New Topography Inversion Using EM Field

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Abstract
The method is developed to model the three dimensional (3D) terrain effect of the electromagnetic fields of the artificial source in the frequency domain. By vector integral theory and boundary conditions, the electromagnetic fields in the air and earth medium are transformed into two vector integral equations just related to the topography. The integral related to the topography is then decomposed into a series of integrals in the triangular element. The electromagnetic fields in the triangular element are decomposed into two parts: the electromagnetic fields in the unlimited air space and in the earth medium. The linear equations come from vector integral equations can be solved by SSOR.

Introduction
It is well-known that rugged surface topography distorted the electromagnetic response of the earth dramatically. When electric-magnetic method are carried out in the mountain land, the effect of topography should be removed from the data before inversion. Wannamaker, Stodt and Rijo (1986) and Chouteau and Bouchard (1988) have modeled 2D topography in magnetotelluric survey (MT) using finite element method. Xu and Zhou (1997) used boundary element method (BEM) to model the MT over the 2D topography. Xu et al. (1997) tried to model the 3D topography effect in MT using BEM. For artificial source, Endo and Noguchi (1999) modeling the 3D electromagnetic field in time domain using the finite difference method (FDM) in the case of 3D topography. Yutaka Sasaki (2003) put forward a method for 3D electromagnetic modeling and inversion with topography using FDM. The BEM is used in this paper to model the electromagnetic response for the 3D topography and artificial source in frequency domain. Firstly, we illustrated the vector Helmholtz equation, boundary conditions and boundary integral equations of electromagnetic field. Secondly, the BEM is used to solve the integral equation. Finally, the electromagnetic responses on 3-D hill topography is shown.

Electromagnetic Field Boundary Integral Equation
It is supposed that the studied region \( \Omega \) is surrounded by a boundary \( \Gamma \), and is occupied by homogenous and isotropic media with a electric conductivity of \( \sigma \) and a medium permittivity of \( \varepsilon \). By using the magnetic permeability \( \mu_0 \) in the vacuum instead of that in medium (\( \mu \)) and the time factor is \( e^{i\omega t} \), the electromagnetic fields in the region, according to Xu’s method (Xu et al., 1997), can be described by the following boundary integral equations,

\[
\begin{align*}
\frac{\omega \mu}{4\pi} H(p) + & \int_{\Gamma} \left[ (H \cdot n) \nabla \varphi + (n \times H) \times \nabla \varphi + (\sigma + i\omega\varepsilon)(n \times E)\varphi \right] d\Gamma \\
= & \int_{\Gamma_c} \left[ (H \cdot n) \nabla \varphi + (n \times H) \times \nabla \varphi + (\sigma + i\omega\varepsilon)(n \times E)\varphi \right] d\Gamma \\
\frac{\omega \varepsilon}{4\pi} E(p) + & \int_{\Gamma} \left[ (E \cdot n) \nabla \varphi + (n \times E) \times \nabla \varphi - i\omega\mu(n \times H)\varphi \right] d\Gamma \\
= & \int_{\Gamma_c} \left[ (E \cdot n) \nabla \varphi + (n \times E) \times \nabla \varphi - i\omega\mu(n \times H)\varphi \right] d\Gamma
\end{align*}
\]

(1)

where

\( H(p) \), \( E(p) \) are magnetic and electric fields at a point \( p \) with the studied region respectively;

\( k = \sqrt{\omega^2\mu\varepsilon - i\omega\mu\sigma} \) is the wave number;

\( \varphi = \frac{1}{4\pi r} e^{-ikr}, \) \( r \) is the distance between \( p \) and integral point;
\( n \) is the outer normal direction of the region \( \Omega \);
\( \omega_p \) is the solid angle from point \( p \) to region \( \Omega \), \( \omega_p = 4\pi \) when the point \( p \) is within the region \( \Omega \);
\( \Gamma_e \) is the small boundary within the source is located.

The right sides of equation (1) can be deduced to equal the primary magnetic \( H_p(p) \) and electric field \( E_p(p) \) in the unlimited space with electric conductivity \( \sigma \) respectively. Thus

\[
\begin{align*}
\frac{\omega_p}{4\pi} H(p) + \oint_{\Gamma} [(H \cdot n) \nabla \varphi + (n \times H) \times \nabla \varphi + (\sigma + i\omega \varepsilon)(n \times E) \varphi] d\Gamma &= H_p(p) \\
\frac{\omega_p}{4\pi} E(p) + \oint_{\Gamma} [(E \cdot n) \nabla \varphi + (n \times E) \times \nabla \varphi - i\omega \mu(n \times H) \varphi] d\Gamma &= E_p(p)
\end{align*}
\]

(2)

These are the boundary integral equations for the homogeneous medium within the region \( \Omega \).

**Electromagnetic Boundary Integral for 3D Topography**

In the case of 3D topography, there are two regions, \( \Omega_1 \) above the earth surface including a source and \( \Omega_2 \) underneath the surface.

