The Optical Angular Momentum in a Vector Vortex Optical Field

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Abstract— We study the propagation dynamics of vortex vector field with azimuthally locally uniform and inhomogeneous polarization states by using angular spectrum the electromagnetic beam. The evolution of the polarization states in the field cross section is analyzed numerically in detail during propagation by using the Stokes polarization parameters. The results indicate that the polarization states in the field cross section rotate along the propagation axis due to the existence of vortex field. In particular, the interaction between the central phase singular point (i.e., dislocation) and the polarization singular point (i.e., disclination) leads to the creation or annihilation of optical field in the center of optical field, depending on the number of vortex and polarization topological charges. The transverse energy flux distributions and both spin and orbital optical angular momentum of vector vortex optical field with the different states of polarization in the cross-section of the field are computed and analyzed. Our results indicate that the different states of polarization in the cross-section of the optical vector vortex field can influence and reconstruct the transverse Poyting vector (energy flux) and both spin and orbital optical angular momentum flux distribution in the cross-section of the field. These results, therefore, provide useful information on how to spatially manipulate the angular momentum of laser beams by choosing appropriate states of polarization in the cross-section of the field. This work provides further insight into dynamic behaviors of the vector vortex optical field and sheds light on a new approach in manipulating micro-particles.

Light carries angular momentum comprised of both a spin component associated with polarization, and an orbital component arising from the spatial profile of light intensity and the phase [1]. Considerable interest in orbital angular momentum arises from its potential use in multiple applications including quantum information processing, atomic manipulation, micro-manipulation and the biosciences [2–5].

In the coordinate system, the z-axis is taken to be the propagation axis. A cylindrical optical vector field is expressed as [6, 7]

$$E(r,\theta) = A(r,\theta)\left[\cos(m\theta+\theta_0)\mathbf{e_x} + \exp(i\Delta\theta)\sin(m\theta+\theta_0)\mathbf{e}_y\right] \tag{1}$$

where $r = \sqrt{x^2+y^2}$ and $\theta = \arctan(y/x)$ are the polar radius and azimuthal angle in the polar coordinate system, respectively. m is the topological charge, and θ is the initial phase. $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors in x and y-direction, respectively. In Eq. (1). $\Delta\theta = 0$ implies that the x- and y-components have the same phase. In this case, it is seen from Eq. (1) that the SoP depends only on the azimuthal angle φ. The vector optical field is linearly polarized at any position in the field cross section. When $m = 1$ with $\theta = 0$ and $\pi/2$, the vector fields describe the radially and azimuthally polarized vector fields, respectively. On the other hand, Eq. (1) degenerates to the linearly-polarized fields if $m = 0$.

By using the Fourier transform, the transverse components of the vector angular spectrum $A_x(\rho\cos\theta, \rho\sin\theta)$ and $A_y(\rho\cos\theta, \rho\sin\theta)$ are given by the Fourier transform of the initial field:

$$\begin{aligned}&\begin{pmatrix} A_x(\rho\cos\phi,\rho\sin\phi) \\ A_y(\rho\cos\phi,\rho\sin\phi)\end{pmatrix} = \left(\tfrac{k}{2\pi}\right)^2 \\ &\begin{pmatrix} \int_0^\infty\int_0^{2\pi} A(r)\cos(m\theta+\theta_0)\exp\left[-ikr\rho\cos(\theta-\phi)\right] rd\theta dr \\ \int_0^\infty\int_0^{2\pi} A(r)\exp(i\Delta\theta)\sin(m\theta+\theta_0)\exp\left[-ikr\rho\cos(\theta-\phi)\right] rd\theta dr \end{pmatrix}\end{aligned} \tag{2}$$

where k is the wavenumber. For the Gaussian distribution with the n-th vortex and an arbitrary polarized electromagnetic field (see Eq. (1)) in the source plane $z = 0$, $A(r,\theta) = \exp(-r^2/w^2)\exp(in\theta)$

where w is beam-width, the angular spectrum is

$$\begin{aligned} A(\rho\cos\phi, \rho\sin\phi) &= \pi i^{m+n}\left(\frac{k}{2\pi}\right)^2 \frac{\sqrt{\pi}kw^3\rho}{8}\exp\left(-k^2w^2\rho^2/8\right) \\ &\quad \times\Big\{P\exp\left[i(m\phi+n\phi+\theta_0)\right](\mathbf{e_x}-i\exp(i\Delta\theta)\mathbf{e}_y) \\ &\quad +T\exp\left[-i(m\phi-n\phi+\theta_0)\right](\mathbf{e_x}+i\exp(i\Delta\theta)\mathbf{e}_y) \\ &\quad -\Big[P\exp\left[i(m\phi+n\phi+\theta_0)\right](\cos\phi-i\exp(i\Delta\theta)\sin\phi) \\ &\quad +T\exp\left[-i(m\phi-n\phi+\theta_0)\right](\cos\phi+i\exp(i\Delta\theta)\sin\phi)\Big]\rho/\gamma\mathbf{e}_z\Big\} \end{aligned} \tag{3}$$

