Effective Parameters of Artificial Material Composed of Dielectric Particles

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Abstract — In this research, a theory has been developed to estimate the effective parameters of artificial material composed of dielectric bars placed in a periodic fashion in dielectric host medium (air) from equivalent circuit point of view and verified with HFSS simulation. The theory provides a relationship between effective permittivity and the filling factor of the constituent particle and explains the anisotropic property of the artificial material macroscopically.

1. INTRODUCTION
In previous research it has been shown theoretically that effective material parameter (permittivity $\varepsilon_r$, and permeability $\mu_r$) of artificial material, composed of spherical metal or dielectric balls embedded in host dielectric medium in a regular three dimensional fashion, depend on the filling factor [1]. This theoretical concept has been verified by simulation [2–4] too. In this research, a theory is developed to predict effective material parameters macroscopically from lumped element circuit parameters, inductance and capacitance, resulted from constituent particle. As inductance and capacitance are geometry-dependent parameters, so effective parameters, sensitivity of effective permittivity to particle permittivity and anisotropic behavior, which are discussed in this research, can be explained from particle geometry and particle permittivity.

2. THEORY
When rectangular dielectric bars are arranged in space to form an artificial material, a unit cell of that material is shown in Fig. 1. The dimensions of the bar is $x \times y \times h$ and cell is $\Delta x \times \Delta y \times H$ as shown in Fig. 1. The total capacitance of the cell $C_{cell}$ is the parallel combination of two of capacitances; one is the contribution from the shaded area $C_s$ and other is form the unshaded area $C_{us}$ as shown in Fig. 1(b). Again $C_s$ is the series combination of three capacitances $C_a$, $C_d$ and $C_a$ as shown in Fig. 1(c).

Now,

\begin{align}
C_{us} &= \varepsilon_0(\Delta x\Delta y - xy)/H \\
C_a &= 2\varepsilon_0 xy/(H - h) \\
C_d &= \varepsilon_r\varepsilon_0 xy/h
\end{align}

\(1\)

Figure 1: (a) Unit cell with dielectric bar as constituent particle, (b) top view, (c) front view.

Figure 2: A parallel plate waveguide of unit length with width $W$ and height $H$, filled with unit cells showing total cell capacitance $C_{cell}$ and cell inductance $L_{cell}$.
Therefore,

\[ C_s = \frac{1}{1/C_a + 1/C_d + 1/C_a} = \frac{\varepsilon_0 \varepsilon_r x y}{(H - h) \varepsilon_r + h} \]  

(2)

Therefore total cell-capacitance

\[ C_{cell} = C_{us} || C_s = \frac{\varepsilon_0 (\Delta x \Delta y - xy)}{H} + \frac{\varepsilon_0 \varepsilon_r x y}{(H - h) \varepsilon_r + h} \]  

(3)

As there is no magnetic material in the cell, total cell-inductance

\[ L_{cell} = \mu_0 H \Delta y / \Delta x \]  

(4)

Now, to calculate the effective parameter, it is sufficed to consider that a Parallel Plate Waveguide (PPWG) with width \( W \) and height \( H \) as shown in Fig. 2 is loaded with the unit cells as described in Fig. 1, where electric field is polarized in \( z \)-direction, magnetic field is polarized in \( x \)-direction and direction of wave propagation is in \( y \)-direction. Total capacitance per unit length of PPWG is

\[ C_0 = C_{cell} \times \left( \frac{W}{\Delta x \Delta y} \right) \]  

(5)

and total shunt admittance per unit length is

\[ Y_0 = j \omega C_0 \]  

(6)

Total inductance per unit length of PPWG is

\[ L_0 = (L_{cell} / \Delta y) / (W / \Delta x) \]  

(7)

and total series impedance

\[ Z_0 = j \omega L_0 \]  

(8)

Therefore, propagation constant

\[ \gamma = \alpha + j \beta = \sqrt{Z_0 Y_0} = j \omega \sqrt{L_0 C_0} \]  

