RCS Computation of 3D-wake Vortex Using Method of Moments

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Abstract— An efficient and accurate 3D full wave numerical solution model for computing the radar cross section (RCS) of wake-vortex is presented, by solving Euler equation to compute the pressure in the aircraft wake region by computational fluid dynamics model and parallel integral equation (IE), based on method of moments (MoM) is used to compute the volume polarization currents, under the plane wave incidence. Then the currents are used to compute far-field from the vortex region.

1. INTRODUCTION

The scattering of radio waves in the atmosphere has drawn attention of many researchers in recent times, particularly in the areas of radio acoustic sounding system (RASS) [1–5] and wake-vortex studies [4–9]. In clear air, the radar reflectivity of wake vortex is mainly caused by Bragg scattering from the refractive index variations. Since wake turbulence is inevitable to all flying aircrafts, remote detection and tracking of wake vortices is important for hazard avoidance especially near airports during landing and take-off phases. Aircrafts that fly through these hazardous vortices experience sudden induced roll. In order to avoid the wake vortex encountering hazard, efforts have been made in the past to monitor and detect wake vortices, and the existing technologies include lidar, sodar and radar. Among them, the radar detection is considered to be the potential candidate for its long working range and under different weather conditions [10, 11].

In the past studies, aircraft wake is evolved by known vortex model derived from experiment based on aircraft geometry and inertial properties [6]. Also vortex wake is based on wing alone model; body and tail vortex effect are not considered and these models are confined to 2D cross sections in the wake region. And the RCS is computed using Born approximation with highly oscillatory integrand, with the aid of quadrature methods. The validity and applicability of Born approximation is well studied in the low frequency regime, and more recently even at high frequency regime [7].

However, there is still a need to improve the accuracy and efficiency of wake vortex scattering problem, which demands for a rigorous full wave solution method Owing to huge disparity in EM and acoustic frequencies, a quasi-stationary approximation using an adiabatic approach enables EM propagation at different instants of acoustic-flow cycle to be posed as scattering problem of a frozen inhomogeneous media [8], which then can be effectively solved through integral equation using MoM.

2. FORMULATION

The Euler equations which are derived from conservation laws of mass, momentum and energy applicable for compressible inviscid flows are expressed in vector form in Cartesian coordinate system as follows [10]:

$$\frac{\partial \mathbf{m}}{\partial t} + \frac{\partial \mathbf{f}_x}{\partial x} + \frac{\partial \mathbf{f}_y}{\partial y} + \frac{\partial \mathbf{f}_z}{\partial z} = 0 \tag{1}$$

$$\mathbf{m} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ E \end{pmatrix}; \quad \mathbf{f}_x = \begin{pmatrix} \rho u \\ p + \rho u^2 \\ \rho uv \\ \rho uw \\ u(E+p) \end{pmatrix}; \quad \mathbf{f}_y = \begin{pmatrix} \rho v \\ \rho uv \\ p + \rho v^2 \\ \rho vw \\ v(E+p) \end{pmatrix}; \quad \mathbf{f}_z = \begin{pmatrix} \rho w \\ \rho uw \\ \rho vw \\ p + \rho w^2 \\ w(E+p) \end{pmatrix} \tag{2}$$

There are five equations and six unknowns. To close the system one more equation is required which is supplemented by equation of state:

$$p = \rho\,(\gamma - 1)\,e \tag{3}$$

Where $\mathbf{m}$ is a vector of conservative variables, which are to be computed, $\mathbf{f}_x$, $\mathbf{f}_y$ and $\mathbf{f}_z$ are convective fluxes in x, y and z directions respectively, p is the pressure, ρ is the density, γ is the adiabatic index and e is the internal energy per unit mass of the fluid, also we have total energy per unit volume, E as

$$E = \rho e + \frac{1}{2}\rho(u^2 + v^2 + w^2) \tag{4}$$

Let V be the inhomogeneous dielectric volume with relative permittivity $\epsilon_r(\vec{r})$ and relative permeability $\mu_r(\vec{r}) = 1$. The permittivity and permeability of free space are ϵ_0 and μ_0 respectively. Let $\vec{E}^i$ be the incident field and $\vec{E}^s$ be the scattered field; then the total field $\vec{E}$ can be expressed as the sum of $\vec{E}^i$ and $\vec{E}^s$ [14]

$$\vec{E}\left(\vec{r}\right) = \vec{E}^i\left(\vec{r}\right) - \vec{E}^s\left(\vec{r}\right) \tag{5}$$

