Electromagnetic Analysis of Propagation and Scattering Fields in Dielectric Elliptic Cylinder on Planar Ground

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Abstract—Radio wave technologies using electromagnetic waves of microwaves and ultra-high frequencies have been rapidly developed for high bit rate wireless communications and RFID application in out-door and in-door spaces. Signal detection evaluation is very important factor in these systems including several objects along propagation paths of urban streets and country suburb, and, in houses and building.

In order to estimate scattering and interference fields in many environments, typical useful models for scattering objects are dielectric elliptic cylinders on the planar ground. Field intensity distributions of electromagnetic waves in these circumstances are studied by Mathieu function expansion with addition theorems. Incident and scattering fields are expanded by Mathieu function series and coefficients of series are derived by electromagnetic field continuity equations of boundary conditions on the elliptic cylinder and planar surface.

Reflection and scattered fields by elliptic cylinders on planar ground are shown by these series coefficients, and also fields in shadow regions between dielectric elliptic cylinders and planar ground. Field intensities in the elliptic cylinder including resonance cases for particular frequencies are evaluated. Reflection and scattered fields are compared with computer simulation results derived by numerical calculation of the FDTD method. Based on these field calculations optimum system design of WiMAX wireless communications and RFID systems in out-door and in-door space regions are investigated. Positions and locations of radiating and receiving antenna for RF stations and RF readers may be decided from field results of this theory.

1. INTRODUCTION

Recently mobile communication is used in many applications such as WiMAX wireless communications and RFID systems. WiMAX wireless communication has been rapidly developed for broadband mobile communication of image and TV transmission. Mobile WiMAX communication system uses microwave carrier of 2.5 GHz frequency band and modulation system is mainly OFDM for transmission of signals. To improve coverage and provide high-data-rate services in a cost-effective manner, Femto cell is proposed for ubiquitous indoor and outdoor communication, using a single access technology such as WiMAX. Particularly, femto access point can improve indoor coverage, where the signal from macro base station may be weak. When the same carrier frequency is used by macro/micro base station and femto access point, the effect of the co-channel interference becomes an important factor for design of excellent wireless communication system.

To evaluate pass loss and received level of the electric field in microcell and femto cell environment, wave reflection, scattering and diffraction due to the presence of obstacles on the ground are studied [1]. Here the scattering model consists of an infinite cylindrical conductor near an infinite planar ground. This scattering problem involving the conducting plane can be exchanged with that of two elliptic cylinders, using the theory of images. Expansion of fields in terms of Mathieu functions is used to analytically describe the scattered field. To apply the boundary conditions, we have used the addition theorem of Mathieu functions. We have reported the plane wave scattering from a girder bridge on the plane analytically by replacing the problem with scattering by two elliptic cylinders using the image theory [2]. Particularly, numerical results showed the variation of field distribution in shadow region when the height of the elliptic cylinder was different.

Also, we have studied the received level of the electric field by presence of forest in WiMAX wireless communications and street walls in RFID systems [3–6]. In this paper, we consider an elliptic cylinder on the planar ground as a simple object and study the electromagnetic scattering analytically using field expansion in terms of Mathieu function. Reflection and scattering fields are compared with the numerical results by FDTD method.
2. ANALYSIS METHOD USING FIELD EXPANSION BY MATHIEU FUNCTION

The elliptic cylinder located at an arbitrary height \( h \) from the perfectly conducting plane is considered. The parameters of the elliptic cylinder are its focal \( c_0 \), the length of major axis \( a \) and the length of minor axis \( b \). The coordinate system is represented by an elliptic cylindrical coordinate system \((\xi_1, \eta_1, z)\) and \((\xi_2, \eta_2, z)\).

For the incident wave and scattered wave, we have, for \( i = 1, 2 \)

\[
E_y = A \sum_{n=0}^{\infty} \left\{ C_n U_n(\xi_i) ce_n(\eta_i) ce_n(\alpha) + S_n We_n(\xi_i) se_n(\eta_i) se_n(\alpha) \right\}
\]

(1)

where \( ce_n \) and \( se_n \) are Mathieu functions of order \( n \) and, \( U_n = Ce_n, Me_n(2) \) and \( W_n = Se_n, Ne_n(2) \) are the modified Mathieu functions of order \( n \). \( \alpha \) is the angle of incidence. For the scattered wave, \( C_n \) and \( S_n \) are the unknown coefficients and are determined by boundary condition on the elliptic cylinder and the plane.

