Leakage Inductance Determination for Transformers with Interleaving of Windings

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Abstract—A diversity of formulas is given in literature for leakage inductance calculation of transformers with interleaving of windings. In the paper, the meaning of included terms and details of the application for concentric and pie windings transformers shall be discussed. Considering the limitation of common formulas to magnetically symmetric transformers an alternative approach shall be demonstrated, which enables leakage inductance prediction also in the case of transformers with magnetically asymmetric arrangements of windings.

1. INTRODUCTION

Several aspects of the operation of power-electronic circuits with transformers are significantly influenced by the leakage inductance of the respective transformer. For instance, it concerns power transfer capability of the circuit and power semiconductor stress parameters. The tendency to higher switching frequencies of power-electronic apparatuses results in an increasing relevance of low transformer leakage inductance values. Mostly, the so-called interleaving of windings (i.e., the fragmentation of the primary and the secondary windings into a certain number of sub-windings and alternate interleaving of primary and secondary sub-windings) is used to meet this requirement.

Using an appropriate approximation method it is possible to determine the leakage inductance of a transformer already in the design phase on the basis of predicted geometry parameters. Thus, circuit simulation can be used to shorten the development process of power-electronic apparatuses. Formulas which are given in literature generally rely on an approximation method for leakage inductance determination which has been established by Rogowski [1]. It is based on the consideration of the energy of the leakage magnetic field and applies to a transformer design with windings arranged on the same leg. The basic formula of this method is

\[ L_L = \mu_0 \cdot N^2 \cdot l_m \cdot \lambda \cdot k_\sigma \]  

(1)

with \( L_L \) — leakage inductance, \( \mu_0 \) — absolute permeability, \( N \) — number of turns of the winding to which the leakage inductance refers, \( l_m \) — mean length per turn for whole arrangement of windings, \( \lambda \) — relative leakage conductance (depending on geometry parameters and on the degree of interleaving of windings), \( k_\sigma \) — Rogowski factor (also depending on geometry parameters).

The formulas which can be found in contemporary technical literature mostly represent simplified variants of the mentioned basic formula. Usually the Rogowski factor (whose value is nearly 1 in most arrangements) is omitted and the formula describing the relative leakage conductance is included in the main formula. Often, constant factors like absolute permeability and numerical values are merged into one coefficient, so that the physical background of the formula is not recognizable anymore, e.g., in [2]. In some cases different formulas are given for a transformer design without interleaving of windings and for transformer arrangements with interleaving of windings, e.g., in [3]. The degree of interleaving is considered in literature by the number of the couples of sub-windings [4, 5] or by the number of interfaces (insulating interspaces) between the single sub-windings of an arrangement of windings [6, 7]. Furthermore, the aim to present a universal formula which is valid for transformer versions with concentric and also with pie windings arrangements often causes some confusion. It also results from the diversity of terms which are used for the dimensions of windings (e.g., breadth, width, traverse, build, height, length) comparing the formulas and their descriptions which are given in different books and papers.

2. LEAKAGE INDUCTANCE CALCULATION FOR MAGNETICALLY SYMMETRIC ARRANGEMENTS

To enable a correct application of a formula for leakage inductance calculation both for transformers with concentric and transformers with pie windings, the attribution of the chosen symbols for
The respective dimensions of the arrangement of windings has to be made quite clear. Therefore, based on variants which are valid for transformer versions without and with interleaving of windings, the following representation of the formula is proposed:

$$L_L = \mu_0 \cdot N^2 \cdot \frac{l_m}{n_{lf}^2} \cdot \frac{X_{par-lf}}{X_{par-lf}} \cdot \left( \frac{\Sigma X_{perp-lf}}{3} + \Sigma \delta \right)$$

(2)

with $\Sigma X_{perp-lf}$ — sum of the dimensions of all sub-windings which are orientated perpendicular to the leakage flux, $X_{par-lf}$ — dimension of the sub-windings which is orientated parallel to the leakage flux, $\Sigma \delta$ — sum of the thicknesses of all insulating interspaces between the sub-windings, $n_{lf}$ — number of insulating interspaces between the sub-windings (no consideration of Rogowski factor).