The scattered field due to the volume polarization current $\vec{J}_v(\vec{r}) = j\omega(\varepsilon(\vec{r}) - \epsilon_0)\vec{E}(\vec{r})$ in V can be written as

$$\vec{E}\left(\vec{r}\right) = \vec{E}^i\left(\vec{r}\right) + j\omega\mu_0 \int_V \bar{\bar{G}}\left(\vec{r}, \vec{r}'\right) \cdot \vec{J}\left(\vec{r}'\right) dv' \tag{6}$$

$$\text{where } \bar{\bar{G}}\left(\vec{r}, \vec{r}'\right) = \left(\bar{\bar{I}} + \frac{1}{k_0^2}\nabla\nabla\right) \cdot G\left(\vec{r}, \vec{r}'\right) \tag{7}$$

$\bar{\bar{I}}$ is a unit dyad and $G(\vec{r}, \vec{r}')$ is the free space Green's function, with $k_0 = \omega\sqrt{\mu_0\epsilon_0} = \frac{2\pi}{\lambda_0}$.

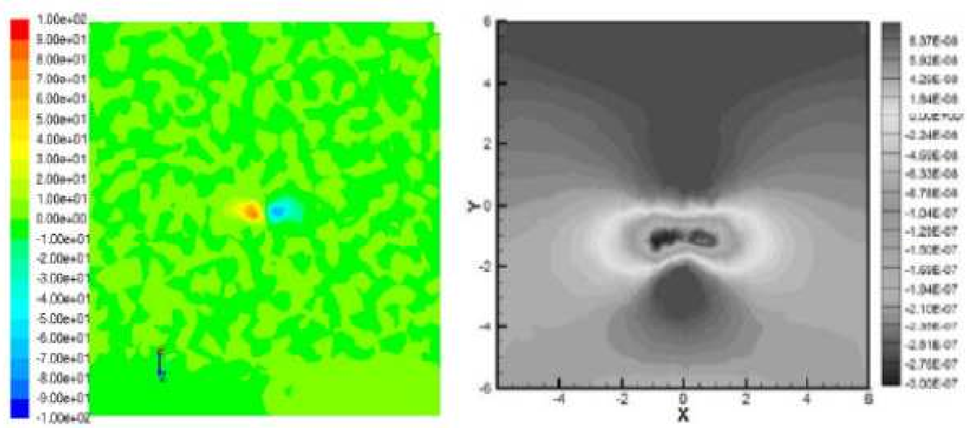
The use of method of moments requires the total field $\vec{E}(\vec{r})$ or unknown electric current density $\vec{J}_v(\vec{r})$ inside domain to be expanded in terms of a set of N orthogonal basis functions $\vec{f}_n$ which reduce Equation (6) into system of linear equations, of the form [15, 16]

$$\mathrm{ZI} = \mathrm{V} \tag{8}$$

3. NUMERICAL RESULTS

In this section, we present numerical results of dielectric constant variation due to pressure in the wake from a typical aircraft model of wing span 2.16 m and fuselage 2.5 m, and hence RCS computation. The flow parameters of density, pressure and velocity, around the aircraft are obtained by solving for inviscid compressible flow by a finite volume formulation using ANSYS FLUENT solver. The rectangular parallelepiped flow domain with embedded aircraft is discretized with unstructured tetrahedral mesh.

The case analysed is for the landing configuration with velocity 133 m/s, angle of attack of 10 degrees, so the boundary condition set at inlet is velocity prescribed at 133 m/s with $\alpha = 10$ degrees. Outlet Boundary condition is set as pressure boundary conditions and velocity is computed. Other four boundaries, two sides and top and bottom boundaries are set as characteristic boundary



(a) (b)

Figure 1: (a) X-Vorticity contour from flow solver. (b) Dielectric Constant variation due to pressure.

