Fast Integral Equation Solution Techniques for Planar-3D Structures in Multilayered Media

Thomas Vaupel
Fraunhofer Institute for High Frequency Physics and Radar Techniques FHR
Wachtberg, Germany

Abstract—In this contribution, a hybrid space/spectral domain integral equation approach is presented for the characterization of quasi-3D structures in multilayered media. The vertical conductors for the quasi-3D components like vias, through holes, airbridges, finite dielectric regions etc. can cross an arbitrary number of dielectric layers with arbitrary vertical discretization. The electrodynamic behavior of the vertical currents is incorporated by extended spectral domain Green’s functions derived by analytical integrations over the vertical direction and by a summation over the Cartesian wavenumbers which must be computed only once and stored in a database. For short lateral distances, the couplings are computed with a subtraction technique based on asymptotic representations of the extended Green’s functions, whereas for larger distances and for group coupling computations in context with fast matrix vector product evaluations, adaptive integration path deformations with extended Laguerre quadrature methods are applied. For the solution of the linear systems of equations, a GMRES solver was implemented together with a diakoptic preconditioner based on the group decomposition of larger structures, combined with a reversed Cuthill McKee reordering applied to the submatrix pattern of the system matrix. With all these techniques, the analysis of complex quasi-3D structures can be carried out with the same performance than pure planar microstrip and/or slotline structures.

1. INTRODUCTION

For the analysis and design process of (M)MIC structures and printed antenna systems, electromagnetic simulators based on the Method of Moments (MoM) combined with the Green’s function of the multilayered environment are widely used due to their high accuracy and good modeling capabilities which have been accomplished over the last years. For the incorporation of quasi 3D components within microstrip/stripline and/or slotline/aperture components, spatial domain techniques with mixed potential formulations and the discrete complex image method are used for solving such structures [1–3] showing typically a very good numerical performance for planar metalizations. However, for quasi-3D elements the numerical effort is often significantly increasing using these techniques since the performance may strongly depend on the required number of complex images and problems occur with large source to observation point distances.

In our approach volume currents with vertical direction are employed for the modeling of vertical interconnects, via- or through holes, coaxial feedings as well as finite dielectric regions, where arbitrary vertical discretizations in combination with arbitrary crossings of layer interfaces are possible. For the interactions of the vertical currents under each other and with the planar components, extended Green’s functions are derived by analytical space domain integrations of the pertinent Green’s functions over the vertical direction. After sampling the Cartesian wavenumber plane, all Green’s functions are computed and stored at the selected sampling points in a database. In case of crossings of vertical basis functions over several dielectric layers, a further summation over the wavenumber plane is employed. With this database technique, the characterization of complex Planar-3D structures with large vertical extensions (e.g., LTCC structures) or electromagnetic bandgap (EBG) structures with a large number of vias is possible using enhanced spectral domain techniques [5] in the same way and with the same high performance than pure planar microstrip and/or slotline structures. For the computation of couplings with small lateral distances of the basis functions, an asymptotic subtraction technique for drastical convergence acceleration of the reaction integrals is used [4]. For larger distances, an adapted integration path deformation is used leading as well to a fast convergence of the integrals. For large structures, a group decomposition is made combined with diagonalized translation operators on the Cartesian wavenumber plane [6].
2. FORMULATION

For the self and mutual coupling of vertical currents, the pertinent Green’s function component can be advantageously formulated by

\[ G_{jzz}^{E,TM,z\leq z',i\leq j} = -\frac{1}{2\omega \varepsilon_i} \frac{k_x^2 + k_y^2}{k_z} e^{-jk_z z'} + \Gamma_i^{TM} e^{-jk_z (2d_{i-1} - z')} \]

\[ \times \left[ e^{jk_z z} + \Gamma_j^{TM} e^{jk_z (2d_{i-1} - z')} \right] - \frac{1}{j\omega \varepsilon_i} \delta(z - z'), \tag{1} \]

here for observation points below the source points. For the transmission factor from an upper to a lower dielectric layer, the compact formulation not depending on \( z \) and \( z' \) reads:

\[ T_{i,j}^{TM,mod}(k_p) = \frac{1}{2\pi j} e^{-j \sum_{n=i}^{j-1} (k_{n+1} - k_n) d_n} \prod_{n=i}^{j-1} \frac{\varepsilon_n}{\varepsilon_{n+1}} + \Gamma_n^{TM} \varepsilon(-\gamma_n^{TM}) \tag{2} \]