(9)

Considering the lossless case

\[ \beta^2 = \omega^2 \varepsilon_0 \varepsilon_{r} \mu_{0} \mu_{r} = \omega^2 L_0 C_0 \]  

(10)

Using Eqs. (3), (4), (5) and (7) in Eq. (10) yields

\[ \varepsilon_{r} \mu_{r} = \left[ \frac{(\Delta x \Delta y - xy)}{H} + \frac{\varepsilon_r x y}{(H - h) \varepsilon_r + h} \right] \times \frac{H}{\Delta x \Delta y} \]  

(11)

Now characteristic impedance of PPWG, filled with particles, is

\[ Z_c = \frac{H}{W} \sqrt{\frac{\mu_{r}}{\varepsilon_{r}}} = \sqrt{\frac{Z_0}{Y_0}} \]  

(12)

Using Eqs. (3), (4), (5) and (7) in Eq. (12) yields

\[ \frac{\varepsilon_{r} \mu_{r}}{\mu_{r} \varepsilon_{r}} = \left[ \frac{(\Delta x \Delta y - xy)}{H} + \frac{\varepsilon_r x y}{(H - h) \varepsilon_r + h} \right] \times \frac{H}{\Delta x \Delta y} \]  

(13)

Introducing the filling factor term \( f = xy h / (\Delta x \Delta y H) \) and ratio of particle-height \( (h) \) to cell-height \( (H) \) in the direction of electric field \( a = h / H \) in Eqs. (11) and (13) yields

\[ \mu_{r} = 1 \]  

(14)

\[ \varepsilon_{r} = 1 + f \left( \frac{\varepsilon_r}{a + \varepsilon_r (1 - a)} - 1 \right) \]  

(15)

This is a very general equation for effective permittivity in terms of filling factor \( f \) and \( a (= h / H) \).
3. SIMULATION

3-D electromagnetic simulation software HFSS are used for simulation work. In simulation, the particle is put inside an air-cell as shown in Fig. 3. The side walls of air-cell are assumed perfect magnetic conductors, whereas the upper and bottom walls are perfect electric conductors. The dimension of the air-cell is 2.1 mm in each side. The electric and magnetic field of the incident electromagnetic plane wave is polarized in the direction of the z-axis and x-axis respectively. When incident plane wave falls on the unit cell, a part of the wave power is reflected back and the rest are transmitted. Numerically calculated scattering matrix parameter, \( S_{11} \) and \( S_{21} \) are used to calculate characteristic impedance \( Z_c \) and refractive index \( n \) from the following equations [5]:

\[
Z_c = \sqrt{\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2}} 
\]

(16)

\[
n = \frac{1}{kl} \cos^{-1} \left[ \frac{1}{2S_{21}} (1 - S_{11}^2 + S_{21}^2) \right] 
\]

(17)

\[
\varepsilon_r = n / Z_c 
\]

(18)

\[
\mu_r = n Z_c 
\]

(19)

de here \( l \) is the cell length and \( k \) is wave vector. Fig. 4 shows the calculated and simulated values of \( \varepsilon_r^{\text{eff}} \) with respect to filling factor \( f \) for \( a = 0.976 \), \( \varepsilon_r = 10 \) and exhibits a very good agreement between those. In fact, total cell capacitance \( C_{\text{cell}} \) increases when filling factor is increased and for this reason \( \varepsilon_r^{\text{eff}} \) increases as predicted from Eq. (10). This equation of effective permittivity of artificial material in terms of filling factor \( f \) and ratio of particle-height to cell-height \( a \) in the direction of electric field is very meaningful from macroscopic point of view to understand the following cases: (1) Justification of special condition, (2) sensitivity of \( \varepsilon_r^{\text{eff}} \) to \( \varepsilon_r \), (3) anisotropic behavior and (4) dependence of \( \varepsilon_r^{\text{eff}} \) on particle shape.

