Modification of Simplified Modal Method for Subwavelength Triangular Grating with Very High Efficiency

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Abstract— A modification of simplified modal method (MSMM) is presented to calculate the transmission efficiencies of subwavelength fused-silica grating. In order to consider the reflection of grating, which is neglected in simplified modal method (SMM). Then the grating region is divided into N thin planer grating slabs and regarded as multilayer films. Then the reflectance is characterized by the effective admittances of air and grating which are complete characteristic of multilayer films. This proposes a manner of reducing the reflection loss of dielectric grating. By analyzing the effective index and admittance of grating, we designed a triangular fused-silica grating with more than 99.9% diffraction efficiency for both TE and TM-polarization. This method offers a clear physical insight of diffraction characteristic: a deep-etched grating would get a high efficiency when the average difference of its mode indices is odd times of π and the admittance of the grating is approximate to that of air. Due to tiny deviation between this method and RCWA, MSMM can be used to design and optimize fused-silica gratings directly. It is better than SMM which only gives starting value of grating parameters. Contrast to RCWA, the starting parameters values of subwavelength dielectric grating can be predicted by the formulas in MSMM when the transmission efficiency is known. However, the starting values are got by cut-and-try procedure in RCWA.

1. INTRODUCTION

Dielectric gratings with high damage threshold are important elements in the pulse compressors in high power laser systems [1, 2]. In these devices, a high efficiency for the dispersive −1st order is usually required. Calculated by rigorous coupled wave analysis (RCWA) [3–5], a triangular dielectric grating can get efficiency 99% for TE polarization [3]. But we can't know the physical mechanism that a dielectric grating with designated parameters can get a high efficiency when grating was designed by RCWA. Recently, Clausnitzer et al. introduced the simplified modal method (SMM) [6, 7], which analyzes the transmission efficiency by the phase shift of different modes, to explain the diffraction in deep-etched dielectric grating. This provides a new method for designing highly-efficient grating. But the reflection is neglected in SMM, thus the diffraction efficiency of a real grating is always less than the predicted result. Fresnel reflection at interfaces is introduced into SMM by a symmetrical embedded grating and a 99.9% efficient grating was designed for TE polarization [8]. Sun et al. proposed multireflection modal method (MRMM) [9] by introducing multiple Fresnel reflection into SMM. An enhanced simplified modal method (ESMM), considering the reflection of modes, was presented by Jing et al. [10]. But these methods only discussed the TE-polarized illumination. Actually, TM-polarized light is different with TE-polarized light. It is impossible to deflect both TE and TM-polarized light to −1st order with 100%-efficiency for a rectangular grating [8].

In this paper, a modification of simplified modal method (MSMM) is presented to calculate the transmission efficiencies of subwavelength dielectric grating. The total reflection of a grating is characterized by the effective admittances of air and grating. The admittance of triangular grating is analyzed to decide when a triangular grating with some particular parameters will get a low reflection. Together with the average difference of mode indices fulfilling a certain condition, a grating with efficiency more than 99.9% for both TE and TM-polarized light is designed.

2. MODIFICATION OF SIMPLIFIED MODAL METHOD

For a rectangular grating, the effective index n_{eff}^{m} is given by the dispersion equation [11]:

$$\cos(\alpha d) = \cos(\beta b)\cos(\gamma g) - \frac{\beta^2 + \tau^2\gamma^2}{2\tau\beta\gamma}\sin(\beta b)\sin(\gamma g) \tag{1}$$

where $\alpha = k_0 \sin\theta_{in}$, $\beta = k_0\sqrt{n_b^2 - n_{eff}^2}$, $\gamma = k_0\sqrt{n_g^2 - n_{eff}^2}$, $\tau = n_b^2$ for TM polarization or 1 for TE polarization, b and g are the width of the grating ridge and groove respectively, n_b and n_g are the corresponding refractive indices. For a dielectric grating with depth h, approximated by a stack of lamellar gratings and neglecting the reflection, the -1st and 0th transmission efficiencies could be approximated as [12]:

$$\eta_{-1T} = \sin^2\left(\frac{k_0 h \Delta\overline{n}_{eff}}{2}\right) \tag{2a}$$

$$\eta_{0T} = \cos^2\left(\frac{k_0 h \Delta\overline{n}_{eff}}{2}\right) \tag{2b}$$

where $\Delta\overline{n}_{eff} = \sum_m (n_{0eff}^m - n_{0eff}^m) h_m/h$ is the average difference of mode effective indices, h_m is the depth of mth layer, n_{0eff}^m and n_{1eff}^m are the corresponding effective indices of mode 0 and mode 1.

