Dynamics of Many-soliton Molecules in Dispersion-managed Optical Fiber

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Abstract— We study the stability and the dynamics of many-soliton molecules in dispersion-managed optical fiber with focus on 4- and 5-soliton molecules by analytical and numerical means. We calculate in particular their binding energy, pulse width and their chirp using a variational approach. We found that our variational calculations agreed favorably with the numerical solutions of the nonlinear Schrödinger equation and they well describe the intensity profiles of the experimental molecules.

1. INTRODUCTION

Few years ago, a stable bound state of two DM solitons in opical fibers was realized experimentally [1] and most recently three-soliton molecules in DM optical fibers were also realized by the same group [2, 3]. The main motivation behind creating such molecules is to increase the bit-rate of data transfer in optical fibers. Coding with two or more solitons per clock period increases the alphabet beyond the binary system of a single soliton: logical zero, one (single soliton), two (two-soliton molecule) and three (three-soliton molecule) in time slot. Clock period of commercial systems are now often 25 ps corresponding to 40 GBits/s transmission rate.

The main preoccupation in soliton molecules is their stability against disintegration. Hence, intensive interest in their stability has emerged [4–10]. The existence of a nonzero binding energy in terms of the width of the soliton molecules is an indication on its stability. Indeed, the stability of the many soliton molecule can be problematic since in [2, 3], 3-soliton molecule was found to be less stable than 2-soliton molecule. For many soliton molecules, matters appeared more denser with rising number. Therefore, the stability of such molecules become a difficult task.

The aim of this paper is to study the existence regimes and dynamics of many-soliton molecules in DM optical fibers. Initially we develop a variational approximation to describe the periodic dynamics of a soliton molecule. The obtained system of coupled equations for the pulse width and chirp allows to find the parameters of DM soliton molecules for the given dispersion map and pulse energy. The binding energy and equilibrium separations of the molecules of larger molecules is calculated in the frame of the variational approximation. The predictions of the variational approximation are compared with results of numerical simulations of the nonlinear Schrödinger equation (NLSE) and good agreement is found for both few and many soliton-molecules. All numerical simulations are performed using the parameters of the existing DM fiber setup [2, 3].

2. DISPERSION-MANAGED NONLINEAR SCHRÖDINGER EQUATION

Solitons in dispersion-managed dissipative optical fibers are described by the following NLSE:

$$iE_z - \frac{d(z)}{2}E_{tt} + S(z)|E|^2E = -ig(z)E, \tag{1}$$

where $E(t,z)$ $|E|^2[W]$, $z[m]$ and $t[s]$ are the complex envelope of the electri field, the propagation distance, and the retarded time, respectively. $S(z)[1/\mathrm{W}\cdot\mathrm{m}]$ and $g(z)\,[1/\mathrm{m}]$ represent the nonlinearity and the gain/loss parameters, respestively. $d(z)\,[\mathrm{s}^2/\mathrm{m}]$ corresponds to the dispersion management map defined by

$$d(z) = \begin{cases} d^+, & 0 \le z \le L^+, \\ d^-, & L^+ < z \le L^+ + L^-, \end{cases} \tag{2}$$

where $d^{+,-}$ are constant group velocity dispersions of the fiber segments $L^{+,-}$, respectively.

Using the transformation $E(t,z) = a(z)\,u(t,z)$, $a(z) = a_0 \exp\left(-\int_0^z g(x)\,dx\right)$ where a_0 is dimensionless parameter.

This moves the loss term to the coefficient of the nonlinear term. Thus, Equation (1) reduces to

$$iu_z - \frac{d(z)}{2}u_{tt} + \gamma(z)|u|^2 u = 0, \tag{3}$$

where $\gamma(z) = S(z)a(z)^2$ is the fiber's effective nonlinearity.

In general, for any $\gamma(z) > 0$, we then introduce a new coordinate z' [1/W] defined by $z'(z) = \exp\left(-\int_0^z \gamma(x)\,dx\right)$

$$i\,u_{z'} - \frac{d'(z')}{2}\,u_{tt} + |u|^2\,u = 0, \tag{4}$$

where $d'(z') = d'(z')/\gamma(z')[\mathrm{W}\cdot\mathrm{s}^2]$represents fibers effective dispersion including the variations both of fibers GVD and effective nonlinearity.