Figure 3: Dielectric particle in an air-cell, polarization of input fields and reflection and transmission of wave.

Figure 4: Dependence of relative effective permittivity of artificial material on filling factor for \( a = 0.976 \).

Figure 5: Sensitivity of effective permittivity of material to particle permittivity.

Figure 6: Variation effective permittivity with particle shape with same values of filling factor \( f \), \( a \) and particle’s relative permittivity (= 10).
Table 1: Anisotropy in artificial material.

<table>
<thead>
<tr>
<th>Electric field orientation with respect to particle with $a = 50$</th>
<th>$a = b/H$</th>
<th>$f$</th>
<th>$\varepsilon_r^{\text{eff}}$ (experimental)</th>
<th>$\varepsilon_r^{\text{eff}}$ (calculation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.1</td>
<td>5.97</td>
<td>5.90</td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>0.1</td>
<td>1.13</td>
<td>1.11</td>
<td></td>
</tr>
</tbody>
</table>

(1) Justification of special condition: When there is no particle in the cell, then $f = 0$, $\varepsilon_r^{\text{eff}} = 1$ and when unit cell is filled with material of particle, then $f = 1$, and $\varepsilon_r^{\text{eff}} = \varepsilon_r$. Thus proves the validity of the derived Eq. (15) for effective permittivity.

(2) Sensitivity of $\varepsilon_r^{\text{eff}}$ to $\varepsilon_r$: Effective permittivity Equation (15) predicts that for small value of $a$, relative effective permittivity $\varepsilon_r^{\text{eff}}$ is insensitive to $\varepsilon_r$. Fig. 5 demonstrates both the simulation and calculated values of $\varepsilon_r^{\text{eff}}$ to show this insensitiveness. Again, when $a = 1$, Eq. (15) reduces to $\varepsilon_r^{\text{eff}} = 1 + f(\varepsilon_r - 1)$ and predicts that $\varepsilon_r^{\text{eff}}$ is very much sensitive to $\varepsilon_r$. Fig. 5 shows this sensitiveness by simulation and calculation both.

(3) Anisotropic behavior: The term $a$, which is the ratio of particle-height to cell-height in the direction of electric field, in Eq. (15) explains the anisotropy behavior of the artificial material. When the orientation of electric field is changed with respect to the particle, $a$ is changed but filling factor $f$ remains same in Eq. (15) and thus give rise to different values of $\varepsilon_r^{\text{eff}}$ for different direction of electric field. Anisotropy is demonstrated by the configuration of electric field excitation with respect to the particle as shown in the Table 1.

(4) Dependence of $\varepsilon_r^{\text{eff}}$ to particle shape: For given values of $a$ and $f$, $\varepsilon_r^{\text{eff}}$ is not supposed to change according to Eq. (15). But simulation result shows that shape of particle affects slightly. Fig. 6 displays various shapes of particles with given values of $a (= 0.975)$ and $f (= 0.50)$ and indicates the simulated $\varepsilon_r^{\text{eff}}$ values which are slightly different from each other. This may be attributed to the fact that although $f$ and $a$ remain constant but due to change in particle shape, the total cell capacitance $C_{\text{cell}}$ might be changed. Investigation is underway to find the cause of this.

4. CONCLUSION

A generalized equation for effective permittivity of artificial material composed of dielectric bar has been developed from lumped element circuit point of view. This generalized theory explains the dependence of effective permittivity on filling factor $f$ and $a$, the ratio of particle-height to the cell-height in the direction of electric field. Macroscopically, it has been shown that total cell capacitance plays the vital role to determine the effective permittivity of dielectric-filled artificial material. The derived permittivity equation successfully estimates the effective permittivity of two special conditions (no particle in the cell and cell is filled with particle) and thus validates the proposed model. This study shows that effective permittivity is very sensitive to filling factor when $a = 1$ and thus macroscopically explains the anisotropic behavior of this artificial material.

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REFERENCES