Deriving the Geometry of Frequency Selective Surfaces (FSS) and Metamaterials (MTM) Elements from Transmission Lines by Using Surrogate Meta-modeling Techniques

Fabrizia Ghezzo, **Loris Serafino**, **Chunlin Ji**, **Xigeng Miao**, **and Ruopeng Liu**
Kuang-Chi Institute of Advanced Technology
Gaoxin Zhong 1st Road, High-Tech Industrial Park, Shenzhen 518057, China

Abstract— In this work we have implemented a multi response model based on the Kriging surrogate modeling techniques to determine the correspondence between the resistance, capacitance, inductance, and admittance (RCLG) of a generalized transmission line and the dimensions of the geometrical features of a specific frequency selective surface (FSS) or metamaterial (MTM) structure - expressed in parametric form — operating in predefined frequency ranges. Specifically, this study was conducted considering the model for the application of FSS and MTM in the microwave frequency range. To demonstrate the potentiality of the newly developed model, an example of design of a metamaterial structure in a simplified case of perpendicular incidence and single layer configuration is analyzed and the preliminary results obtained are discussed.

1. INTRODUCTION

Frequency selective surfaces (FSS) and metamaterials (MTM) consist of surfaces made of periodically distributed elements whose size is much smaller than the wavelength [1]. These conductive elements are engineered in order to enhance the electromagnetic performance of the host material. In fact, while each individual element behavior cannot be perceived by the propagating wave, these elements collective behavior can considerably change the overall electromagnetic response of the medium. The behavior of a material with integrated sub-wavelength conductive resonant structures can be considered equivalent to a homogeneous material with effective relative permittivity and permeability. FSS have been used for decades in order to enhance the performance of the materials of many microwave devices. Areas where FSS have been applied since the 1970s [2] include antenna components for modern communications systems, frequency separation in beam splitters, transparent windows also known as radomes, artificial dielectrics, reflectors, absorbers, phase screens for beam steering and beam width equalization [3].

Very well known FSS elements can be identified with conductive open lines, solid patches, center connected lines and their combinations. Metamaterials are also periodic conductive elements of sub-wavelength size that differently from FSS can allow a medium to achieve properties that cannot be found in Nature. Materials with simultaneously negative permittivity and permeability also known as left handed materials or negative index materials, where theoretically presented by Victor Veselago in 1968 [4]. Negative index materials give rise to completely new electromagnetic phenomena. A part from the possibilities presented by Veselago, Pendry [5] was the first to propose the use of artificial materials, opening a new area of research. To practically realize these structures and achieve the desired performance it is necessary to first simulate their behavior. The computational effort that goes into the design of these elements is quite fundamental and accounts for most of the work to be completed to achieve the desired result and performance. Most of the structures studied in the literature, both in the case of FSS and metamaterials are generally derived by changing some of the geometrical features of resonant elements already studied in the literature. By doing so a new structure is found. After this step, a series of simulations need to be carried out to characterize its behavior at the varying frequency. This step is time consuming and computationally expensive. Varying the size and the shape of these elements brings the most diverse range of results and behaviors in a large frequency spectrum. It appears therefore necessary to develop some design tool in order to help the designer to select the right element geometry according to the requirements saving a lot of computational time. Many works in the literature are currently trying to solve the barrier imposed by the complexity of this design problem. For instance, genetic algorithms (GA) [6] are one of these proposed solutions. The disadvantage of the GA method consists on the large computational time required for obtaining the solution and also on the need of adding some selection criteria for filtering the results. Some of the structures found by simulations are often not practical and not feasible from the point of view of the fabrication.

Transmission line theory has also been used for the design and analysis of microwave networks [7]. Circuit models for metallic meshes and capacitive metallic plates in filter design have been intensively studied by TL which is seen as a quite promising tool for the design of these elements. Recently, several efforts have been also presented in metamaterials area [8–10]. The application of TL to metamaterials design is not straightforward. Moreover, if the circuit is representing a multilayer structure it will remain the same only if no interaction occurs between the layers [11] otherwise the interaction between these artificial atoms must be taken into account since it influences heavily the final material's performance. Currently, when a new FSS or metamaterial structure is designed, it is possible to calculate the capacitance and inductance of the circuit of very simple models otherwise the calculation can be extremely difficult due to the complexity of the element's geometry. The progress on the study of FSS and metamaterials by using TL theory seems to be limited by the complexity of the geometry of the individual elements. To fully utilize the potential of this field it seems necessary to develop a method which allows to obtain the structure with the desired transmission and reflection properties in a more general and efficient way. In this work we attempted to link a specific TL model to a FSS or metamaterial structure in the most general case of arbitrary angles of incidence and polarization by using a meta-modeling technique. Specifically, the method consists on the application of the Kriging surrogate modeling technique.