Equation (4) permits us to describe the variations of dispersion, nonlinearity, and optical power due to loss or gain by a single variable on the distance which is measured with the accumulation of nonlinearity. For a negative constant, (4) is called the NLSE, can be analytically solved for any initial input by using the inverse scattering transformation, and has soliton solution [11, 12].

For numerical purposes, it is useful to reduce Equation (4) into a dimensionless form. First we introduce $Z = z'/L'$, $T = t/\tau_m$ and $Q(Z,T) = u(z',t)\cdot\sqrt{L'}$ where τ_m is the characteristic time scale equal to the pulse duration of the laser source and $L' = (L^+ + L^-)\gamma$ is the length of the dispersion map period. In terms of these parameters, the dimensionless NLSE takes the form

$$iQ_Z - \frac{D(Z)}{2}Q_{TT} + |Q|^2 Q = 0, \tag{5}$$

where $D = d'L'/\tau_m^2$.

We use the experimental parameters for the DM map corresponding to the setup of [2]. The pulse duration $\tau_m = 0.25\,\mathrm{ps}$, $d^- = -4.259\,\mathrm{ps}^2/\mathrm{km}$, $d^+ = 5.159\,\mathrm{ps}^2/\mathrm{km}$, $\gamma = 1.7\,\mathrm{W}^{-1}\mathrm{km}^{-1}$, $L^+ = 24\,\mathrm{m}$, $L^- = 22\,\mathrm{m}$, $L' = 0.078\mathrm{W}^{-1}$ and $D = 0.521$.

3. VARIATIONAL CALCULATION FOR: SINGLE AND TWO SOLITON-MOLECULES

The variational approach is based on restating the governing equation in terms of a variational problem, where the Lagrangian, generating the original equation, is minimized for a particular trial function. As a result, one obtains a set of coupled ordinary differential equations for parameters of the corresponding ansatz. The technique is proved to be efficient for the analysis of non-integrable soliton bearing equations in different areas of physics [13].

We employ the following trial wavefunction:

$$Q(Z,T) = A\sum_{j=1}^{2}\exp\left[-\frac{(T-\eta_j)^2}{q_j^2} + i\alpha_j T^2 + i\varphi_j\right], \tag{6}$$

where A guarantees the normalization of Q, the number of solitons in the molecule, namely $N = 2$. The variational parameters $q(Z)$, $\varphi(Z)$ and $\eta(Z)$ and $\alpha(Z)$ correspond respectively to the width, the phase, the peak position and the chirp of the soliton. The Lagrangian corresponding to Equation (5) reads

$$L[Q,Q^*] = \int_{-\infty}^{\infty}\frac{i}{2}\left(QQ_Z^* - Q^*Q_Z\right)dT - E. \tag{7}$$

where

$$E = -\int_{-\infty}^{\infty}\left[\frac{D}{2}|Q_T|^2 + \frac{1}{2}|Q|^4\right]dT. \tag{8}$$

is the energy functional.

For the sake of completeness we start with the development of the variational approach for a single DM soliton. The energy of pulse is given by

$$E_s = \int_{-\infty}^{\infty}|Q|^2 dT = A^2 q\sqrt{\pi/2}. \tag{9}$$

In real units, this energy can be expressed as $E[J] = (\tau_m/L')\mathcal{E}$, for the above experimetal parameters $\mathcal{E} = 20\,[\mathrm{pJ}]$.

Equations of motion for the differents variational parameters are then derived from the Euler equations, namely

$$\frac{d}{dZ}\cdot\frac{\partial L}{\partial F_Z}-\frac{\partial L}{\partial F}=0. \tag{10}$$

where F and F_Z denote a variational parameter and its time derivative, respectively.

