Analytical Formulation for Electromagnetic Leakage from an Apertured Rectangular Cavity

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Abstract— An analytical formulation has been developed for the electromagnetic leakage from an apertured rectangular cavity excited internally by an electric dipole. The formulation consists of three parts: the interior problem of determining the field distribution within a closed cavity with the eigen-mode expansion method; the problem of characterizing the leakage mechanism of aperture by using the Bethe's small hole coupling theory; the exterior problem of determining the transmission field by using antenna theory. Theoretical values of shielding effectiveness are in good agreement with full-wave simulations. Analytic approaches, disregarding limited scope of application, are still of realistic interest in both science and engineering aspects due to clear physical meaning, high computation efficiency and easy to be programmed. In this paper, an analytic formulation is implemented to investigate the electromagnetic leakage from an aperture rectangular cavity. The formulation is based on the Bethe's small aperture coupling theory, which uses equivalent electric and magnetic dipoles to describe the relationship between the internal field of a closed cavity excited by an internal source and the leakage field from apertures. This Bethe's theory had been employed by some authors in earlier papers, and it is shown that the approach can work very well provided that the size of aperture is much smaller than one wavelength.

1. INTRODUCTION

With the rapid development of electronic technologies and the wide applications of wireless communication technologies, electromagnetic environment becomes more and more subtle and complex. So, various protection measures to prevent electromagnetic interference from both conducting and field coupling mechanisms have been studied and developed [1–4].

Electromagnetic shielding is the primary technology for suppressing electromagnetic interference via field coupling channel. It can be implemented by using a metallic enclosure to enclose an interference source to lower its field leakage, or to screen a sensitive object by reducing external field strength. Although a closed metallic enclosure generally has very high shielding effectiveness (SE) for electromagnetic waves, apertures that exist inevitably on the enclosure for some practical functions can result in dramatic reduction of SE. Thus, lots of work has given attention to the effect of apertures on shielding effectiveness, and some numerical, analytical [5] or experimental results have been reported.

Usually, in standard method for SE measurement, the SE of a shielding enclosure is defined as the ratio of the field strength at a given point with the enclosure removed to that at the same point with the enclosure applied. Where, the field source is required to be placed outside the enclosure. In other words, the field observation point is inside the enclosure, which has the advantage of reducing the potential influence of other unwanted interference source on the received field strength. As a result, in lots of literatures the SE of an enclosure is evaluated against an external field source.

However, the opposite case with source inside an enclosure is also in practical interest and deserves to be concerned. In principle, the two cases can be transformed into each other according to the reciprocity principle. However, since the "external source" cases have not been studied overall and thoroughly, we can not find appropriate "dual problem" for each of the "internal source" cases. For example, field distribution in near field zone is the primary concern for the latter case but for the former the exciting source (frequently, a plane wave is assumed) locates obviously in the far field zone [6, 7].

2. ANALYTICAL MODEL

Figure 1 shows an empty metallic rectangular cavity excited by an electric dipole placed inside it. The x, y, and z dimensions of the cavity are x_e, y_e, and z_e. A circular aperture is cut in the $x = x_e$ wall with its center at (x_e, y_0, z_0). The z-directed electric dipole source is located at the point (x_s, y_s, z_s). It is assumed that the cavity wall is perfectly conducting with a vanishing thickness.

According to the Bethe's theory, the aperture can be represented by equivalent aperture dipole moments $\mathbf{p}$ and $\mathbf{m}$ [8, 9]. The electric dipole moment is perpendicular to the aperture, and the

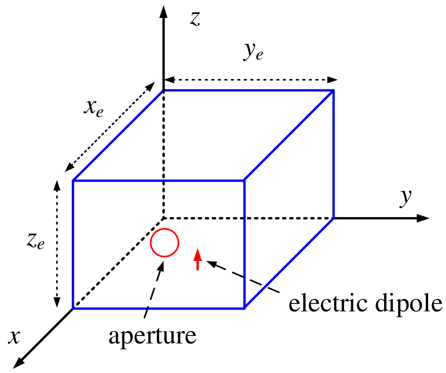

Figure 1: A rectangular cavity with a circular aperture and excited by an electric dipole placed inside the cavity.