3. FDTD ANALYSIS OF ELECTROMAGNETIC SCATTERING

Microwave scattering and diffraction by shielding objects are very important phenomena for wireless broadband communication such as mobile WiMAX. Computer simulation using FDTD method is useful to evaluate these characteristics numerically. Two-dimensional analysis model for microwave scattering by elliptic cylinder on the ground is shown in Fig. 1. Analysis region is defined as \( \ell_x \times \ell_z \).

In FDTD simulations, the incident wave is assumed to be a traveling wave from base station antenna at a far distance. The incident wave is y-polarized Gaussian beam wave with angular frequency \( \omega = 2\pi f \), beam waist \( z = z_0 \) and beam spot size \( S \). In the simulation model, the electromagnetic fields at point \((i, j)\) and time \( n\Delta t \) are calculated by difference equations. The elliptic cylinder is located at an arbitrary height \( h \) on the planar ground. The parameters of the elliptic cylinder are the length of major axis \( a \) and the length of minor axis \( b \).

Table 1: Numerical parameters for FDTD analysis.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ell_x ): Length of analysis space (x)</td>
<td>13 m (108.3( \lambda ))</td>
</tr>
<tr>
<td>( \ell_z ): Length of analysis space (z)</td>
<td>20 m (166.7( \lambda ))</td>
</tr>
<tr>
<td>( \ell_g ): Width of planar ground</td>
<td>1 m (8.3( \lambda ))</td>
</tr>
<tr>
<td>( \Delta S ): Length of a cell</td>
<td>10^{-2} m (( \lambda /12 ))</td>
</tr>
<tr>
<td>( \Delta t ): Time increment</td>
<td>0.02 ns (( T_0 /20 )), ( T_0 = 1/f )</td>
</tr>
<tr>
<td>( f ): Frequency of incident wave</td>
<td>2.5 GHz</td>
</tr>
<tr>
<td>( \lambda ): Wavelength of incident wave</td>
<td>0.12 m (( c/f ))</td>
</tr>
<tr>
<td>( t_0 ): Peak time of incident pulse</td>
<td>5 ns</td>
</tr>
<tr>
<td>( T ): Pulse width of incident wave</td>
<td>2 ns</td>
</tr>
<tr>
<td>( x_0 ): Center point of the beam (x)</td>
<td>6.0 m</td>
</tr>
<tr>
<td>( S ): Beam spot size at ( z = 0 )</td>
<td>3.0 m (25( \lambda ))</td>
</tr>
<tr>
<td>( a ): Major axis length/( 1/2 ) rectangle length</td>
<td>4.8 m</td>
</tr>
<tr>
<td>( b ): Minor axis length/( 1/2 ) rectangle height</td>
<td>Case 1 Case 2 Case 3</td>
</tr>
<tr>
<td>( h ): Height of elliptic cylinder</td>
<td>2.1 m 2.4 m 2.7 m 0 m</td>
</tr>
<tr>
<td>( \varepsilon_1 ): Dielectric constant of objects</td>
<td>4( \varepsilon_0 ) 4( \varepsilon_0 ) 4( \varepsilon_0 )</td>
</tr>
<tr>
<td>( \varepsilon_2 )/Plane</td>
<td>Case 1-1, 2 Case 2-1, 2 Case 3-1, 2</td>
</tr>
<tr>
<td>( \sigma_1 ): Conductivity of objects</td>
<td>( \infty 10^{-4} S/m \infty 10^{-4} S/m \infty 10^{-4} S/m )</td>
</tr>
<tr>
<td>( \sigma_2 )/Plane</td>
<td>( \infty 10^{-4} S/m \infty 10^{-4} S/m \infty 10^{-4} S/m )</td>
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</table>
The incident wave is generated by current density in FDTD equations,

$$J_y^0(i, 1) = J_0 \exp \left\{ - \left( \frac{i\Delta s - x_0}{S} \right)^2 \right\} \exp \left\{ - \left( \frac{n\Delta t - t_0}{T} \right)^2 \right\} \sin (2\pi f n\Delta t)$$  \hspace{1cm} (2)

where $f$ is the frequency of incident wave, $x_0$ is the center point of incident beam, $S$ is the beam spot size at $z = z_0(j = 1)$, $T$ is the parameter for transmission pulse width.