Figure 1, left shows a vertical electric current within two dielectric layers, (here schematically sketched as a line current). The actual basis functions have a rectangular cross-section in the \( x-y \) plane, leading to the volume current description

\[ j_{vm}(x, y, z) = \text{rect} \left( \frac{x-x_m}{w_{xm}/2} \right) \text{rect} \left( \frac{y-y_m}{w_{ym}/2} \right) \text{rect} \left( \frac{z-z_v}{\Delta z/2} \right) \varepsilon_z \tag{3} \]

with \( w_{xm}, w_{ym} \) the lateral dimensions and \( \Delta z \) the height of the basis function. For the later computation of the self-coupling of this current, the following integral (assuming a constant current distribution over the length) is formulated:

\[ I(z, d_2 \leq z \leq d_1) = \sum_{z_{v1}}^{z_{v2}} G_{jzz}^{E,TM,z\geq z',i=2} (z, z') dz' + \int_{z}^{d_1} G_{jzz}^{E,TM,z\leq z',i=j=2} (z, z') dz' \]

\[ + \int_{d_1}^{z_{v2}} G_{jzz}^{E,TM,z\leq z',i=1,j=2} (z, z') dz' \tag{4} \]

This integral is relevant for the computation of the \( E_z \)-component of the electric field within the lower layer with permittivity \( \varepsilon_2 \). For the first and second integral in Eq. (4), the \( \delta \)-function in Eq. (1) is relevant which results in the contribution \(-1/j\omega \varepsilon_2\). Furthermore a \( z \)-independent term \( j(k_x^2 + k_y^2)/(k_z^2 \omega \varepsilon_2) \) occurs, which can be combined with the \(-1/j\omega \varepsilon_2\) term to \( jk_z^2/(\omega \varepsilon_2 k_z^2) \) decaying with \( \sim 1/(k_z^2 + k_y^2 + k_z^2) \) using the condition \( k_x^2 + k_y^2 + k_z^2 = k_0^2 \).

After the integration over the source coordinates \( z' \) in Eq. (4), the integration over the observation coordinates \( z \) must be performed. For this goal, a vertical current basis function is decomposed into several segments, if it crosses through several layers. Accordingly a vertical current segment is defined to be restricted to one layer of the multilayered environment. Thus the current in Fig. 1, left is decomposed into two segments whereas the vertical currents in the general coupling situation

![Diagram showing vertical current crossing a dielectric interface and general coupling situation of vertical currents in a multilayer environment.](image-url)
sketched in Fig. 1, right are decomposed into an arbitrary number of segments depending on the number of crossed layers. The extended Green’s function which is necessary for the self or mutual coupling computation between segments on the same level is denoted with \text{Intvii}(k_x, k_y, z_{i1}, z_{i2}). It is derived e.g., for the self coupling of the lower segment in Fig. 1, left by the integration \( \int I(z, d_2 \leq z \leq d_1)dz \). The analytical result is very extensive thus only the mathematical structure is given here by

\[
\text{Intvii}(k_x, k_y, z_{i1}, z_{i2}) = \frac{jk_i^2 k_x^2}{k_z^2 \omega \varepsilon_i} (z_{i2} - z_{i1}) - \frac{k_x^2 + k_y^2}{2\omega \varepsilon_i k_z^3 (1 - \Gamma_i^T M \Gamma_i^T M e^{-2jk_z(d_{i-1} - d_i)})} \cdot (2 + \sum_{n=1}^{9} V_n e^{-j(a_n k_x + b_n k_y)})
\]

(5)

illustrating the extended Green’s function for the mutual coupling of vertical current segments (or self coupling of one segment) extending over the same vertical region from \( z_{i1} \) to \( z_{i2} \) within a layer of permittivity \( \varepsilon_i \). The coefficients \( a_n \) and \( b_n \) depend on the layer interface coordinates as well as \( z_{i1} \) and \( z_{i2} \). If the vertical current basis functions extent over several layers, the extended overall Green’s function is derived as for example the couplings of vertical currents basis functions (self or mutual couplings) between \( z_{v1} \) and \( z_{v2} \) (source region) as well as \( z_{v1} \) and \( z_{v2} \) (test or observation region) by the summation:

\[
IG_{VV}(k_x, k_y, z_{v1}, z_{v2}, z_{v1}, z_{v2}) = \sum_{i=1}^{\text{isegiq}} \sum_{j=1}^{\text{isegiq}} \text{Intvul/ul}(z_{i1}, z_{i2}, z_{j1}, z_{j2}, k_x, k_y)
\]

\[
+ \sum_{i=1}^{\text{isegiq}} \text{Intvii}(z_{i1}, z_{i2}, k_x, k_y)
\]