Supposed that the electric and magnetic fields are \( E_1 \) and \( H_1 \) in the region \( \Omega_1 \), respectively, the electromagnetic responses at point \( p_1 \) can be written from equation (2) as follows:

\[
\begin{align*}
\frac{\omega_1}{4\pi} H_1(p_1) + \oint_{\Gamma} [(H_1 \cdot n_1) \nabla \varphi_1 + (n_1 \times H_1) \times \nabla \varphi_1 + i\omega \varepsilon(n_1 \times E_1) \varphi_1] d\Gamma &= H_p(p_1) \\
\frac{\omega_1}{4\pi} E_1(p_1) + \oint_{\Gamma} [(E_1 \cdot n_1) \nabla \varphi_1 + (n_1 \times E_1) \times \nabla \varphi_1 - i\omega \mu(n_1 \times H_1) \varphi_1] d\Gamma &= E_p(p_1)
\end{align*}
\]

(3)

where \( \Gamma \) is the boundary of the topography, \( n_1 \) is the normal direction of the surface downward, and \( \varphi_1 = \frac{1}{4\pi r} e^{-ik_1 r}, k_1 = \omega \sqrt{\mu_\varepsilon} \).

Similarly, in the region \( \Omega_2 \) which is homogeneous and isotropic with the electric conductivity of \( \sigma \), if the electric and magnetic fields are \( E_2 \) and \( H_2 \), respectively, at point \( p_2 \) then we have

\[
\begin{align*}
\frac{\omega_2}{4\pi} H_2(p_2) + \oint_{\Gamma} [(H_2 \cdot n_2) \nabla \varphi_2 + (n_2 \times H_2) \times \nabla \varphi_2 + (\sigma + i\omega \varepsilon)(n_2 \times E_2) \varphi_2] d\Gamma &= 0 \\
\frac{\omega_2}{4\pi} E_2(p_2) + \oint_{\Gamma} [(E_2 \cdot n_2) \nabla \varphi_2 + (n_2 \times E_2) \times \nabla \varphi_2 - i\omega \mu(n_2 \times H_2) \varphi_2] d\Gamma &= 0
\end{align*}
\]

(4)

where \( \Gamma \) is the boundary of the topography, \( n_2 \) is the normal direction of the surface upward, and \( \varphi_2 = \frac{1}{4\pi r} e^{-ik_2 r}, k_2 = \sqrt{\omega^2 \mu_\sigma - i\omega \mu \sigma} \).

Noted that the right sides are zeros in equations (4) because there is no source in the lower space.

When the points \( p_1 \) and \( p_2 \) are at the same point \( p \) on the surface, according to the continuity of electromagnetic field in transversal direction and electric current density in the normal direction across the surface,

\[
\begin{align*}
n_2 \times E_2 &= -n_1 \times E_1 = -n \times E \\
n_2 \times H_2 &= -n_1 \times H_1 = -n \times H \\
n_2 \cdot E_2 &= -(1 + \alpha)n_1 \cdot E_1 = -(1 + \alpha)n \cdot E \\
n_2 \cdot H_2 &= -n_1 \cdot H_1 = -n \cdot H \\
\omega_p + \omega_p &= 4\pi
\end{align*}
\]

(5)\(\text{ to } (7)\)

where: \( \alpha = \frac{i\omega \varepsilon}{\sigma + i\omega \varepsilon} - 1 \).

Added the electromagnetic field equations (3) and equation (4) together and used equation (5) and (6) and (7), respectively, the electromagnetic boundary integral equations for 3D topography can be written as below:

\[
\begin{align*}
H(p) + \oint_{\Gamma} \{ H_n \nabla(\varphi_1 - \varphi_2) + (n \times H) \times \nabla(\varphi_1 - \varphi_2) \\
+ (n \times E) [i\omega \varepsilon(\varphi_1 - \varphi_2) - \sigma \varphi_2] \} d\Gamma &= H_p(p) \\
E(p) + \frac{\omega_p}{4\pi} E_p(p) + \oint_{\Gamma} \{ E_n [\nabla(\varphi_1 - \varphi_2) - \alpha \nabla \varphi_2] \\
+ (n \times E) \times \nabla(\varphi_1 - \varphi_2) - i\omega \mu(n \times H) (\varphi_1 - \varphi_2) \} d\Gamma &= E_p(p)
\end{align*}
\]

(8)
where $H_n$ and $E_n$ are the magnetic and electric field component in the normal direction, respectively.