Taking the corresponding derivatives, the Lagrangian (7) reads

$$L_1=E_s\left\{\frac{1}{4}(q^2+4\eta^2)\alpha_Z+\varphi_Z-\frac{D(Z)}{2}\left[\frac{1}{q^2}+(q^2+4\eta^2)\alpha^2\right]-\frac{E_s}{2\sqrt{\pi}q}\right\}. \tag{11}$$

Using (10), equations of motion for variational parameters take the explicit form:

$$E_Z=0, \tag{12a}$$

$$\alpha_Z-2D(Z)\left(\frac{1}{q^4}+\alpha^2\right)+\frac{E_p}{\sqrt{\pi}q^3}=0, \tag{12b}$$

$$\alpha_Z-2D(Z)\alpha^2=0, \tag{12c}$$

$$qq_Z+4\eta\eta_Z-2D(Z)(q^2+4\eta^2)\alpha=0. \tag{12d}$$

Interestingly, Equation (12a) shows the conservation of the energy. Equations (12b) and (12c), allow us to find the chirp by two ways, all depends on the data that we have. Equation (12d) describes the time evolution of both the width and pick position of the soliton. For fixed η, Equation (12d) reduces to $q_Z-2D(Z)\alpha q=0$.

For two soliton-molecule, the resulting coupled equations of motion, which turn out to be lengthy and hence will not be shown here for convenience, are then solved numerically.

The stationary point of Equations (12b)–(12d) for a given energy E_p determines the width and chirp of a single DM soliton at some point of a DM map and can be found using the numerical technique of Nijhof et al. [14, 15]. Once the width is found, the amplitude of the DM soliton is determined form Equation (9).

Figure 1 shows that the center of mass and the chirp exhibit periodic dynamics, while the DM soliton propagates along the fiber line over 20 map periods. The amplidue of both center of mass and chirp is reduced for two soliton molecules.

In Figure 2, we compare the intensity profile obtained by our variational calculation with our simulation results for both single and two-soliton molecules. Our numerical solution was performed by the split-step fast Fourier transform method, with 2048 Fourier modes and the step size was $dz=5\times10^{-4}$. These figures clearly show the good agreement between the variational calculation on the one hand and the direct numerical solution of the NLSE on the other. Our curves were calculated using the experimental values of [2] apart from the phases where here we have worked with anti-phases solitons.

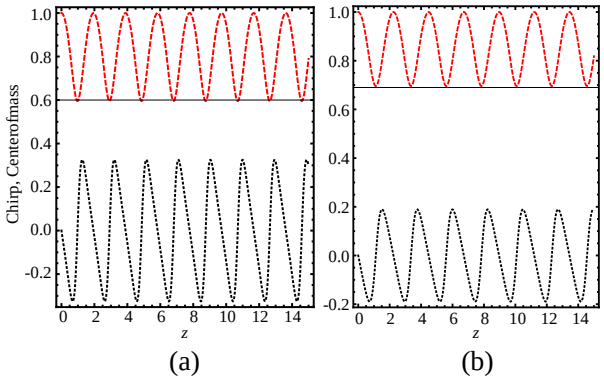

Figure 1: Center of mass and chirp for (a) single soliton and (b) two soliton-molecule with same experimental parameters of Refs. [2, 3].

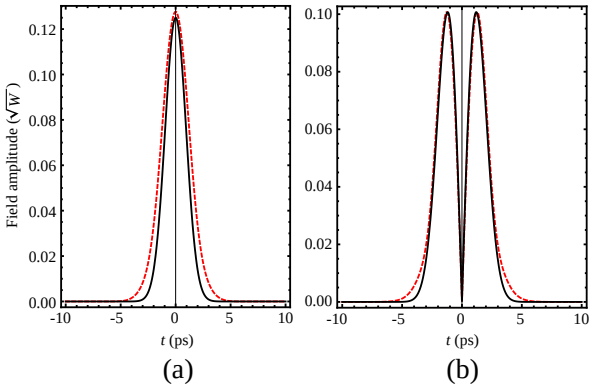

Figure 2: Field amplitude envelopes along dispersion managed fiber of (a) single soliton and (b) two-soliton molecule. Solid line: our variational calculation and Red dashed line: our numerical simulation.

4. MANY-SOLITON MOLECULES

In this section, we investigate the formation and existence of many-soliton molecules in DM optical fiber. We then extend the variational ansatz of the previous section to four and five soliton molecules. To check the validity of our variational calculations for many soliton molecules, we solve again numerically our NLSE.

Figure 3 shows that 5-soliton molecule has indeed nonzero binding energy in terms of the width of solitons. The depth of the minimum gives an estimate to the strength of the bond in the molecule.