(6)

Here \( \text{isegiq} \) denotes the number of segments of the source region and \( \text{isegiq} \) the number of segments of the test region, the expression \( \text{Intvul/ul} \) denotes the use of the function \( \text{Intvul} \) in case of the coupling of an upper to a lower segment and \( \text{Intvul} \) in case of a coupling from a lower to an upper segment (without vertical overlapping). The mathematical structure of \( \text{Intvul} \) and \( \text{Intvul} \) is similar as Eq. (5).

In case of mutual couplings of basis functions without overlapping in vertical direction, only the first summation with the \( \text{Intvul/ul} \)-functions in Eq. (6) is relevant. Further extended Green’s functions are computed for the interactions of vertical currents with planar basis function of electric (microstrip structures) and magnetic surface currents (slot structures) with similar but less extensive expressions than Eqs. (5) and (6). The effort for the computation of all these extended Green’s functions is always negligible.

After evaluating all necessary components of the extended Green’s function, the overall couplings are computed by

\[
V_{nm} = \int \int [IG(k_x, k_y, z_{v1}, \ldots, z_{v2}) - IG_{\text{Asy}}(k_x, k_y, z_{v1}, \ldots, z_{v2})]
\]

\[
\cdot \tilde{F}_{m}(k_x, k_y, z_{v1}) \cdot \tilde{F}_{n}(k_x, k_y, z_{j1}) dk_x dk_y + V_{nn}^{\text{Asy}}
\]

(7)

in the case of couplings with overlapping or small distance between the basis functions (< \( \lambda/4 \)). Here \( IG \) is the tensor containing the extended Green’s functions like Eq. (6) and the \( \tilde{F}_{m,n}(k_x, k_y, z_{nm}) \) are the spectral domain representations of the basis functions.

The \( IG_{\text{Asy}} \) are asymptotic representations of the \( IG \)-functions for large wavenumbers \( \sqrt{k_x^2 + k_y^2} = k_p \to \infty \). The subtraction of the \( IG_{\text{Asy}} \) in Eq. (7) typically leads to a strong exponential decay of the integrands with a corresponding fast convergence of the numerical integration.

The asymptotic representations for couplings with only planar basis functions can be found in [4].
The derivation of the corresponding asymptotic representations becomes significantly more complicated, if basis functions for vertical currents according to Eq. (3) are involved. As an example we consider the derivation for couplings between vertical basis functions. First we consider the case of a mutual coupling between the lower basis function and the upper basis functions in Fig. 1, right having the common plane \( z_{v2} \). A careful analysis of the limit values of \( \text{Intvu}l_{u,v}^{\text{Asy}} \) and \( \text{Intvu}l_{v,u}^{\text{Asy}} \) for large \( k_\rho \) using the asymptotic transmission factors \( T_{u,l}^{\text{Asy}} \) and \( T_{l,u}^{\text{Asy}} \) finally leads to:

\[
\text{Intvu}l_{u,v}^{\text{Asy}} = -\frac{j}{2\omega k_\rho \varepsilon_u} \cdot \frac{2\varepsilon_u}{\varepsilon_l + \varepsilon_u} \cdot \hat{\Gamma}_u^{\text{Asy}}
\]

\[
\text{Intvu}l_{v,u}^{\text{Asy}} = -\frac{j}{2\omega k_\rho \varepsilon_l} \cdot \frac{2\varepsilon_l}{\varepsilon_u + \varepsilon_l} \rightarrow \text{Intvu}l_{u,v}^{\text{Asy}} = \text{Intvu}l_{v,u}^{\text{Asy}} = -\frac{j}{\omega k_\rho (\varepsilon_u + \varepsilon_l)}.
\]

with \( \varepsilon_u, \varepsilon_l \) see Fig. 1, left, i.e., the asymptotic representations are the same for both coupling directions as it is expected in context with the reciprocity theorem. For self- or mutual couplings between vertical current segments on the same level within a layer of permittivity \( \varepsilon_i \) it can be derived:

