Semi Analytical Model for Non-Resonant Layered Frequency Selective Surfaces (FSS)

Poojali Jayaprakash and Kavitha Arunachalam
Department of Engineering Design, Indian Institute of Technology Madras, 600036, India

Abstract— The proposed approach employs transmission line theory to model electromagnetic wave propagation through FSS comprising of non-resonant metal grids and patches. The equivalent reactance of the individual metal layers is initially estimated by minimizing the error between scattering parameters obtained from 3D full wave simulations and $ABCD$ matrix of the free standing structure. With the knowledge of the individual metal layer's reactance, a simple analytical model is adopted to predict the frequency response of multi-layered FSS for different configurations without the need for 3D full wave simulations. Performance of this semi-analytical approach is studied for varying unit cell size, D ($\lambda/5$–$\lambda/15$), metallization dimensions, and number of FSS layers in X-band (8–12 GHz). The frequency response of the semi-analytical model is in good agreement with 3D full wave simulations suggesting that the proposed approach is faster and simpler for design of multi-layered non-resonant FSS.

1. INTRODUCTION

Miniaturized FSS are periodic metallic structures comprising of non-resonant inductive grids and capacitive patches stacked on thin low loss dielectric substrate. These are widely used in satellite communication and remote sensing systems for selective reception or rejection of a band of frequencies [1]. In non-resonant FSS, the size, D of an unit cell, i.e., building block of the periodic structure is typically smaller compared to the operating wavelength (λ) [2]. Equivalent circuit models provide a quick and easy means to predict and understand the frequency response of layered non-resonant unit cells [2, 4, 5]. In [2], a transmission line model was introduced to explain the frequency response of non-resonant FSS comprising of inductive grids and capacitive patches. Analytical expressions for inductance (L) and capacitance (C) in [2] which were based on [3] do not hold good for design and analysis of layered FSS. Hence, a generalized equivalent circuit method was presented in [4]. However, this method requires further numerical simulations for parameter optimization. Furthermore, it models the dielectric layer in the FSS as a LC circuit instead of a delay line. In this work, we propose a computationally simple and realistic model for design of multi-layered non-resonant FSS. Section 2 presents the analytical model, L and C parameter estimation for the individual metal layers and model validation for layered non-resonant FSS. The performance of the proposed model is validated using a finite element method based EM simulation software, HFSS® for varying unit cell size, D ($\lambda/5$–$\lambda/15$), metallization dimensions and FSS layers (1–3) in the X band (8–12 GHz). Results and discussion are presented in Section 3 followed by conclusion.

2. METHODOLOGY

2.1. Analytical Model

Figure 1 shows the generalized cascaded transmission line representation of a multi-layered FSS, where M_i refers to the ith non-resonant metal layer and D_i is the ith low loss dielectric substrate, d_i is the thickness of ith dielectric substrate and and $i = 1, 2, \ldots, n$. The cascaded transmission line is terminated by ports 1 and 2 with free space impedance (Z_0). The measurement plane for the input (port 1) and output (port 2) ports is the defined at a distance, d_0 from the layered FSS as in the

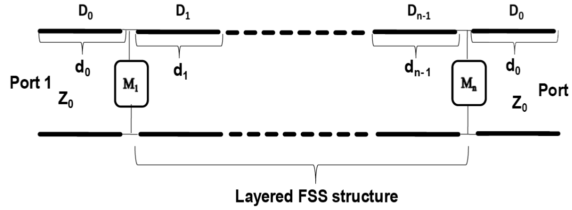

Figure 1: Generalized Transmission line model approach for Non-resonant FSS structures.

3D full wave simulations. It should be noted that the metal layers are modeled as either inductive or capacitive elements depending on the unit cell's physical structure, and dielectric substrates are modeled as delay lines. The $ABCD$ parameters of the cascaded structure is given by [6],

$$\begin{bmatrix} A^C & B^C \\ C^C & D^C \end{bmatrix}_{cascaded} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{Air} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{FSS} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{Air}, \quad (1)$$

where,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{FSS} = \prod_{i=1}^{n} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{M_i} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{D_i}; \quad \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{Air} = \begin{bmatrix} \cos(\beta_0 d_0) & jZ_0 \sin\beta_0 d_0 \\ \frac{j\sin(\beta_0 d_0)}{Z_0} & \cos(\beta_0 d_0) \end{bmatrix} \quad (2a)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{D_i} = \begin{bmatrix} \cos(\beta_i d_i) & jZ_0 \sin\beta_i d_i \\ \frac{j\sin(\beta_i d_i)}{Z_0} & \cos(\beta_i d_i) \end{bmatrix}; \quad \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{M_i} = \begin{bmatrix} 1 & 0 \\ Y_i & 1 \end{bmatrix}; \quad i = 1, 2, 3, \ldots, n \quad (2b)$$