Thus, in this formula the respective dimensions of the sub-windings are identified based on a comparison between their orientation and the orientation of leakage flux within the core window. The meaning of the geometry parameter symbols and the path of leakage flux $\Phi_l$ within the core window are illustrated in principle in Fig. 1 assuming the leakage flux to be concentrated in the insulating interspaces between the windings or sub-windings.

![Figure 1](image-url)

The proposed representation and also the variants of the formula which are given in literature enable a relatively uncomplicated approximate calculation of the leakage inductance of a transformer even in the case of interleaving of windings. However, it has to be stated that the scope is limited to magnetically symmetric arrangements of windings. These transformer versions are characterized by an odd number of primary sub-windings and an even number of secondary sub-windings or vice versa. Furthermore, the number of turns of outer sub-windings is half of the number of turns of the inner sub-windings which belong to the same group of sub-windings (inner primary sub-windings if the outer sub-windings belong to the primary winding; inner secondary sub-windings if the outer sub-windings belong to the secondary winding). In connection with this, it can be assumed that the outer sub-windings show a dimension perpendicular to the leakage flux which is half of the same dimension of an inner sub-winding which belongs to the same group of sub-windings.

### 3. UNIVERSAL LEAKAGE INDUCTANCE CALCULATION

In the majority of applications magnetically symmetric transformer versions are used because they show the lowest leakage inductance values considering a certain degree of interleaving of windings. However, also magnetically asymmetric designs of windings are used in praxis. For instance in transformers for capacitor discharge welding machines a magnetically asymmetric design of windings is often applied to enable a variation of the turns ratio by means of alteration between series and parallel connection of the primary sub-windings. Also transformer versions, whose primary sub-windings have identical numbers of turns independent of the location of the respective sub-windings in inner or outer position, represent a typical example of magnetically asymmetric transformers.

In formula (2) and its variants which are given in literature it is not possible to consider the real design and arrangement of sub-windings of a magnetically asymmetric transformer because only...
the sum value of the relevant dimension of the single sub-windings and the sum of thicknesses of all insulating interspaces between the sub-windings can be taken into account. Therefore, in this case the mentioned formulas will deliver a result less than the real leakage inductance value which can only represent a rough approximation.

Prediction of leakage inductance with a higher degree of accuracy even in the case of magnetically asymmetric transformers is possible based on the calculation of the individual leakage inductance values of all existing couples of sub-windings of the transformer arrangement which have finally to be merged together to the total leakage inductance of the transformer. In [8], Petrov described an abstract method to establish the required combination formulas.

This approach shall be illustrated by means of leakage inductance determination for different arrangements of windings of an experimental transformer. The basic elements of the windings of the transformer are single pie-shaped coils (number of turns: 11). Furthermore the transformer (displayed in Fig. 2 (left)) is characterized by the following features:

Core: PM 114/93 [9], material N27
Primary winding:
Consisting of 3 sub-windings ($W_{11}$, $W_{12}$, $W_{13}$) realized using 8 coils
Total number of turns: $N_1 = 88$
Different arrangements of sub-windings and distribution of turns considered
Series connection of the sub-windings and the coils forming the sub-windings
Secondary winding:
Consisting of 2 sub-windings ($W_{21}$, $W_{22}$), realized using 8 coils
Each sub-winding consisting of 4 coils connected in parallel
Sub-windings connected in series
Total number of turns: $N_2 = 22$ ($W_{21}$: $N_{21} = 11$; $W_{22}$: $N_{22} = 11$)
Thickness of insulating interspaces between the sub-windings: 0.1 mm

In Fig. 2 (right), the location of the sub-windings is illustrated. Due to the height of the outer primary sub-windings ($W_{11}$ and $W_{13}$) which is half of the height of the inner primary sub-winding ($W_{12}$) the displayed version represents a magnetically symmetric arrangement. This classification is founded by the curve of the magnetomotive force (mmf) between the center leg and the outer leg of the core which shows the same maximum values in positive and negative range.