2. KRIGING SURROGATE MODEL

In material science optimization problems, often no analytical formula exists for accurately evaluating the following functional response $f : \mathrm{H} \to \Re$ where H represents the space of the input variable, also called *design space*. In these cases the response to be optimized is accessible only as discrete sets of pairs of input and output from a simulation or from an experiment. Therefore, given a set of input (from a given sample space) $x = (x_1, \dots, x_n)^T$ and a response $y = (y_1, \dots, y_n)^T$ such that $y_i = f(x_i)$ with $i = 1, \dots, n$ the primary goal of meta-modeling is to predict the response $f(x_{new})$ at a new point by constructing an approximate (also called *surrogate*) model of the true functional response.

Surrogate models are convenient, computationally light to run and most importantly — for the case here studied — they can also yield to insights about the functional relationship between a set of input parameters and a target response. In general, meta-modeling consists of two steps: a) the design of experiment (DOE) and b) the meta-modeling procedure. The purpose of DOE techniques is to generate a suitable training set $\{(x_i, y_i) : y_i = f(x_i),\ i = 1, \dots, n\}$ that will be used for the construction and validation of the surrogate model. For the purpose of this work, we adopted the Latin hypercube sampling (LHS) space-filling strategy. The rationale behind LHS is to subdivide the design space into n intervals along each dimension. The samples are then chosen randomly such that each interval contains exactly one point [12]. Many meta-modeling procedures are available in the literature [13, 14]. The Kriging meta-modeling is particularly suitable when the functional response to be studied is deterministic and highly nonlinear [15]. A technical in-depth discussion of the Kriging techniques can be found in the works [16, 17]. Kriging techniques can be successfully extended to multiple response functional relation, characteristic aspect of the type of design required in this work. The DACE Matlab toolbox was used in this work to construct the Kriging surrogate model. *Pros* and *cons* of the Kriging approach with respect to other meta-modeling procedures are reported in the literature [18]. The Maximum Likelihood Estimators used to estimate the required parameters involves a constrained maximization step. This results in a number of complicated calculations including matrices inversion. The correlation matrix can become singular when the training points are too close to each other. Such problems may eventually overweight the approximation benefits. Nonetheless Kriging (together with LHD experimental design) resulted well suited for the simulation of the model here considered especially for the possibility of modeling a multivariate functional response. In this work we focus on the solution of the problem of linking a specific TL model with a FSS or metamaterial structure in the most general cases of oblique incidence and arbitrary polarization angles. The main idea of this work was to generate a functional model capable of describing the relation between the TL circuit parameters (RCLG values) and the parameters describing the geometry of the structure under consideration using one of the surrogate modeling techniques.

3. PRELIMINARY RESULTS

In the model, the design procedure is carried out in the following way. The geometry of the FSS or MTM element of interest is selected and expressed in a suitable parametric form characterized by a specific number of geometrical parameters. These parameters include all the dimensions that are necessary to describe the element, i.e., the geometry of the structure including the size of the unit cell and the characteristics of the propagation of the wave (angle of incidence and polarization). An initial data set, determined by applying the Latin hypercube DoE, is solved by running full wave simulations in CST Microwave Studio software. The MatLab code developed for the Kriging runs automatically the simulations in CSTMW Studio and stores the results for subsequent analysis. Once the solutions in terms of transmission and reflection characteristics (S-parameters) are found for an established range of frequencies, the algorithm extracts the RCLG equivalent circuit characteristics from the S-parameters. The procedure for the extraction of the circuit characteristic values is different for the case of right-hand and left-hand media [19] and this is a key aspect that requires future work and investigations in order to include into the model's design abilities more complex cases. This step builds the correspondence between the RCLG circuit characteristics and the geometrical parameters of the physical model. Once the training of the Kriging model is completed, the code is ready for the inverse procedure. In a second phase in fact, by assigning the desired values of RCLG of a transmission line the model generates as an output the values of the geometrical parameters of the structure. It is necessary to point out that, currently, the model is limited to finding the correspondence of the generalized TL model in the case of arbitrary wave incidence and polarization angles with a pre-selected FSS or metamaterial element.