magnetic dipole moment lies in the aperture plane. The strength of $\mathbf{p}$ and $\mathbf{m}$ are, respectively, proportional to the E and H fields existing on the inner wall with the aperture filled. Essentially, these fields are the "unperturbed" field of a closed cavity. Since the cavity walls are assumed to be perfectly conducting, the fields consist only of a normal E field and a tangential H field. The relationships between the dipole moments and the unperturbed fields are given through the aperture polarizability coefficients. Especially, for the configuration in Fig. 1, we have

$$\mathbf{p} = \alpha_e \varepsilon_0 E_x \mathbf{e}_x, \quad \mathbf{m} = -\alpha_{my} H_y \mathbf{e}_y - \alpha_{mz} H_z \mathbf{e}_z \tag{1}$$

Various expressions and curves of polarizability coefficients are available for some simple aperture shapes. Especially, for a circular aperture with a diameter of d, the coefficients are

$$\alpha_e \approx d^3/6, \quad \alpha_{my} = \alpha_{mz} = d^3/3 \tag{2}$$

Once the dipole moments are known, the field outside the cavity due to aperture leakage can be described by the free space field of the equivalent dipole antenna. Since the analytic formula of both electric and magnetic dipoles radiation can be found easily from textbooks, they are omitted here.

The remaining problem is the determining of unperturbed field distribution. Fortunately, the problem can be solved by using the eigen-mode expansion methods [9]. For the configuration displayed in Fig. 1, the expression for E-field is

$$\begin{aligned}
\mathbf{E}(\mathbf{r}_0) \;=\; & \frac{j\omega\mu_0}{k^2} I\mathrm{d}l\,\delta(\mathbf{r}_0-\mathbf{r}_s)\mathbf{e}_z \\
& -\frac{j\omega\mu_0}{k^2} I\mathrm{d}l \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{l=0}^{\infty} \frac{\varepsilon_{0n}\varepsilon_{0m}\varepsilon_{0l}\sin\frac{n\pi x_s}{x_e}\sin\frac{m\pi y_s}{y_e}\cos\frac{l\pi z_s}{z_e}}{x_e y_e z_e[k^2-(n\pi/x_e)^2-(m\pi/y_e)^2-(l\pi/z_e)^2]} \\
& \times\left\{\left[\left(\frac{n\pi}{x_e}\right)^2+\left(\frac{m\pi}{y_e}\right)^2\right]\sin\frac{n\pi x_0}{x_e}\sin\frac{m\pi y_0}{y_e}\cos\frac{l\pi z_0}{z_e}\mathbf{e}_z\right. \\
& -\frac{m\pi}{y_e}\left(\frac{l\pi}{z_e}\right)\sin\frac{n\pi x_0}{x_e}\cos\frac{m\pi y_0}{y_e}\sin\frac{l\pi z_0}{z_e}\mathbf{e}_y \\
& \left.-\frac{n\pi}{x_e}\left(\frac{l\pi}{z_e}\right)\cos\frac{n\pi x_0}{x_e}\sin\frac{m\pi y_0}{y_e}\sin\frac{l\pi z_0}{z_e}\mathbf{e}_x\right\}
\end{aligned} \tag{3}$$

wherein, $\varepsilon_{0n} = 1$ for $n = 0$, $= 2$ for $n \neq 0$, and so on. The singularity resulted from the term containing the δ-function can be neglected since only the fields far away from the source are concerned.