**Boundary Element Method**

To solve the integral equations (8), the topography $\Gamma$ is divided into $M$ triangular elements. Taking the center in each triangular element as the node at which the electromagnetic response will be calculated and $\omega_p=2\pi$, the integrals in equations (8) over the surface $\Gamma$ are equal to the sum of integrals in each triangular element $\Gamma_j$. Taking $p$ as the node of the ith element, the equations (8) can be rewritten as below:

\[
\begin{align*}
H_i + \sum_{j=1}^{M} \left\{ H_n \nabla(\varphi_{1i} - \varphi_{2i}) + (n \times H) \times \nabla(\varphi_{1i} - \varphi_{2i}) \right\} \\
+ (n \times E)[i\omega\varepsilon(\varphi_{1i} - \varphi_{2i}) - \sigma\varphi_{2i}]d\Gamma = H_{pi} \\
E_i + \frac{\alpha}{2} E_{ni} + \sum_{j=1}^{M} \left\{ E_n [\nabla(\varphi_{1i} - \varphi_{2i}) - \alpha \nabla \varphi_{2i}] \right\} \\
+ (n \times E) \times \nabla(\varphi_{1i} - \varphi_{2i}) - i\omega\mu(n \times H)(\varphi_{1i} - \varphi_{2i})d\Gamma = E_{pi}
\end{align*}
\]  

where the subscript $i$ used in the basic solutions of $\varphi_1$ and $\varphi_2$, means that $r$ is the distance from ith node to the integral point.

In each element $\Gamma_j$, the electromagnetic response should be the stack of the electromagnetic field in unlimited air space and the distribution of the earth medium. In order to simplify the computation, it is supposed that the distribution of the earth medium is a constant within the element, which means that the electromagnetic fields at any point $p$ in each element satisfy the relationships

\[
E(p) = E_j + [E_p(p) - E_{pj}]
\]

\[
H(p) = H_j + [H_p(p) - H_{pj}]
\]

by equations (9) and (10), we get

\[
\begin{align*}
H_i + \sum_{j=1}^{M} \left\{ H_n \int_{\Gamma_j} \nabla(\varphi_{1i} - \varphi_{2i})d\Gamma + (n_j \times H_j) \int_{\Gamma_j} \nabla(\varphi_{1i} - \varphi_{2i})d\Gamma \right\} \\
+ (n_j \times E_j) \int_{\Gamma_j} [i\omega\varepsilon(\varphi_{1i} - \varphi_{2i}) - \sigma\varphi_{2i}]d\Gamma = H_{pi} + H_{ci} \\
E_i + \frac{\alpha}{2} E_{ni} + \sum_{j=1}^{M} \left\{ E_n \int_{\Gamma_j} [\nabla(\varphi_{1i} - \varphi_{2i}) - \alpha \nabla \varphi_{2i}]d\Gamma \right\} \\
+ (n_j \times E_j) \times \int_{\Gamma_j} \nabla(\varphi_{1i} - \varphi_{2i})d\Gamma - i\omega\mu(n_j \times H_j) \int_{\Gamma_j} (\varphi_{1i} - \varphi_{2i})d\Gamma = E_{pi} + E_{ci}
\end{align*}
\]  

where

\[
\begin{align*}
H_{ci} = \sum_{j=1}^{M} \left\{ (H_{n_j} - H_p) \nabla(\varphi_{1i} - \varphi_{2i}) + n \times (H_j - H_p) \times \nabla(\varphi_{1i} - \varphi_{2i}) \right\} \\
+ n \times (E_{p_j} - E_p)[i\omega\varepsilon(\varphi_{1i} - \varphi_{2i}) - \sigma\varphi_{2i}]d\Gamma
\end{align*}
\]

\[
\begin{align*}
E_{ci} = \sum_{j=1}^{M} \left\{ (E_{n_j} - E_p) [\nabla(\varphi_{1i} - \varphi_{2i}) - \alpha \nabla \varphi_{2i}] \right\} \\
+ n \times (E_{p_j} - E_p) \times \nabla(\varphi_{1i} - \varphi_{2i}) - i\omega\mu n \times (H_j - H_p)(\varphi_{1i} - \varphi_{2i})d\Gamma
\end{align*}
\]

The boundary integral equations (11) are vector equations and can be decomposed into 6 linear equations in the directions of $x, y$ and $z$ in the Cartesian coordinate. Since there are 6 linear equations decomposed from the integral equations (11) for each node, $6 \times M$ linear equations will be formed for $M$ elements (nodes). The coefficient matrix of the line equations is a diagonally dominant matrix, so the equations can be solved by the SSOR method.

Calculated the electromagnetic fields in each element on the surface, the electromagnetic fields either in the air space or earth medium can be obtained by the equations (3) and (4).
Electromagnetic Field on 3-D Hill Topography

The 3-D hill topography is shown in Figure 1a and 1b, and the vertical magnetic dipole source is located by the center of the 3-D hill topography. The non-normalized and normalized vertical magnetic responses are given Figure 2. The results indicate that the vertical magnetic responses is decreased in both the real and imaginary part due to the topography effect of the uplift topography, i.e, the uplift topography makes the vertical magnetic responses smaller above the source.

![Figure 1: 3-D hill topography (Tx is the location of the source)](image)

![Figure 2: The vertical magnetic responses for an uplift topography (circle and cross are calculated by BEM, solid and dash lines are calculated in a homogeneous earth with horizontal surface)](image)

Conclusion

The boundary element modeling method in this paper for electromagnetic field of artificial source in frequency domain with 3D topography is proven to be effective.

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REFERENCES