The model was validated by comparing the results obtained for the case presented in Fig. 1, also known as the Jerusalem Cross (JC). Figs. 2(a) and 2(b) show the transmission and phase characteristics of the JC element whose dimensions were found by applying the Kriging meta-model after specific values of RCLG were assigned. The RCLG values assigned to the model were the same calculated *a priori* after solving the specific case — the reference case — (with defined values of the geometrical parameters) in CST Studio. In other words, the JC reference case was

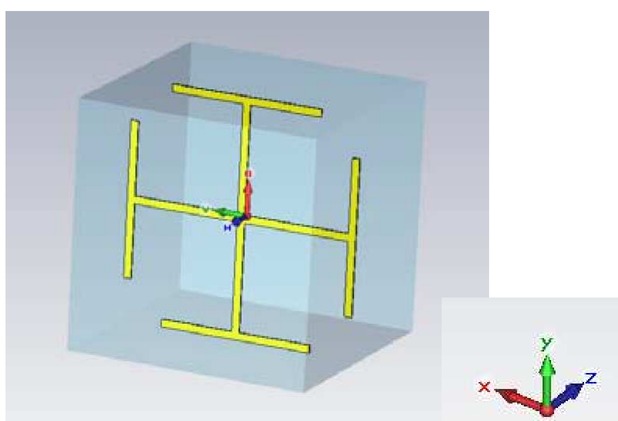

Figure 1: Jerusalem cross (JC) CST Microwave Studio model. The wave is propagating along the z-axis with the electric field component oriented along the y-axis and the magnetic component along the x-axis.

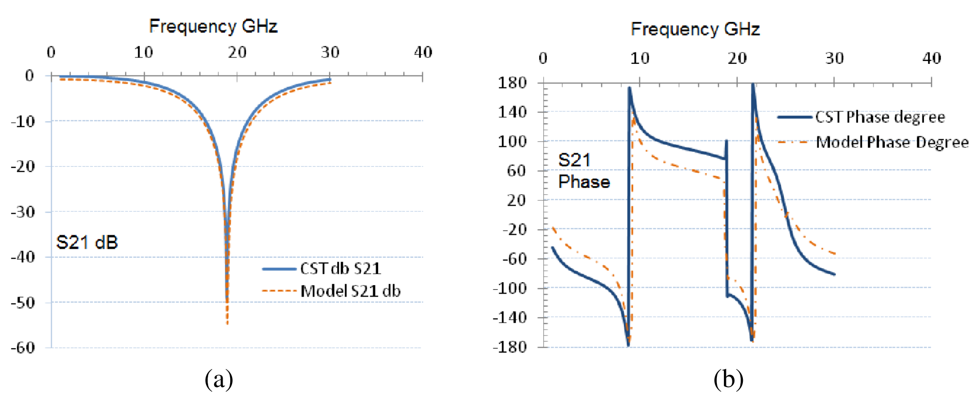

Figure 2: (a) Model's transmission characteristics S_{21} and (b) phase behavior compared with the results obtained by using CST Microwave studio.

modeled and firstly solved by using CST Microwave Studio in the frequency range [0, 40] GHz. The reason for the wide frequency spectrum is simply due to the interest of analyzing the behavior of the element in this entire frequency range. The resulting transmission behavior of the structure as obtained from the numerical simulation is presented by the blue curves. Secondly, the transmission and reflection characteristics were determined via application of the meta-model. For its successful application, the model was first trained on a data set (a number of cases where the dimensions of the structure varied) consisting of 100 samples. The results of the model are presented by the dash-orange lines in Figs. 2(a) and 2(b).

4. CONCLUSIONS

In this work we implemented a model based on the Kriging technique to generate the correspondence between the characteristic values of a generalized transmission line model (RCLG) extracted from assigned or desired S-parameter curves and the dimensions of a specific frequency selective surface (FSS) or metamaterial (MTM) element. Although this work focused on the application of the model for a simple case of perpendicular incidence on a well known metamaterial based FSS element, the model is built in a way that cases of arbitrary incidence and polarization angles can be solved. This part is currently under validation. Progress in this study can be remarkably helpful for the solution of several problems in metamaterials and frequency-selective surfaces design by using transmission lines or simply starting from given trends of the S-parameters. Its future generalization can be useful for the design and optimization of microwave devices based on FSS and metamaterials structures derived by simply assigning the desired S-parameters responses. The efficiency of the model depends on the ability of describing parametrically the geometry of the structure in the most general way. Besides the encouraging results obtained so far, further analysis is in progress to complete and verify the limits for the application of this methodology, especially for the case of left-hand media.