\[
IG_{VV}^{\text{Asy}} = \frac{j}{2\omega \varepsilon_i k_\rho} (2 - \Gamma_i^{\text{Asy}} - \hat{\Gamma}_i^{\text{Asy}})
\]

where the asymptotic reflection factors

\[
\Gamma_i^{TM,\text{Asy}} = \frac{\varepsilon_{i+1} - \varepsilon_i}{\varepsilon_{i+1} + \varepsilon_i}, \quad \hat{\Gamma}_i^{TM,\text{Asy}} = \frac{\varepsilon_{i+1} - \varepsilon_i}{\varepsilon_{i-1} + \varepsilon_i}
\]

must be only considered, if the lower and/or upper ending of the segment falls together with the corresponding layer interface of the assigned reflection factor. Otherwise the asymptotic reflection factor is set to zero. If we compute the complete extended asymptotic Green’s function for vertical basis functions extending from \( z_{v1} \) to \( z_{v2} \) as in Fig. 1, left, it can be written according to Eq. (6) and Eqs. (8)–(10):

\[
IG_{VV}^{\text{Asy}}(z_{v1}, z_{v2}) = \text{Intvi}^{\text{Asy}}(d_2, d_1) + \text{Intvi}^{\text{Asy}}(d_1, d_0) + \text{Intvu}l^{\text{Asy}}(d_1, d_0, d_2, d_1)
\]

\[
+ \text{Intvu}l^{\text{Asy}}(d_2, d_1, d_0) = \frac{j}{2\omega \varepsilon_2 k_\rho} (2 - \hat{\Gamma}_2^{\text{Asy}} - \Gamma_2^{\text{Asy}}) + \frac{j}{2\omega \varepsilon_1 k_\rho} (2 - \hat{\Gamma}_1^{\text{Asy}} - \Gamma_1^{\text{Asy}})
\]

\[
- \frac{j}{2\omega \varepsilon_1 k_\rho \varepsilon_2 + \varepsilon_1} - \frac{j}{2\omega \varepsilon_2 k_\rho \varepsilon_2 + \varepsilon_2} = \frac{j}{2\omega k_\rho} \left( \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - \frac{\hat{\Gamma}_1^{\text{Asy}}}{\varepsilon_1} - \frac{\hat{\Gamma}_2^{\text{Asy}}}{\varepsilon_2} \right)
\]

due to the fact that \( \hat{\Gamma}_2^{\text{Asy}} = -\hat{\Gamma}_1^{\text{Asy}} = (\varepsilon_1 - \varepsilon_2)/(\varepsilon_1 + \varepsilon_2) \). It can be quickly seen that Eq. (11) converts to Eq. (9) for the case \( \varepsilon_2 = \varepsilon_1 \).

If this analysis is done for a vertical current basis function crossing an arbitrary number of layers, it can be found that Eq. (11) is still valid for this general case if \( \varepsilon_1 \) is replaced by the permittivity at the upper ending of the vertical current and \( \varepsilon_2 \) with the permittivity at the lower ending because the contributions of all the other layers in between are compensating each other.

The \( V_{nm}^{\text{Asy}} \) in Eq. (7) are integral representations formulated in polar coordinates containing the \( \tilde{IG}_{nm}^{\text{Asy}} \) and the \( \tilde{F}_{m,n}(k_x, k_y, z_m) \). These integrals can be completely evaluated analytically [4], thus, Eq. (7) is evaluated for all applied basis functions with the same high efficiency and accuracy.

In the case of couplings with larger separations of the basis functions and for group coupling computations in context with fast iterative solvers, partly adaptive integration path deformations are used for the \( k_x \) or the \( k_y \) wavenumber, depending on the lateral separation of the basis functions or the groups. If we e.g., look at the coupling computation between the observation and source groups \( g_l \) and \( g_r \) during a fast matrix vector product evaluation \( \vec{b} = Z_{far} \cdot \vec{I} \) we get (see also [6]):

\[
b_{n,l} \approx \sum_{k_{x1}} \sum_{k_{y1}} w_{k_{x1}k_{y1}} F_{ln}^{*}(k_{x1}, k_{y1}) \cdot \tilde{T}_{ll}(k_{x1}, k_{y1}, z_{v1}, \ldots, z_{v2}) \cdot \tilde{J}_{r}(k_{x1}, k_{y1})
\]