According to the existence of 3 primary sub-windings and 2 secondary sub-windings in the considered transformer we call it a 3-2 version. Applying the method described in [8] we derived the following formula which enables the calculation of the total leakage inductance of a transformer with a 3-2 arrangement of its windings whose sub-windings are connected in series at the primary

![Figure 2: Left: Experimental transformer. Right: Illustration of the location of sub-windings within the core window (only right hemisphere of the transformer displayed) and curve of the magnetomotive force (mmf) across the core window.](image-url)
side and at the secondary side as well:

\[
L_{L_{\text{total}}} = \frac{N_{\text{ref}}^2}{N_{\text{ref}}^2} \left( \frac{N_{11}N_{21}}{N_{12}N_{22}} \cdot L_{L11/21} + \frac{N_{12}N_{21}}{N_{11}N_{22}} \cdot L_{L12/21} + \frac{N_{13}N_{21}}{N_{12}N_{22}} \cdot L_{L13/21} + \frac{N_{12}N_{23}}{N_{12}N_{22}} \cdot L_{L12/22} + \frac{N_{13}N_{23}}{N_{12}N_{22}} \cdot L_{L13/22} + \frac{N_{11}N_{23}}{N_{11}N_{22}} \cdot L_{L11/12} - \frac{N_{11}N_{23}}{N_{11}N_{22}} \cdot L_{L11/13} - \frac{N_{12}N_{23}}{N_{12}N_{22}} \cdot L_{L12/13} - \frac{N_{13}N_{23}}{N_{12}N_{22}} \cdot L_{L13/22} \right)
\]

(3)

\(N_{\text{ref}}\) is the number of turns to which the leakage inductance values of the couples of sub-windings refer (advantageously \(N_{\text{ref}} = 1\) should be chosen). The single leakage inductance values of the couples of sub-windings have been calculated using the basic formula of the method of Rogowski (1) omitting the Rogowski factor. Due to the number of primary turns in the coefficient preceding the term in parentheses, the resulting total leakage inductance refers to the primary side of the transformer. As it can be seen comparing the formulas (2) and (3), in contrast to leakage inductance calculation for magnetically symmetric transformers it is a more complex procedure in the case of magnetically asymmetric versions. However, today the calculation of the single leakage inductance values of the existing couples of sub-windings of the transformer arrangement and merging them together using the respective combination formula can be realized conveniently implementing the algorithms into a PC [10]. The values of total leakage inductance have been determined using an RLC meter (inductance measurement at the primary side during short-circuiting the output terminals of the secondary side, measuring frequency 50 Hz). Due to the given transformer design showing a closed core (no air gap intended) and hence a very high magnetizing inductance, the measured short-circuit inductance values will nearly coincide with the respective leakage inductance values. Therefore, the measured values are considered to be the sought leakage inductance values. The results are presented in Table 1.

<table>
<thead>
<tr>
<th>Arrangement of the primary sub-windings (W_{11}, W_{12}, W_{13} (N_{11} - N_{12} - N_{13}))</th>
<th>Calculation</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>22–44–22</td>
<td>67.88 µH</td>
<td>74.2 µH</td>
</tr>
<tr>
<td>33–22–33</td>
<td>117.69 µH</td>
<td>116.0 µH</td>
</tr>
<tr>
<td>11–66–11</td>
<td>117.69 µH</td>
<td>121.4 µH</td>
</tr>
<tr>
<td>22–22–44</td>
<td>167.5 µH</td>
<td>159.4 µH</td>
</tr>
</tbody>
</table>

In the case of the magnetically symmetric arrangement of the primary sub-windings with the numbers of turns 22–44–22 the calculated result is identical to the value which is delivered by formula (2). The leakage inductance values calculated by means of formula (3) show an average deviation from the measured values of about 4.5%. In the case of thicker insulating interspaces between the sub-windings (0.4 mm) the deviation is even slightly lower.

4. CONCLUSIONS

The calculation of leakage inductance of transformers with interleaving of windings has been discussed. The origin of certain terms in approximation formulas for magnetically symmetric transformers and the attribution of these terms to the geometry of concentric and pie windings arrangements has been clarified. An approach which enables leakage inductance prediction also in the case of magnetically asymmetric arrangements of windings has been demonstrated yielding good coincidence with results of measurement in the considered example.

REFERENCES