In Eq. (2), $\beta_0 = \frac{2\pi}{\lambda_0}$, is free space propagation constant, $\beta_i = \frac{2\pi}{\lambda_i}$, is propagation constant in the dielectric substrate and $Y_i = jwL_i$ or $1/(jwC_i)$ depending on the type of metal layer. A simpler estimate for L_i and C_i based on the theory of infinite wire grids and metal patches given by [3],

$$C_i = \varepsilon_0 \varepsilon_{eff} (2D/\pi) \log[csc(\pi s/2D)]; \quad L_i = \mu_0 (D/2\pi) \log[csc(\pi w/2D)] \quad (3)$$

was adopted in [2, 4, 5] for equivalent circuit representation of non-resonant FSS. In Eq. (3), D is unit cell size, s is gap between patches, w is width of the wire grid, ε_{eff} is effective permittivity of the medium and ε_0, μ_0 are free space permittivity and permeability respectively. Using Eqs. (1)–(2), transmission (S_{21}^C) and reflection (S_{11}^C) scattering parameters of the cascaded system in Figure 1 can be written as [6],

$$S_{11}^C = \frac{A^C + \frac{B^C}{Z_0} - C^C Z_0 - D^C}{\left(A^C + \frac{B^C}{Z_0} + C^C Z_0 + D^C\right)}; \quad S_{21}^C = \frac{2}{\left(A^C + \frac{B^C}{Z_0} + C^C Z_0 + D^C\right)} \quad (4)$$

For a given estimate of the circuit parameters, L_i and C_i for the ith metal layer, Eqs. (1), (2), (4) were used to calculate the scattering parameters of the layered non-resonant FSS.

2.2. 3D EM Simulation

Scattering parameters of the individual metal layers in free space were obtained using 3D EM simulation software, HFSS®. Due to the periodicity of the unit cell, 3D EM wave simulations were studied for the FSS unit cell by enforcing periodic boundary conditions. Scattering parameters were calculated for a free standing non-resonant unit cell of size, D for an incident plane wave with TM polarization and metal thickness of 17 microns. Simulations were carried out for varying cell size (D), patch gap (s) and grid width (w) summarized in Table 1 for signal transmission in the X band (8–12 GHz). Transmission parameter, obtained for the inductive and capacitive unit cell FSS models in Table 1 were used to estimate the equivalent inductance (L) and capacitance (C) respectively based on the model presented in Section 2.1.

2.3. Circuit Parameter Estimation

The equivalent circuit parameter of the free standing single layers in Table 1 was estimated by solving the following minimization problem,

$$\min\left(\left\|S_{21}^M - S_{21}^C(y)\right\|^2\right), \quad y \in R^1 \quad (5)$$

Table 1: Single layer inductive grids and capacitive patches simulated in HFSS®.

Unit cell	D (mm)	s (mm)	w (mm)
1	2	0.25	0.25
2	3	0.5	0.5
3	4	0.3	0.3
4	5	0.5	0.5
5	7	1.0	1.0

where S_{21}^M and S_{21}^c are complex one dimensional vectors of size $N \times 1$ representing frequency domain transmission parameters obtained using 3D EM simulations and Eq. (4) respectively over 8–12 GHz (X band) and, y is the real valued circuit parameter which is either L or C depending on the metal layer. This optimization problem was solved using golden section algorithm which is one of the efficient methods for single variable optimization [7]. It should be noted that the complex transmission vectors, S_{21} in Eq. (4) are represented as real valued vectors of size $2N \times 1$ in the optimization routine. The search space was bounded between $y_{\min}$ and $y_{\max}$ estimated based on Eq. (3) and the search was continued until the cost function in Eq. (5) was below a predetermined threshold, δ.