For neglecting reflection of the grating, there exit deviations between the efficiencies calculated by RCWA and those calculated by Eq. (2). Now we consider a grating with period p and wavelength λ. And the incidence angle is the first Bragg angle $\theta = \arcsin[\lambda/(2n_g p)]$. In the next analysis, the grating region is divided into N thin planer grating slabs as shown in Fig. 1(a).

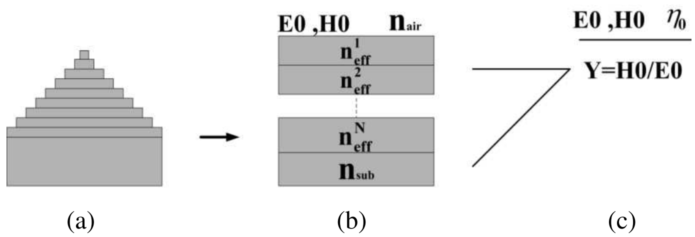


Figure 1: (a) The division of triangular grating; (b) slabs are regarded as multilayer films; (c) equivalent interface of the multilayer films.

Based on the optical thin film theory [13], the characteristic matrix of this N layer structure is:

$$\begin{bmatrix} B \\ C \end{bmatrix} = \left\{ \prod_{m=1}^{N} \begin{bmatrix} \cos\delta_m & \frac{i}{\eta_m}\sin\delta_m \\ i\eta_m \sin\delta_m & \cos\delta_m \end{bmatrix} \right\} \begin{bmatrix} 1 \\ \eta_{sub} \end{bmatrix} \tag{3}$$

where $\delta_m = 2\pi h n_{eff}^m/(N\lambda)$ is the phase shift of mth layer, $\eta_0 = n_g\cos\theta$, $\eta_m = n_{eff}^m$, $\eta_{sub} = n_{sub}$ for TE-polarization and $\eta_0 = n_g/\cos\theta$, $\eta_m = [(n_{eff}^m)^2 + n_g^2\sin^2\theta]/n_{eff}^m$, $\eta_{sub} = n_b^2/n_{sub}$ for TM-polarization, $n_{sub} = (n_b^2 - n_g^2\sin^2\theta)^{1/2}$ is the effective index of the substrate. Then the optical admittance of the grating is $Y = H_0/E_0 = C/B$, and the reflectance can be calculated by:

$$R = \left(\frac{\eta_0 - Y}{\eta_0 + Y}\right)\left(\frac{\eta_0 - Y}{\eta_0 + Y}\right)^* \tag{4}$$

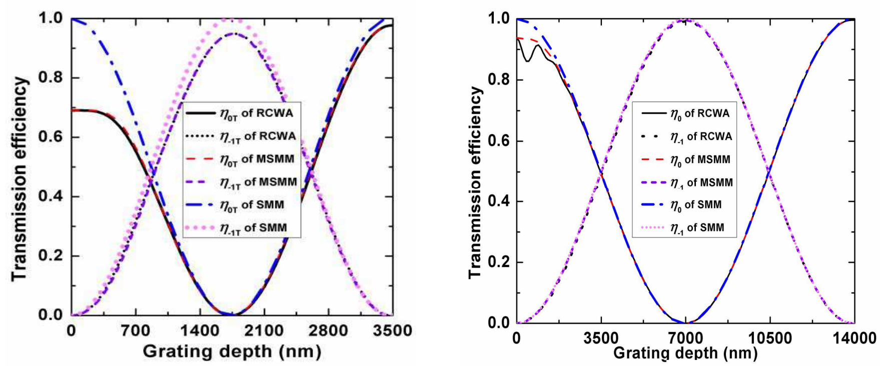
If the grating period fulfills the condition $\lambda/2 < p < 3\lambda/(2n_b)$ and the incidence is first Bragg angle, there are only two transmitted modes in grating region and each of them carries nearly half of the energy of the incident wave [14]. Thus the -1st and 0th transmission efficiency formulas could be modified as:

$$\eta_{-1T} = \left(1 - \frac{R_0}{2} - \frac{R_1}{2}\right)\sin^2\left(\frac{k_0 h \Delta\overline{n}_{eff}}{2}\right) \tag{5a}$$

$$\eta_{0T} = \left(1 - \frac{R_0}{2} - \frac{R_1}{2}\right)\cos^2\left(\frac{k_0 h \Delta\overline{n}_{eff}}{2}\right) \tag{5b}$$

where R_0 and R_1 are the corresponding transmission of mode 0 and mode 1.