ACKNOWLEDGMENT

This research was financially supported by the Guangdong High-End Electronic Information Special in Strategic Emerging Industry (No. 2012556019); the Shenzhen Key Laboratory of Ultrahigh Refractive Structural Material (No. CXB201105100093A) and the Shenzhen Innovative R&D Team Program (Peacock Plan) (No. KQE201106020031A).

REFERENCES

1. Kozakoff, D. J., *Analysis of Radome-enclosed Antennas*, 2nd Edition, Artech House, Norwood, 2010.
2. Anderson, I., "On the theory of self-resonant grids," *The Bell System Technical Journal*, Vol. 54, No. 10, December 1975.
3. Munk, B. A., *Frequency Selective Surfaces: Theory and Design*, Jon Wiley & Son, New York, 2000.
4. Veselago, V. G., "The electrodynamics of substances with simultaneously negative values of ε and μ," *Soviet Physics UpsekHI*, Vol. 10, 509–514, English Translation, 1968.
5. Pendry, J. B., A. J. Holden, D. J. Robbins, and W. J. Stewart, "Magnetism from conductors and enhanced non linear phenomena," *IEEE Transaction on Microwave Theory and Techniques*, Vol. 47, No. 11, 2075–2084, November 1999.
6. Mitchell, M., *An Introduction to Genetic Algorithms*, MIT Press, Cambridge, MA, 1998.
7. Miano, G. and A. Maffucci, *Transmission Lines and Lumped Circuits*, Academic Press, 2001.
8. Gil, M., J. Bonache, J. Selga, J. Garcia-Garcia, and F. Martin, "Broadband resonant type metamaterial transmission lines," *IEEE Microwave and Wireless Components Letters*, Vol. 17, No. 2, February 2007.
9. Eleftheriades, G. V., A. K. Iyer, and P. C. Kremer, "Planar negative refractive index media using periodically L-C loaded transmission lines," *IEEE Transactions on Microwave Theory and Techniques,* Vol. 50, No. 12, 2702-12, 2865-73, December 2002.
10. Yasar-Orten, P., E. Ekmekci, and G. Turhan-Sayan, "Equivalent circuit models for split ring resonator arrays," *PIERS Proceedings*, 534–537, Cambridge USA, July 5–8, 2010.
11. Simovsky, C. R., "Bloch material parameter of magneto-dielectric metamaterials and the concept of Bloch lattices," *Metamaterials*, Vol. 1, No. 2, 62–80, 2007.
12. Park, J. S., "Optimal Latin-hypercube designs for computer experiments," *J. Stat Plan Inference*, Vol. 39, 95–111, 1994.

13. Sacks, J., W. J. Welch, T. J. Mitchell, and H. P. Wynn "Design and analysis of computer experiments," *Statistical Science*, Vol. 4, No. 4, 409–435, 1989.
14. Sudjianto, A. K. T. Fang, and R. Li, *Design and Modeling for Computer Experiments (Computer Science & Data Analysis)*, Chapman & Hall/CRC, 2005.
15. Jones, D. R., M. Schonlau, and W. J. Welch, "Efficient global optimization of expensive black-box functions," *Global Optimization*, Vol. 13, 455–492, 1998.
16. T. W. Simpson, T. M. Mauery, J. J. Korte, and F. Mistree, "Kriging models for global approximation in simulation-based multidisciplinary design optimization," *AIAA Journal*, Vol. 39, No. 12, 2233–2241, 2001.
17. Kleijnen, J. P. C. and E. Mehdad, "Kriging in multi-response simulation, including a Monte Carlo laboratory," CentER Discussion Papers Series No. 2012-039, May 16, 2012, Available at SSRN: http://ssrn.com/abstract=2060891 or http://dx.doi.org/10.2139/ssrn.2060891.
18. Simpson1, T. W., J. D. Peplinski, P. N. Koch, and J. K. Allen, "Metamodels for computer-based engineering design: Survey and recommendations," *Engineering with Computers*, Vol. 17, 129–150, 2001.
19. Caloz, C. and T. Itoh, *Electromagnetic Metamaterials: Transmission Lines Theory and Microwave Applications*, J. Wiley & Sons, New Jersey, 2006.