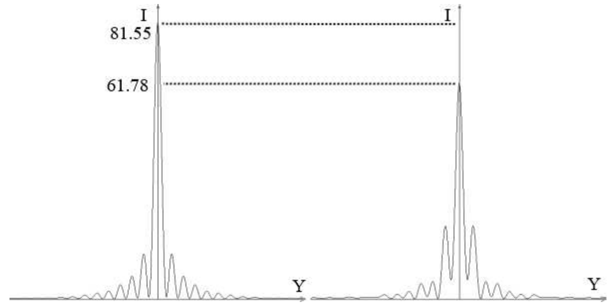

Figure 5: $|Ez|^2$ distribution for gradient and EBG lenses in focus plane.

Next results were achieved for lens with $\mathbf{L} = 100 * 10^{-3}$ m and wave with wavelength $\lambda = 6.7 * 10^{-3}$ m and amplitude 1.0. From previous formulas $\mathbf{R} = 67 * 10^{-3}$ m. Lens has 80 periods in every symmetrical part, thickness of one period is $l = 0.84 * 10^{-3}$ m. Ratio of wavelength to thickness of one period is 7.98 and the grid was set to have 20 points in each period. Figure 4 shows planar wave field amplitude E_z distribution for gradient lens on the left side and for EBG lens on the right side. The brighter the point is, the higher field value it has. White rectangle is a lens itself.

Focus point of EBG lens is moved $0.125*10^{-3}$ m forward by O_x axis from focus point of gradient lens, it is also blured by O_x axis, i.e., EBG lens has a bit higher focus length. Focus intensity of EBG lens is 76 percent of focus intensity of gradient lens. Focus spots have the same diameter. Figure 5 shows intensity distribution in focus plane.

6. CONCLUSION

In this work EBG approximation of Mikaelian lens was studied. In order to simulate such structures, large amounts of memory are required, which will allow to set both significant parameters to acceptable values. First significant parameter is ratio of wavelength to the thickness of one period, which should be greater than 2 to get focusing of electromagnetic wave. Second significant parameter is number of grid points per period; it should be high enough to simulate all structure properties.

EBG Mikaelian lens definitely has focusing features, but lens has to have large amount of periods to get close to focusing capabilities of gradient lens. EBG Mikaelian lens field distribution will get closer to field distribution of gradient lens during increasing of number of periods, and it was shown that total 160-period lens has good focusing features.

ACKNOWLEDGMENT

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