4. NUMERICAL RESULTS

The results of numerical calculations of the electric fields near the elliptic cylinder on planar ground are shown in this section. We carried out calculations using parameters $f = 2.5$ GHz, $\lambda = 0.12$ m, $E_0 = 1$ V/m, $\Delta s = 0.01$ m, $\Delta t = 0.02$ ns. In the analysis region of Fig. 1, $\ell_x = N_x\Delta s = 1300\Delta s = 13$ m and $\ell_z = N_z\Delta s = 2000\Delta s = 20$ m, where $N_x$ and $N_z$ are the number of the cell in $x$ and $z$ direction, respectively. $\ell_g = 100\Delta s = 1$ m is the width of the planar ground. In FDTD analysis, three structure models are considered. In conductor case, the elliptic cylinder and the plane are perfectly electric conductor and in dielectrics case, those objects have dielectric constant $\varepsilon_1 = \varepsilon_2 = 4\varepsilon_0$ and conductivity $\sigma_1 = \sigma_2 = 10^{-4}$ (S/m). The electric field amplitude in shadow region due to the presence of an obstacle is studied using simple models.

Figure 2 shows the analysis model in Case 1. The electric field of the incident wave is shown in Fig. 3. Fig. 4 shows that the amplitude in shadow region of $x < b + h$ is very weak in both Case 1-1 and 1-2. However, in conductor case of Case 1-1, many reflected waves are observed at $x = 4$ m and 2m, compared with the dielectric case of Case 1-2 as shown in Fig. 5. Fig. 6 shows the envelope of the electric field at propagation distance $z = 10$, 18 and 20 m. In Case 1-2, strong amplitude is observed in the dielectric cylinder at $t = 73$ ns. Around $x = 2$ m in shadow region, the amplitude is highly attenuated due to the elliptic cylinder.
Figure 4: Electric field at \( t = 3000 \Delta t = 60 \) ns (Case 1-1 and 1-2).

Figure 5: Envelope of electric field at \( t = 56 \) ns and 73 ns (Case 1-1 and 1-2).

Figure 6: Envelope of electric field at \((t, z) = (66 \text{ ns}, 18 \text{ m}), (73 \text{ ns}, 10 \text{ m}) \text{ and } (73 \text{ ns}, 20 \text{ m})\) in Case 1-1 and 1-2.

Figure 7: Cylinder on ground (Case 2).

Analysis model in Case 2 is shown in Fig. 7. There is no space between the elliptic cylinder and the plane. Fig. 8 and Fig. 9 show the electric field amplitude in Case 2-1 and 2-2. Figs. 11 and 12 shows the wave scattering and diffraction by a rectangular object in Case 3. Stronger amplitude of transmitted wave is observed in rectangular object at \( z = 10 \text{ m}, t = 73 \) ns as shown in Fig. 13. However, diffracted wave is very weak in these cases compared with the results in Cases 1 and 2.
Figure 8: Envelope of electric field at \( t = 56 \) ns and 73 ns (Case 2-2).

Figure 9: Envelope of electric field at \( (t, z) = (66 \text{ ns}, 18 \text{ m}), (73 \text{ ns}, 10 \text{ m}) \) and \( (73 \text{ ns}, 20 \text{ m}) \) in Case 2-1 and 2-2.

Figure 10: Rectangle on ground (Case 3).

Figure 11: Electric field at \( t = 3000 \Delta t = 60 \text{ ns} \) (Case 3-2).

Figure 12: Envelope of electric field at \( t = 56 \) ns and 73 ns (Case 3-2).

Figure 13: Envelope of electric field at \( (t, z) = (66 \text{ ns}, 18 \text{ m}), (73 \text{ ns}, 10 \text{ m}) \) and \( (73 \text{ ns}, 20 \text{ m}) \) in Case 3-1 and 3-2.

5. CONCLUSION

The phenomenon of scattering from an infinitely long elliptic cylinder on the planar ground is considered. We obtained numerical solution for conductor case and dielectrics case, using FDTD
Numerical results were shown for near fields of elliptic cylinder and rectangular object near the plane. In future, we calculate the exact solution by field expansion using Mathieu function and investigate the characteristics of shadow effect of the elliptic conductor on the ground precisely.

REFERENCES