Figures 4 depicts the formation and the existence of four-and five-soliton molecules in agreement with our recent theoretical work [16]. It is seen also from the same figure that our numerical simulation coincides with our variational calculation. The stability of 5-soliton molecule during its spatiotemporal evolution is clearly visible in Figure 5.

The existence of four- and five-soliton molecules is indeed an important result because it makes alphabets of five and six different symbols complete: no pulse, single pulse, two pulse molecule, three pulse molecules, four pulse molecules and five pulse molecules in a time slot. We theoretically showed that the same fiber can support all these six symbols. Consequently, a fiber-optic transmission of three bits per clock period is strongly possible. This highly confirms that soliton molecules in DM fiber can be used for enhancing the capacity of the communication system via extension of the coding alphabet.

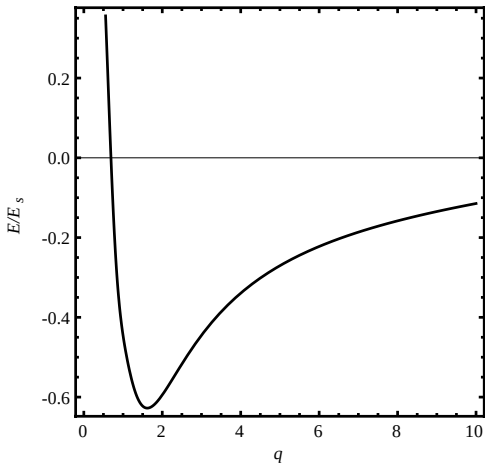

Figure 3: Binding energy of five solitons relative to that of the single-soliton energy E_s as function of the separation of the width q with same experimental parameters of Ref. [3].

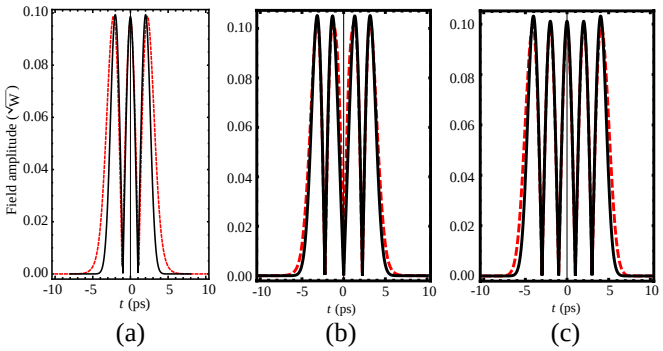

Figure 4: Field amplitude envelopes along dispersion managed fiber of (a) three-soliton molecule, (b) four-soliton molecule and (c) five-soliton molecule. Solide lines: our variational calculation and Red dashed lines our numerical simulation. The parameters are the same as in Figure 2.

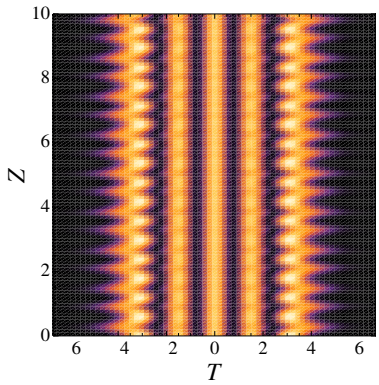

Figure 5: Spatiotemporal evolution of the field amplitude of a 5-soliton molecule. The parameters are the same as in Fig. 2.

5. CONCLUSIONS

In this paper, we have considered many-soliton molecules propagating in DM optical fibers. Using a variational calculation, the binding energy of the 5-soliton molecule was calculated. The coupled system of variational equations for the pulse width and chirp parameter appears to be structurally similar for all considered cases, namely for a single DM soliton, few soliton molecule and many soliton molecule. The difference is seen only in altered numerical coefficient, originating from nonlinearity of the fiber. Furthermore, we have solved numerically the NLSE and verified the stability of the both few-and many-solitons molecules. In addition, the field amplitudes of our trial function agreed favorably with the numerical solution. We theoretically proved that a fiber-optic transmission of beyond two bits per clock period is possible. Therefore, this extended coding alphabet leads to increase the capacity of communication systems.

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