with \( \tilde{J}_{r}(k_{x1}, k_{y1}) = \sum_{m \in g_r} I_{m} \tilde{F}_{lm}(k_{x1}, k_{y1}) \),

\[
b_{n,l} \approx \sum_{k_{x1}} \sum_{k_{y1}} w_{k_{x1}k_{y1}} F_{ln}^{*}(k_{x1}, k_{y1}) \cdot \tilde{T}_{ll}(k_{x1}, k_{y1}, z_{v1}, \ldots, z_{v2}) \cdot \tilde{J}_{r}(k_{x1}, k_{y1}),
\]
introducing the translation operators \( \mathcal{T}_{ll'}(k_{xl}, k_{yl}, z_{vl1}, \ldots, z_{vl2}) = \mathcal{I}G(k_x, k_y, z_{v11}, \ldots, z_{v22})e^{jk_y(y_{l'} - y_l)} \) containing the extended Green’s functions and \( (x_l, y_l), (x_{l'}, y_{l'}) \) the center points of the groups \( g_l \) and \( g_{l'} \), respectively and \( w_{x_l}, w_{y_l} \) suited integration weights. The spectral currents \( \mathcal{J}_{ll'}(k_{x_l}, k_{y_l}) \) are created during the aggregation step given in Eq. (12) as well. In the case, that \( |x_l - x_{l'}| > |y_l - y_{l'}| \) is valid, the last integration path for \( k_x \) is chosen by \( k_x(t) = \pm k_x - jt, t \geq 0 \), whereas \( k_y \) remains real, what leads to an exponential decay of the term \( e^{jk_y(x_l - x_{l'})} \) within the translation operators. In the case \( |x_l - x_{l'}| < |y_l - y_{l'}| \) the same is done vice versa for the wavenumbers with an exponential decay for \( e^{jk_y(y_l - y_{l'})} \), always guaranteeing a fast convergence of the numerical integration in Eq. (12). However, if the groups contain different levels of volume and/or surface currents, the integration in Eq. (12) must be performed in several steps using the pertinent (extended) Green’s functions between the pertinent current levels.

3. APPLICATION

As an example of a larger structure with many vias, two patch antennas with an electromagnetic bandgap structure (EBG) between the antennas was investigated, already outlined in [7]. In Fig. 2, left, the structure is given with the two antennas and the EBG structure consisting of 56 quadratical patches (size 3 mm, gap width 0.5 mm) with vias to the ground all mounted on a grounded dielectric slab with \( \varepsilon_r = 10.2 \) and thickness 1.92 mm. The used meshing also shown leads to 5008 unknowns. The structure was decomposed into 30 groups. Using the standard MoM, the time for the matrix fill was about 60 sec. (AMD 3 GHz PC) and the linear system solution time amounts to 74 sec. per frequency step. However, the time for a matrix redundancy analysis amounts to 140 sec which must be done only once for all frequency steps. The results show nearly the same matching behavior with an antenna resonance at 5.8 GHz. With the EBG, the mutual coupling is reduced up to ca. 6 dB around the resonance. The effect of the EBG is smaller than outlined in [7]. However, some geometrical parameters like the feeding points of the antennas and exact substrate thickness are not exactly known. Maybe due to this, the antenna length had to be increased from 6.8 to 7.4 mm to get a resonance at 5.8 GHz. Further examples and the detailed behavior of the fast solver are given in the oral presentation.

![Figure 2: Investigation of patch antennas with an EBG-mushroom structure to reduce the mutual coupling. Dimensions in mm.](image)

4. CONCLUSION

In this study, a MoM approach for the characterization of quasi-3D structures in a multilayered environment has been presented which is based on extended spectral domain Green’s functions created by analytical integrations in context with the vertical volume basis functions used for the modeling of the vertical interconnects or finite dielectric regions of the quasi-3D components. Although an arbitrary discretization can be applied for the vertical currents which can furthermore cross an arbitrary number of dielectric layers, the characterization of such quasi-3D structures can be accomplished with the same performance as pure planar structures. At the moment, the new approach is tested with regard to fast matrix-vector multiplication strategies in a similar way as it is done in fast multipole methods in free space.
REFERENCES