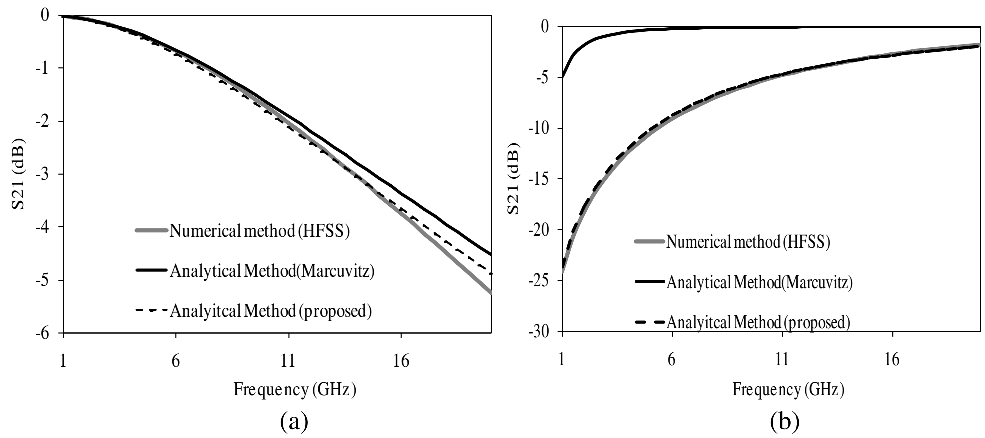

Figure 2: Comparison of the transmission parameter obtained for a free standing FSS metal layer using analytical approach Eq. (3) and the proposed semi-analytical model with 3D EM simulations; (a) patches and (b) wire grids for unit cell #5 in Table 1.

2.4. Multilayered FSS Design

Agreement between the proposed semi-analytical model and 3D EM simulation was studied for a second order X band (8–12 GHz) FSS. For this study, unit cell model 1 was chosen since the idea was to create a miniaturized layered FSS structure. The second order FSS unit cell consisted of the following layers: patch, dielectric, grid, dielectric and a patch. The physical dimensions of multi-layered FSS were $D = 2$ mm ($\lambda/15$ at 10 GHz), $d_1 = 0.64$ mm, $d_2 = 1.28$ mm, $s_1 = 0.1$ mm, $s_2 = 0.35$ mm and $w = 0.35$ mm. The dielectric material chosen was RO3010 with dielectric constant $\varepsilon_r = 10.2$ and loss tangent, 0.0035. The scattering parameters of the multi-layered FSS were calculated using the cascaded 3-layer FSS $ABCD$ matrix (Eq. (1)) where the $ABCD$ matrices of the metallic grid and patches were determined following the approach in Section 2.3.

3. RESULTS AND DISCUSSION

3.1. Equivalent Circuit Model for Single FSS Layer

3D EM simulations were carried out for the unit cell models in Table 1 using HFSS and estimation of the equivalent circuit parameters for the free standing metal layers were carried out in MATLAB. Figure 2 shows signal transmission parameter, S_{21} calculated for unit cell #5 for both inductive grid and capacitive patch models using 3D EM simulations, and estimated L and C values of the proposed equivalent circuit approach. Equivalent circuit approach based on Eq. (3) is also shown in Figure 2 for comparison. It should be noted that the frequency response exhibits low pass behavior for capacitive patches and high pass behavior for inductive grids for both analytical models as explained in [2, 3]. However, the proposed approach yields a better approximation for the 3D EM simulations over the entire frequency band. This enables us to extend the model prediction for design and analysis of multi-layered FSS structures comprising of repetitive capacitive and inductive grids. Table 2 summarizes the equivalent circuit parameters estimated using the proposed semi-analytical model for the single layer FSS unit cells listed in Table 1. Circuit parameters based on Marcuvitz model (Eq. (3)) is also listed in Table 2 for comparison. Interestingly, a comparison between the circuit parameters estimated using the proposed semi-analytical model and Marcuvitz's analytical model indicates an average correction factor of 0.3π for capacitance and 0.03π for inductance to Eq. (3).