with

$$\begin{aligned} P &= I_{(m+n-1)/2}\left(k^2w^2\rho^2/8\right)-I_{(m+n+1)/2}\left(k^2w^2\rho^2/8\right) \\ T &= I_{(m-n-1)/2}\left(k^2w^2\rho^2/8\right)-I_{(m-n+1)/2}\left(k^2w^2\rho^2/8\right) \end{aligned}$$

where $I(\cdot)$ are the Bessel functions of the second kind and $\mathbf{e_z}$ is the unit vector in z direction. The electric field component of the vector cylindrical optical field in z plane can be represented as:

$$\begin{aligned} E(r) &= (-1)^{m+n}\frac{k^3w^3\sqrt{\pi}}{16}\int_0^\infty e^{-\frac{k^2w^2\rho^2}{8}} \\ &\quad \Big\{PJ_{m+n}(-kr\rho)\exp\left[i(m\theta+n\theta+\theta_0)\right](\mathbf{e_x}-i\exp(i\Delta\theta)\mathbf{e}_y) \\ &\quad +TJ_{m-n}(-kr\rho)\exp\left[-i(m\theta-n\theta+\theta_0)\right](\mathbf{e_x}+i\exp(i\Delta\theta)\mathbf{e}_y) \\ &\quad +\Big[PJ_{m+n+1}(-kr\rho)\exp\left[i(m\theta+n\theta+\theta+\theta_0)\right](1-\exp(i\Delta\theta)) \\ &\quad +PJ_{m+n-1}(-kr\rho)\exp\left[i(m\theta+n\theta-\theta+\theta_0)\right](1+\exp(i\Delta\theta))) \\ &\quad +TJ_{m-n-1}(-kr\rho)\exp\left[-i(m\theta-n\theta-\theta+\theta_0)\right](1+\exp(i\Delta\theta)) \\ &\quad +TJ_{m-n+1}(-kr\rho)\exp\left[-i(m\theta-n\theta+\theta+\theta_0)\right](1-\exp(i\Delta\theta))\Big]i\rho/2\gamma\mathbf{e}_z\Big\} \\ &\quad \times\exp\left(ik\gamma z\right)\rho^2d\rho \end{aligned} \tag{4}$$

Under the paraxial approximation, the transverse energy (TE) flow of the vector cylindrical optical field in z plane can be written in the following forms:

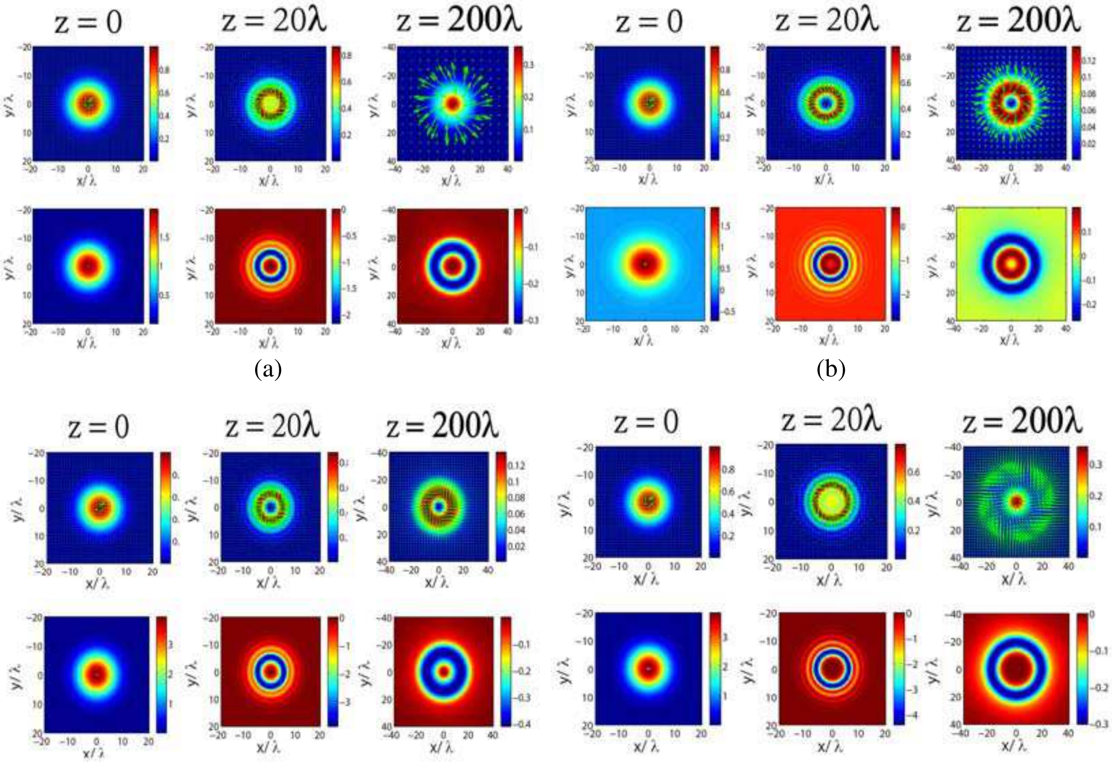
$$\vec{S}=\frac{1}{2}\mathrm{Re}\left[\vec{E}(r)\times\vec{H}^*(r)\right]=\frac{1}{2\mu_0\omega}\mathrm{Im}\left[\vec{E}(r)\times\left(\nabla\times\vec{E}(r)^*\right)\right] \tag{5}$$