To illustrate the accuracy of this modification of simplified modal method (MSMM), the transmission efficiencies of -1st and 0th orders are plotted versus the triangular depth in Fig. 2. we assume that $n_b = 1.45$, $n_g = 1$ and the incidence is first Bragg angle.



(a) (b)

Figure 2: Transmission efficiencies of a triangular grating calculated by MSMM, SMM and RCWA respectively versus the grating depth, with the period $p = 560\,\text{nm}$ and the incident wavelength $\lambda = 1064\,\text{nm}$. (a) TE-polarization; (b) TM-polarization.

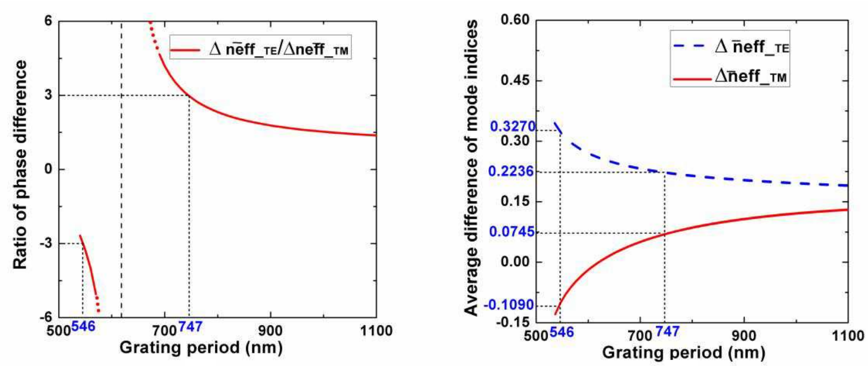
According to Fig. 2, we can conlude that MSMM makes a better result than SMM. The biggest deviation between MSMM and RCWA are 2% for TE-polarization and 5% for TM-polarization. The fine approximation is in that the admittance Y is a complete characteristic of multilayer films, which includes all information of grating layer and the substrate. The little deviation may be due to the evanescent modes and little difference of energy between mode 0 and mode 1 which is considered equal in Eq. (5).

3. DESIGN OF HIGHLY-EFFICIENT GRATING

The guideline for design of a subwavelength highly-efficient grating for both TE and TM-polarization [15] is:

$$\Delta\overline{n}_{\mathit{eff}\_TE}/\Delta\overline{n}_{\mathit{eff}\_TM} = (2l-1)/(2m-1) \tag{6}$$

where l and m are integers. Fig. 3 shows the average difference of mode effective indices as defined in Eq. (2). The triangular grating is divided into a stack of 100 layers with equal depth. As in shown, for $\lambda = 1064\,\text{nm}$ and $n_b = 1.45$, the period $p = 546\,\text{nm}$ fulfills Eq. (6) with $l = 2$, $m = -1$ and period $p = 747\,\text{nm}$ with $l = 2$ and $m = 1$.

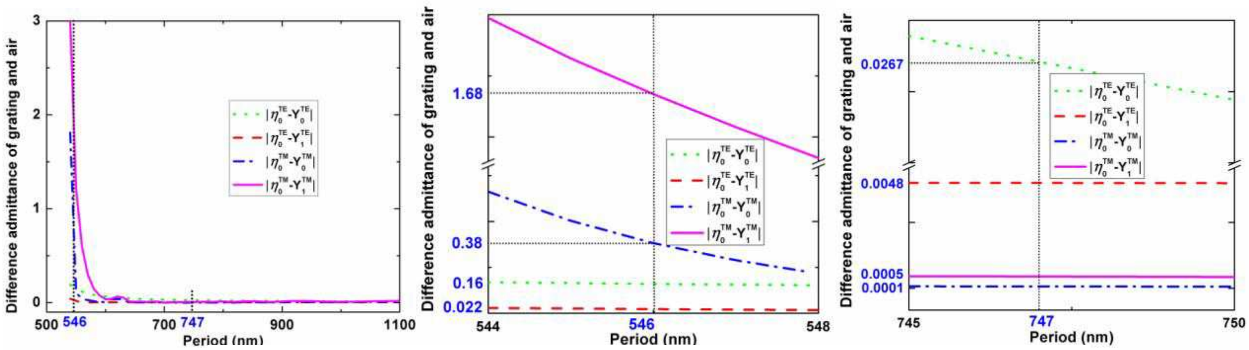


(a) (b)