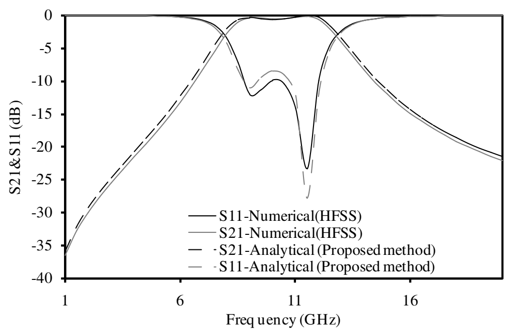

Figure 3: Comparison of the scattering parameters obtained for the second order FSS using the proposed semi-analytical approach with 3D EM simulations.

Table 2: Summary of the equivalent circuit parameters estimated by the proposed model and comparison with analytical Eq. (3) proposed in [2] for free standing inductive and capacitive metal layer of Table 1.

Unit cell	L-Marcuvitz (nH)	L-optimized (nH)	C-Marcuvitz (fF)	C-optimized (fF)
1	6.45	0.56	18	19
2	8.00	0.71	22	23
3	16.91	1.59	47	49
4	18.31	1.73	50	54
5	20.76	1.98	57	59

Table 3: Comparison of proposed method with [2].

SI. No.	**Frequency (GHz)**	**Transmission Loss, S_{21} (dB)**		
		HFSS	**Proposed Method**	**Ref. [2]**
1	8.7	−0.6	−0.8	−4.4
2	9.0	−0.5	−0.7	4.8
3	9.5	−0.4	−0.7	−5.3
4	10.0	−0.5	−0.7	−5.8
5	10.5	−0.4	−0.4	−6.3
6	11.0	−0.2	−0.1	−6.7
7	11.5	−0.1	−0.04	−7.5

3.2. Model Validation for Multi-layered FSS

Figure 3 shows the comparison between the scattering parameters obtained for the second order FSS using the proposed semi-analytical model with 3D EM simulations. It can be observed that the proposed semi-analytical model compares very well with 3D simulations of the cascaded multi-layered FSS. Table 3 summarizes the performance of the proposed equivalent circuit model for the second order FSS with 8–12 GHz transmission window. It should be noted that the semi-analytical model indicated a constant correction factors to the analytical Eq. (3) depending on whether the metal layer comprised of grids or patches. Substitution of the analytical Eq. (3) with the correction factors for L and C in the cascaded transmission line model given by Eqs. (1) and (2) yielded very good results with the full wave simulations. Thus, the proposed semi-analytical model could be used as a quick and easy means to predict the frequency response of multi-layered non-resonant FSS.

4. CONCLUSION

A cascaded transmission line model is proposed for design and analysis of multi-layered non-resonant FSS comprising of metallic grids and patches. The equivalent circuit parameters estimated for the free standing metal layers based on the proposed semi-analytical approach were found to differ from the analytical equations by a constant correction factor. The performance of the proposed cascaded transmission line model obtained using the estimated equivalent circuit parameters for the individual metal layers agreed very well with 3D simulations. Application of the correction factor to the classical model in [3] was shown to work for both single and multi-layered miniaturized FSS. The proposed methods is a quick and easy means to design multi-layered FSS configurations without the need to run time consuming 3D EM simulations for the cascaded structure.

REFERENCES

1. Munk, B. A., *Frequency Selective Surfaces: Theory and Design*, Wiley-Interscience, New York, 2000.
2. Sarabandi, K. and N. Behdad, "A frequency selective surface with miniaturized elements," *IEEE Trans. Antennas Propag.*, Vol. 55, No. 5, 1239–1245, May 2007.
3. Marcuvitz, N., *Waveguide Handbook*, Boston Technical Publishers, Lexington, MA, 1964.
4. Al-Joumayly, M. A. and N. Behdad, "A generalized method for synthesizing low-profile, band-pass frequency selective surfaces with non-resonant constituting elements," *IEEE Trans. Antennas Propag.*, Vol. 58, No. 5, Dec. 2010.
5. Al-Joumayly, M. and N. Behdad, "A new technique for design of low-profile, second-order, bandpass frequencyselective surfaces," *IEEE Trans. Antennas Propag.*, Vol. 57, No. 2, 452–459, Feb. 2009.
6. Ludwig, R. and G. Bogdanov, *RF Circuit Design Theory and Applications*, 2nd Edition, Pearson, 2012.
7. Deb, K., *Optimization for Engineering Design*, PHI Learning Private Limited, 2010.