Analysis of Transient Scattering from 2-D Rough Surface Using Time Domain Integral Equation Method

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Abstract

Because of the need of many engineering applications, the researches of rough surface scattering has attracted many researchers’ attentions for a long time. In this paper, the method of time domain integral equation (TDIE) is used to analyze the characteristics of transient scattering from two-dimensional dielectric random rough surface. The TDIE is computed numerically by using moment method with marching-on-in-time algorithm. Transient scattering from 2-D dielectric random rough surface when illuminated by different polarized pulse tapered wave are computed, and the numerical results are compared with scattering from dielectric plate of the same size.

Introduction

Until relatively recently most researches of rough surface scattering has taken place in the frequency domain (FD) [1]. Since the beginning of computational electromagnetics (CEM), there has been a steady growth in time domain (TD) modeling. Besides physical interpretability, for certain problems, fewer arithmetic operations are required when performed in the TD [2]. Lawrence Carin analyzed the scattering from objects buried below a rough surface using the time domain differential equation method. With the advance of the ultra wide bandwidth (UWB) technology, the research of the characteristics of scattering from rough surface is a project with great value of application. But there are so few papers about this topic, especially for large-scale surface scattering. In this paper, the method of time domain integral equation (TDIE) is used to analyze the characteristics of transient scattering from two-dimensional random rough surface. The TDIE is computed numerically by using moment method with marching-on-in-time algorithm. In this work, an efficient, accurate and stable numerical solution is developed for the TDIE. The two-dimensional rough surface was approximated by a set of triangular patches. We use the triangular patch modeling and The RWG vector basis functions. The solution is based on the Galerkin’s testing in space and marching-on-in time method. It has no matrix storage thereby eliminating the expensive inversion step. Furthermore, the method employs a special averaging procedure to control the late-time oscillations. The diffractions of artificial edges are eliminated by introducing pulse tapered wave as the incident wave. Specifically, a TDIE model for Transient scattering from 2-D dielectric rough surface is developed.

This paper is organized as follows: Section 1 describes the time domain integral equation formulation and the numerical solution procedures are presented. Section 2 describes numerical results. Finally, our conclusions are presented.

Figure 1: Transient scattering from 2-D dielectric rough surface
Section 1

The scatterer has material parameters of $\mu_e$ and $\varepsilon_e$, and exterior to the body is a homogeneous medium with parameters $\mu_e$ and $\varepsilon_e$. Exterior to the body, the total fields are designated by $\mathbf{E}_e$ and $\mathbf{H}_e$, whereas interior to the object the fields are given by $\mathbf{E}_d$ and $\mathbf{H}_d$. Employing the equivalence principle, the rough surface may be replaced with two sets of electric ($\mathbf{J}_e$, $\mathbf{J}_d$) and magnetic ($\mathbf{M}_e$, $\mathbf{M}_d$) currents. By enforcing the continuity of the tangential electric and magnetic fields at the dielectric interface, we derive the following integral equations:

$$
\begin{align*}
\left[ \frac{\partial^2 (\mathbf{A}_e + \mathbf{A}_d)}{\partial t^2} + \nabla (\psi_e^e + \psi_d^e) + \nabla \times \left( \frac{1}{\varepsilon_e} \frac{\partial \mathbf{F}_e}{\partial t} + \frac{1}{\varepsilon_d} \frac{\partial \mathbf{F}_d}{\partial t} \right) \right]_{\tan} &= \left[ \frac{\partial \mathbf{E}^i}{\partial t} \right]_{\tan}, \quad (1) \\
\left[ \frac{\partial^2 (\mathbf{F}_e + \mathbf{F}_d)}{\partial t^2} + \nabla (\psi_e^m + \psi_d^m) - \nabla \times \left( \frac{1}{\mu_e} \frac{\partial \mathbf{A}_e}{\partial t} + \frac{1}{\mu_d} \frac{\partial \mathbf{A}_d}{\partial t} \right) \right]_{\tan} &= \left[ \frac{\partial \mathbf{H}^i}{\partial t} \right]_{\tan}, \quad (2)
\end{align*}
$$