In order to accelerate the convergence of the three-dimension series, the following formulas are applied.

$$\sum_{1}^{\infty} \frac{\cos nx}{n^2 - a^2} = \frac{1}{2a^2} - \frac{\pi}{2a} \frac{\cos (x - \pi) a}{\sin \pi a} \quad (0 \leq x \leq 2\pi) \tag{4}$$

$$\sum_{1}^{\infty} \frac{n \sin nx}{n^2 - a^2} = \frac{\pi}{2} \frac{\sin (x - \pi) a}{\sin \pi a} \quad (0 \leq x \leq 2\pi) \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (3) reduces the sums to a two-dimension series

$$E_z (x_0, y_0, z_0) = \frac{-j\omega\mu I dl}{k^2 (x_e y_e z_e)} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \Gamma_{nm} \left(\frac{z_e}{2k'}\right) \times \frac{\cos k' (z_0 + z_s - z_e) + \cos k' (|z_0 - z_s| - z_e)}{\sin k' z_e} \tag{6}$$

$$E_y (x_0, y_0, z_0) = \frac{-j\omega\mu I dl}{k^2 (x_e y_e z_e)} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \Gamma'_{nm} \left(\frac{z_e}{2}\right) \times \frac{\sin k' [z_e - (z_0 + z_s)] + \operatorname{sgn} (z_0 - z_s) \sin k' (z_e - |z_0 - z_s|)}{\sin k' z_e} \tag{7}$$

$$E_x (x_0, y_0, z_0) = \frac{-j\omega\mu I dl}{k^2 (x_e y_e z_e)} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \Gamma''_{nm} \left(\frac{z_e}{2}\right) \times \frac{\sin k' [z_e - (z_0 + z_s)] + \operatorname{sgn} (z_0 - z_s) \sin k' (z_e - |z_0 - z_s|)}{\sin k' z_e} \tag{8}$$

where, $\operatorname{sgn}(x)$ is the sign function, and

$$k' = \sqrt{k^2 - (n\pi/x_e)^2 - (m\pi/y_e)^2}, \tag{9}$$

$$\Gamma_{nm} = \varepsilon_{0n}\varepsilon_{0m} \left[\left(\frac{n\pi}{x_e}\right)^2 + \left(\frac{m\pi}{y_e}\right)^2\right] \sin \frac{n\pi x_0}{x_e} \sin \frac{n\pi x_s}{x_e} \sin \frac{m\pi y_0}{y_e} \sin \frac{m\pi y_s}{y_e}, \tag{10}$$

$$\Gamma'_{nm} = \varepsilon_{0n}\varepsilon_{0m} \left(\frac{m\pi}{y_e}\right) \sin \frac{n\pi x_0}{x_e} \sin \frac{n\pi x_s}{x_e} \cos \frac{m\pi y_0}{y_e} \sin \frac{m\pi y_s}{y_e}, \tag{11}$$

$$\Gamma''_{nm} = \varepsilon_{0n}\varepsilon_{0m} \left(\frac{n\pi}{x_e}\right) \cos \frac{n\pi x_0}{x_e} \sin \frac{n\pi x_s}{x_e} \sin \frac{m\pi y_0}{y_e} \sin \frac{m\pi y_s}{y_e}. \tag{12}$$

The corresponding H field components can be obtained directly by using the Faraday's electromagnetic induction law:

$$H_x (x_0, y_0, z_0) = \frac{I dl}{(x_e y_e z_e)} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \Gamma'_{nm} \frac{z_e}{2k'} \frac{\cos k' [z_e - (z_0 + z_s)] + \cos k' (z_e - |z_0 - z_s|)}{\sin k' z_e}, \tag{13}$$

$$H_y (x_0, y_0, z_0) = -\frac{I dl}{(x_e y_e z_e)} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \Gamma''_{mn} \frac{z_e}{2k'} \frac{\cos k' (z_0 + z_s - z_e) + \cos k' (|z_0 - z_s| - z_e)}{\sin k' z_e}, \tag{14}$$

By coordinate transform, the above model is also feasible for x-directed or y-directed source. Then, by using superposition principle, the model can be expanded further to account for any direction-directed electric dipole source. With the increase of n and m, k' will becomes an imaginary number, the sine terms in the denominator position will leads to a exponentially attenuation, and the series begins to convergence rapidly. However, for the field point having the same z-coordinator with the source, the convergence becomes slow due to the denominator and the numerator having close exponential growth rate. In this case, the value of n and m must be increased further to achieve stable results.