where Re[·] and Im[·] denote the real and imaginary parts, respectively, and the asterisk corresponds to its complex conjugation. Therefore, Eq. (7) describes the energy flux distribution at the propagation plane of $z\equiv$ constant. The optical angular momentum flux density is [8]

$$J_z=l_z+s_z=\frac{E^*\cdot(r\times\nabla)E\cdot e_z}{\iint E^*\cdot Edxdy}+\frac{E^*\times E\cdot e_z}{i\iint E^*\cdot Edxdy} \tag{6}$$

where the first and second terms represent the flux density of the orbital angular momentum (OAM) flux density Lz and the spin angular momentum flux density Sz, respectively.

The distribution of the TE flux and the OAM flux are depicted in Fig. 1. It is clearly seen from Fig. 1 that the TE flux and the OAM flux depend on SoP, as expected. The results indicate that the different states of polarization in the cross-section of the optical vector vortex field can influence and reconstruct the transverse Poyting vector (energy flux) and both spin and orbital optical angular momentum flux distribution in the cross-section of the field. The results provide a deeper understand to manipulate the angular momentum of laser beams.



(a) (b) (c) (d)

Figure 1: The distribution of TE flows (upper) and OAM flux density (lower) in the cross-section of a local linearly polarized vortex vector optical field ($\Delta\theta = \pi/2$ and $\varphi_0 = 0$) with (a) $n = 1$, $m = 1$; (b) $n = 1$, $m = 2$; (c) $n = 2$, $m = 1$; (d) $n = 2$, $m = 2$ for different propagation distance. Arrows represent the directions of transverse energy flow in the field cross-section.

REFERENCES

1. Allen, L., M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, "Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes," *Phys. Rev. A*, Vol. 45, 8185–8189, 1992.
2. Molina-Terriza, G., J. P. Torres, and L. Torner, "Management of the angular momentum of light: Preparation of photons in multidimensional vector states of angular momentum," *Phys. Rev. Lett.*, Vol. 88, 013601, 2002.
3. Chen, R. P., K. H. Chew, and S. He, "Dynamic control of collapse in a vortex airy beam," *Sci. Rep.*, Vol. 3, 1406, 2013.
4. Berkhout, G. C. G., M. P. J. Lavery, J. Courtial, M. W. Beijersbergen, and M. J. Padgett, "Efficient sorting of orbital angular momentum states of light," *Phys. Rev. Lett.*, Vol. 105, 153601, 2010.
5. Wang, X. L., J. Chen, Y. Li, J. Ding, C. S. Guo, and H. T. Wang, "Optical orbital angular momentum from the curl of polarization," *Phys. Rev. Lett.*, Vol. 105, 253602, 2010.
6. Zhan, Q. W., "Cylindrical vector beams: From mathematical concepts to applications," *Adv. Opt. Photon.*, Vol. 1, 1–57, 2009.
7. Wang, X. L., Y. N. Li, J. Chen, C. S. Guo, J. P. Ding, and H. T. Wang, "A new type of vector fields with hybrid states of polarization," *Opt. Express*, Vol. 18, 10786–10795, 2010.
8. Berry, M., "Paraxial beams of spinning light," *Proc. SPIE*, Vol. 3487, 6, 1998.