where,

$$
\begin{align*}
\mathbf{A}_e(r, t) &= \frac{\mu_v}{4\pi} \int_S \mathbf{J}(r', t - R/c_v) ds', \\
\mathbf{F}_v(r, t) &= \frac{\varepsilon_v}{4\pi} \int_S \mathbf{M}(r', t - R/c_v) ds', \\
\psi_e^e(r, t) &= \frac{1}{4\pi\varepsilon_v} \int_S \frac{\partial q_e^e(r', t - R/c_v)}{\partial t} R ds' = -\frac{1}{4\pi\varepsilon_v} \int_S \nabla' \cdot \mathbf{J}(r', t - R/c_v) ds', \\
\psi_e^m(r, t) &= \frac{1}{4\pi\mu_v} \int_S \frac{\partial q_e^m(r', t - R/c_v)}{\partial t} R ds' = -\frac{1}{4\pi\mu_v} \int_S \nabla' \cdot \mathbf{M}(r', t - R/c_v) ds', \\
\phi_e^e(r, t) &= \frac{1}{4\pi\varepsilon_v} \int_S q_e^e(r', t - R/c_v) \frac{R}{R} ds', \\
\phi_e^m(r, t) &= \frac{1}{4\pi\mu_v} \int_S q_e^m(r', t - R/c_v) \frac{R}{R} ds',
\end{align*}
$$

The formulation described so far is popularly known as PMCHW formulation. $v = e$ or $v = d$. $R = |r - r'|$, the distance from the field point, $r$, to the source point, $r'$, and $c_v$ is the velocity of the wave propagation in medium $v$. $q_e^e$ and $q_e^m$ is the electric and magnetic surface charge density, respectively. This formulation is computed numerically by using moment method with marching-on-in-time algorithm.

First, we approximate the arbitrary body by a set of planar triangular patches and choose the RWG basis functions $f_m$.

Then, we divide the time axis into equal intervals of $\Delta t$. The time basis function $T_j(t)$ is

$$
T_j(t) = \begin{cases} 
\cos^2 \left( \frac{\pi}{2} \frac{t - t_j}{\Delta t} \right) & |t - t_j| \leq \Delta t \\
0 & \text{otherwise}
\end{cases} \quad (t_j = j\Delta t) \quad (9)
$$

We now approximate the electric and magnetic currents as

$$
\begin{align*}
\mathbf{J}(r, t) &= \sum_{j=-\infty}^{+\infty} \sum_{n=1}^{N} I_{n,j} f_n(r) T_j(t) \\
\mathbf{M}(r, t) &= \sum_{j=-\infty}^{+\infty} \sum_{n=1}^{N} M_{n,j} f_n(r) T_j(t)
\end{align*}
$$

We test Eq.(1) and Eq.(2) with the same $f_m$ and approximate the time derivatives of the potential functions by finite differences to obtain

$$
\begin{align*}
< f_m, L_e^E(J, M) + L_d^E(J, M) >= < f_m, \frac{\partial E^{inc}(t_i)}{\partial t} > \\
< f_m, L_e^H(J, M) + L_d^H(J, M) >= < f_m, \frac{\partial H^{inc}(t_i)}{\partial t} >
\end{align*}
$$

(12)

(13)
where,

\[ L^E_{\psi}(J, M) = \frac{A_v(r, t_{i+1}) - 2A_v(r, t_i) + A_v(r, t_{i-1})}{\Delta t^2} + \nabla \psi_v(r, t_i) + \frac{1}{\varepsilon_v} \nabla \times \left[ \frac{F_v(r, t_{i+1}) - F_v(r, t_i)}{\Delta t} \right] \]  

(14)

\[ L^H_{\psi}(J, M) = \frac{F_v(r, t_{i+1}) - 2F_v(r, t_i) + F_v(r, t_{i-1})}{\Delta t^2} + \nabla \psi_{m\psi}(r, t_i) - \frac{1}{\mu_v} \nabla \times \left[ \frac{A_v(r, t_{i+1}) - A_v(r, t_i)}{\Delta t} \right] \]  

(15)