Figure 3: Ratio of accumulated phase difference for TE-polarization to that for TM-polarization and average difference of mode indices as a function of triangular gratings' period.

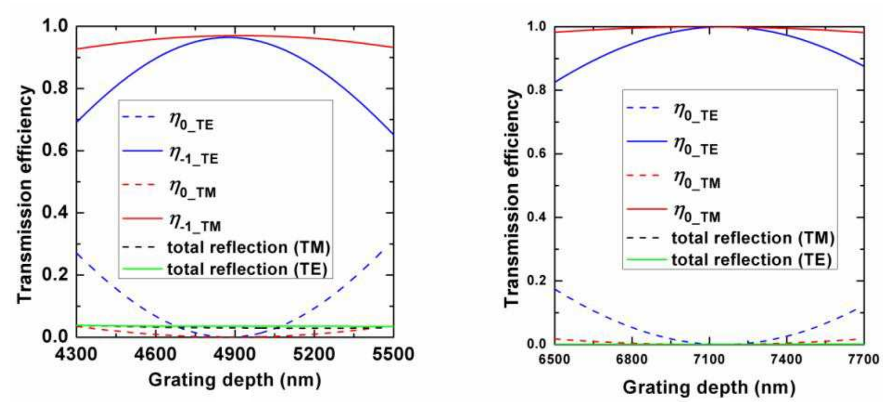
Using Eqs. (2) and (6), the grating can get a high efficiency for both TE and TM-polarization when $p = 546\,\text{nm}$ wit $h = \lambda/(2\Delta\overline{n}_{\mathit{eff}\_TM}) = 4480\,\text{nm}$ and $p = 747\,\text{nm}$ with $h = 7137\,\text{nm}$. SMM can not tell us which one could get a higher efficiency because of neglecting reflection. Now, from Eq. (4), the reflection will vanish while the admittance of grating Y is close to that of air η_0 (Fig. 4(c)). Correspondingly, the transmission efficiency is almost 100% (Fig. 5(b)) due to $T = 1 - R$.

Thus if we want to design a high efficiency grating for incident wavelength $\lambda = 1064\,\text{nm}$, the period should be chosen as $p = 747\,\text{nm}$, and depth $h = 7137\,\text{nm}$. The data in Table 1 exhibits this accurate conclusion.



(a) (b) (c)

Figure 4: Difference admittance of grating and air versus the period. Y_0 and Y_1 are admittances of mode 0 and mode 1 respectively. (b) and (c) are magnifications of (a).



(a) (b)

Figure 5: The efficiency versus the grating depth. (a) $p = 546$ nm; (b) $p = 747$ nm. Total reflection could not see clearly in (b) because they are less than 0.0005.

Table 1: Diffractive efficiencies of the triangular gratings at wavelength $\lambda = 1064$ nm and first Bragg angle for high efficiency at the -1st order by using RCWA.

Period (nm)	Height (nm)	η_{0_TE}	η_{-1_TE}	(TE) total reflection	η_{0_TM}	η_{-1_TM}	(TM) total reflection
546	4880	0.044%	95.666%	4.290%	0.052%	96.006%	3.942%
747	7137	0.005%	99.977%	0.018%	0.040%	99.958%	0.002%

4. DISCUSSION AND CONCLUSION

The simplified modal method (MSMM) was modified by regarding the grating as multilayer thin films. The grating region is divided into N thin rectangular grating slabs and regarded as multilayer films. Then the reflectance is characterized by effective admittances of air and grating. The calculation by MSMM is a very good approximation for RCWA. According to MSMM, a grating can get a high efficiency when its parameters fulfill two conditions: firstly, the average difference of mode indices is odd times of π and secondly the admittance of the grating is approximate to that of air. At last, a grating with more than 99.9% for both TE and TM-polarized light was designed..

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