3. RESULTS AND DISCUSSIONS

In the following contents, it is assumed that $x_e = 300\,\text{mm}$, $y_e = 300\,\text{mm}$, $z_e = 120\,\text{mm}$, and the aperture is circular with a radius of 5 mm. Since the z component is dominate among the three electric field components, the electric shielding effectiveness S_E is define as $S_E = 20\log 10(E_{z0}/E_{zs})$, where E_{z0} and E_{zs} are, respectively, the z-directed E-field with the shielding absent and present. Similarly, the magnetic shielding effectiveness S_H is referred to the y component H field.

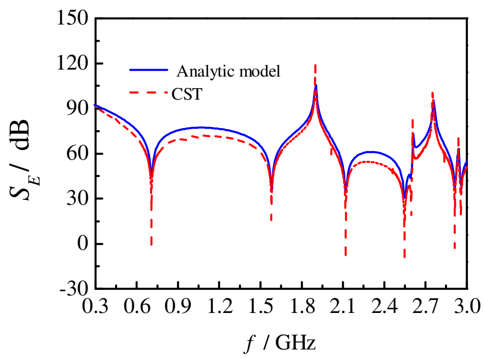

Figure 2: Dependence of the electric field shielding effectiveness S_E on frequency with $r = 0.4\,\text{m}$.

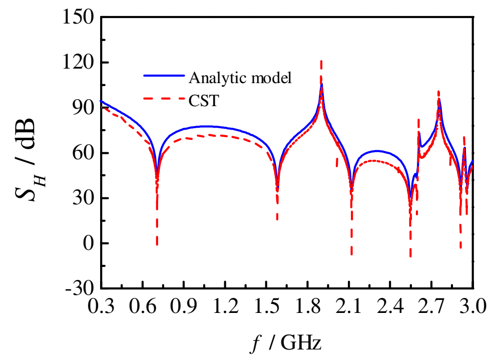

Figure 3: Dependence of the magnetic field shielding effectiveness S_H on frequency with $r = 0.4\,\text{m}$.

The S_E and S_H are shown in Fig. 2 and Fig. 3, respectively, as the function of frequency with $x_s = 150\,\text{mm}$, $y_s = 150\,\text{mm}$, $z_s = 20\,\text{mm}$, and the aperture located at the center of the $x = x_e$ wall. The observation point of shielding effectiveness is chosen at the position (700 mm, 150 mm, 60 mm). In other words, the observation point is at the right ahead of the aperture and the distance between the aperture and the point is $r = 0.4\,\text{m}$. It can be seen that the results obtained by using the present analytic model are in good agreement with those acquired by employing the frequency domain algorithm of the full-wave simulation software CST. Also, the electric shielding effectiveness S_E is approximately equal to the magnetic shielding effectiveness S_H. The underlying reason is that the field point is already within the far-field zone for both the exciting source and the equivalent dipole, and hence the wave impedance at the field point keeps unchanged no matter the shielding applied or removed. Sharp reduction of SE around several frequencies like 0.71 GHz, 1.58 GHz, 2.12 GHz and 2.55 GHz are attributed to, respectively, the resonance effect of the modes TM_{110}, TM_{310}, TM_{330}, and TM_{510}. Where, a mode is classified as a TM (transverse magnetic) mode if it has no the magnetic field component along the z direction. The three subscripts correspond to the values of the three mode index n, m, and l, respectively. It is interesting to note that the resonance effects from some modes are not found. For example, the z-component of the electric field of the TM_{210} mode has a zero point at the position of the source, so this mode can not be stimulated by the source. In addition, sharp increase of SE is also observed near several frequencies. This phenomenon is usually explained as the result of the position of the source or the aperture coincides with the position of the wave node of the standing-wave field within the cavity.

4. CONCLUSIONS

An analytic formulation had been implemented to analyze the electromagnetic leakage from an apertured rectangular cavity. The leakage fields are represented by equivalent electric and magnetic dipoles at the aperture center with their dipole moments expressed by the "closed cavity" field according to the Bethe's small aperture coupling theory. Comparison with the full wave simulation software CST had verified the reliability of the formulation over a broad frequency bandwidth.

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