After the Galerkin’s testing procedure, we follow a few mathematical steps [3]. Furthermore, choosing \( \Delta t \leq \frac{R_{\text{min}}}{\max(c_e, c_d)} \) and taking all known quantities to the right, we may solve the equations explicitly to compute the only unknown \( I_{m,i+1}, M_{m,i+1} \) by marching-on-in-time algorithm without the matrix storage:

\[ \alpha_e I_{m,i+1} = \langle f_m, \frac{\partial E_i}{\partial t} - L^E_{\psi}(J, M) - L^E_{\psi}(J, M) \rangle \]  

(16)

\[ \alpha_m M_{m,i+1} = \langle f_m, \frac{\partial H_i}{\partial t} - L^H_{\psi}(J, M) - L^H_{\psi}(J, M) \rangle \]  

(17)

To overcome the late-time oscillations, we use an averaging scheme [4]. The electric and magnetic current is averaged respectively as

\[ \bar{I}_{m,i} = \frac{1}{4}(I_{m,i-1} + 2I_{m,i} + I_{m,i+1}) \]  

(18)

\[ \bar{M}_{m,i} = \frac{1}{4}(M_{m,i-1} + 2M_{m,i} + M_{m,i+1}) \]  

(19)

Section 2

Fig.1 shows a rough surface(1.5m×1.5m, \( \varepsilon_r = 2.0 \) immersed in air) with rms height 0.2m and the correlation length in the x and y direction both 0.2m. An observation plane(0.75m×0.75m) is set above the rough surface, at a height of \( h=0.1m \).

In this section, we discuss transient scattering from 2-D dielectric rough surface when illuminated by different polarized pulse tapered wave. The diffractions of artificial edges are eliminated by introducing pulse tapered wave as the incident wave[5].

\[ E(x, y, z, t) = 2Re \left\{ \int_0^\infty d\omega X(\omega) e^{-i\omega t} E_i(x, y, z, \omega) \right\} \]  

(20)

Where,

\[ X(\omega) = \frac{\tau_m^3 \omega^2}{8\pi^2 \sqrt{2}} \exp \left( -\frac{\omega^2 \tau_m^2}{8\pi} \right) \quad (\tau_m = 0.6LM) \]  

(21)

\[ E_i(x, y, z, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (E_v v + E_h h) e^{i(k_x x + k_y y - k_z z)} dk_x dk_y \]  

(22)

Fig.2 shows the 3-dimensional pulse tapered wave.

Figure 2: 3-D pulse tapered wave

Figure 3: spatial distributions of the peak (Horizontally polarized, dielectric plate)
Fig. 3 illustrates transient scattering from a dielectric plate (1.5m × 1.5m, \( \varepsilon_r = 2.0 \)) immersed in air when illuminated by horizontally polarized pulse tapered wave and shows the spatial distributions of the peak of the scattered fields on the observation plane. From Fig. 3, we can see the scattered field agrees well with the tapered wave in the shape.

Fig. 4 and Fig. 5 illustrate transient scattering from the rough surface in Fig. 1, when illuminated by horizontally or vertically polarized pulse tapered wave respectively and show the spatial distributions of the peak of the scattered fields on the observation plane. Because of the multi-scattering of the rough surface, the spatial distributions have changed.

Fig. 6 and Fig. 7 show the scattered fields as a function of time at two different points on the observation plane, when illuminated by horizontally polarized pulse tapered wave.

Fig. 8 and Fig. 9 show the scattered fields as a function of time at two different points on the observation plane, when illuminated by vertically polarized pulse tapered wave.

The results show that in some points the scattered fields agree well with the incident wave in shape, while
in some other points wave distortions exist.

**Conclusion**

In this paper, we study the characteristics of transient scattering from 2-D dielectric random rough surface. Specifically, a TDIE model is developed: Transient scattering from 2-D dielectric random rough surface when illuminated by different polarized pulse tapered wave are computed, then the numerical results are compared with scattering from dielectric plate of the same size, and the characteristics of scattering from 2-D dielectric rough surface are analyzed. The results show that in some observation points the scattered fields agree well with the incident wave in shape, but in some other points waveform changes and wave distortions exist.

To analyze the characteristics of transient scattering from large-scale rough surface, it is urgent to study new method and solve the TDIE efficiently and quickly. One way is to employ Parallel Algorithm; another more efficient way is to develop fast algorithm for the TDIE solution.

**REFERENCES**