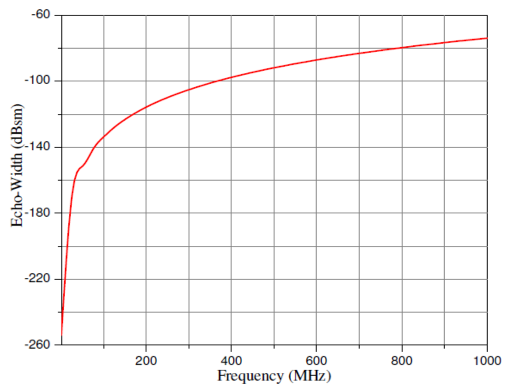


Figure 2: Echo width from the 2D wake profile

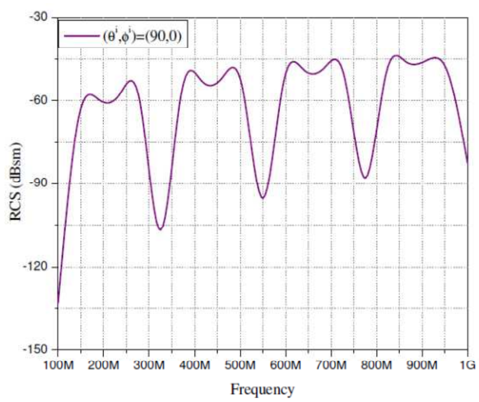


Figure 3: RCS vortex core of size $0.3\,\text{m} \times 6\,\text{m} \times 6\,\text{m}$.

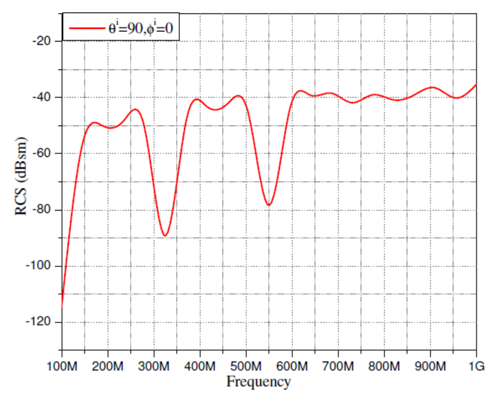


Figure 4: RCS from vortex core of size $0.9\,\text{m} \times 6\,\text{m} \times 6\,\text{m}$.

conditions which are nothing but inflow/outflow boundary condition depending on direction of characteristic waves.

The link between Maxwell and Euler equations is the relationship between the pressure perturbation and the permittivity fluctuations, i.e., the dielectric constant in the integral equation is function of pressure in the wake region. The pressure obtained from the flow solver is used to compute the dielectric constant by using [4]

$$\epsilon_r = 1 + 1.552 \times 10^{-6} P/T \tag{9}$$

And further interpolated over the MoM solver domain. The X-vorticity contour and dielectric constant variation due to pressure are shown in Figure 1. The 2D computational domain is of size $12\,\text{m} \times 12\,\text{m}$, discretized with respect to free space wavelength at highest frequency resulting into 160000 square grids. RCS of two 3D regions about the vortex core viz., near the tail of aircraft $0.3\,\text{m}\times 6\,\text{m}\times 6\,\text{m}$ and the other far from tail $0.9\,\text{m}\times 6\,\text{m}\times 6\,\text{m}$ are computed. The volume is discretized with cubical grids of equal volume. 8 cells per wavelength, with respect to the highest frequency of operation considered, resulting into 440000, 675000 grids and 1.32, 2.025 million unknowns. In both the cases pulse basis and point matching technique are used to convert the integral equations to the system of linear equations. The system of linear equations is solved using a parallel BiCG solver.

4. CONCLUSION

In the present work aircraft wake is computed by solving 3D Euler equations using ANSYS FLUENT solver with FVM method. Complete aircraft flow field is computed including vortex generated by all the components such as body, wing and tail units. CFD based wake flow computation is more accurate than the vortex model based computation. Also, the IE solution using MoM for RCS computation is a full wave method. Thus, for both flow as well as EM fields, accurate methodologies have been employed, enabling the solution to be more accurate than when models or approximations are used for the governing equations.

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