Metamaterials with Tunable Negative Refractive Index
Fabricated from Amorphous Ferromagnetic Microwires:
Magnetostatic Interaction between Microwires

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Abstract—For inhomogeneous mediums the optical Magnus effect has been derived. The metamaterials fabricated from amorphous ferromagnet Co-Fe-Cr-B-Si microwires are shown to exhibit a negative refractive index for electromagnetic waves over wide scale of GHz frequencies. Optical properties and optical Magnus effect of such metamaterials are tunable by an external magnetic field, magnetic field from neighbourhood microwires and mechanical stress.

1. INTRODUCTION

For a few recent years, new developments in artificially structured materials giving rise to negative refractive index \( n = \sqrt{\varepsilon(\omega) \cdot \mu(\omega)} < 0 \) with simultaneously negative real parts of frequency dependent permittivity \( \varepsilon(\omega) \) and permeability \( \mu(\omega) \) in some frequency ranges have been attracting much attention. These materials are consistent with causality and with the well established properties of group velocity in isotropic media. They are named left-handed mediums or left-handed materials, negative-index mediums, negative phase-velocity mediums (NPVM), backward wave mediums or even double negative media. The nonconflicting possibility of negative phase–velocity was theoretically discussed (see Refs. [1–3]). By now, after invention of first NPVM in microwave range around 7 years ago, the modern NPVM leveled the red edge of a visible spectra [4]. Recently developed NPVM have been paid much attention in journals and press. In homogeneous NPVM anomalous effects such as negative refraction, Doppler shift, Cherenkov-Vavilov radiation, light pressure, invisibility effect have been discovered in different frequency ranges. For them the gyrotropic phenomena are possible as well [5–7]. The other polarized electromagnetic effects such as optical Rytov and Magnus effects are given by a circular polarization of propagating waves, \( \Xi = \pm 1 \), and \( \nabla n \). Are they anomalous in inhomogeneous NPVM and is it possible to realize them per se? This paper tries to provide answers to these problems.

2. OPTICAL MAGNUS EFFECT

When a circularly polarized electromagnetic wave propagates in an inhomogeneous medium, the direction of its kinetic momentum does not vary. Then, according to the kinetic momentum conservation law, the ray trajectory becomes twisted, which means the optical Magnus effect. In Ref. [8], the Magnus optical effect has been described as the topological spin transport of photons using Berry phase. In this paper we apply the geometric optics approximation. For propagating electromagnetic wave its forces \( E \) and \( H \propto \exp \{ i (\bar{k} \cdot \bar{r} - \omega \cdot t + \psi(s)) \} \), where \( \psi(s) \) is a wave phase (eikonal) on the way \( s \). The direction of rays \( l_0 \) is given by \( \nabla \psi = k l_0 \) (\( \nabla \psi \cdot E = 0 \) and \( \nabla \psi \cdot H = 0 \)). Then from Maxwell’s equations after very time consuming calculations one can get the ray equation of a circular polarized electromagnetic wave as follows:

\[
\frac{\partial \bar{S}}{\partial s} = \bar{l}_0 \times \nabla \ln n \times \bar{S} + \frac{\Xi}{k} \left[ \frac{\partial}{\partial s} (\nabla \ln n \times \bar{S}) + \frac{1}{2} \frac{\partial \ln \varepsilon}{\partial s} \nabla \ln \mu \times \bar{S} + \frac{1}{2} \frac{\partial \ln \mu}{\partial s} \cdot \nabla \ln \varepsilon \times \bar{S} \right]
\]

\( \bar{S} = \frac{c(E_0 \times B_0^* + E_0^* \times B_0)}{16\pi} \) is the Umov-Poynting vector. As would be expected, Eq. (1) is symmetric with respect to replacement \( \varepsilon \leftrightarrow \mu \).
3. TUNABLE NPVM FROM AMORPHOUS FERROMAGNETIC MICROWIRES

It is difficult to reach tunable gradient inhomogeneity essential for the optical Magnus effect (1) in NPVM formed from artificial nanoresonators or natural materials. At present the glass coated amorphous ferromagnetic microwires are of interest both from fundamental and applied points of view [9–12]. Based on them metamaterials can be constructed [13]. This section is devoted to the ordered system of amorphous ferromagnetic microwires. Let us take microwires with diameter 2a which are parallel to z-axis and occupy sites of a square lattice with a lattice constant b in xy-plane. Then consider a linear polarized electromagnetic wave, propagating along y-axis with electric force along z-axis in an external magnetic field \( \{0, \, 0, \, H_0\} \). The frequency dispersion of the material had been taken into account in non-renormalised permeability tensor \( \tilde{\mu} = (\begin{array}{ccc} \mu & -im & 0 \\ im & \mu & 0 \\ 0 & 0 & 1 \end{array}) \), where \( \mu = \frac{\omega_0(\omega_0 + \omega_m) - \omega^2}{\omega_0^2 - \omega^2} \), \( \omega_0 = \gamma H_0 \), \( \omega_M = 4\pi \gamma M \), \( m = \frac{\omega_0 \omega_M}{\omega_0^2 - \omega^2} \) (\( \gamma = \frac{\mu_0}{\rho} \) magnetomechanical constant). One can derive the effective permeability of this system [14]:

\[
\mu_{\text{eff}} = \frac{1}{2} \frac{(\omega_0 + \omega_m)^2 - \omega^2}{\omega_0 (\omega_0 + \omega_m) - \omega^2} \left( 1 + \sqrt{\frac{\sigma}{\omega \varepsilon}} \left( \frac{2a}{b} \right)^2 \right) + 1 \tag{2}
\]

Here \( \omega_m = (2\pi \gamma)^2 \gamma M \) (\( \omega_0 = \gamma H_0 \gg \omega_m \), \( M \) is a saturation magnetization of amorphous ferromagnetic with a bulk conductivity \( \sigma \)). The approach developed in Ref. [15] can be extended for the case of ferromagnetic wires. Then the effective permittivity is

\[
\varepsilon_{\text{eff}} = \varepsilon_0 - \left( \frac{c}{\omega b} \right)^2 \frac{2\pi}{\ln \frac{b}{a} \left( 1 + i \frac{\xi_{zz}}{\omega \mu_0 \ln \frac{b}{a}} \right)} \tag{3}
\]

Here \( \xi_{zz} = (1 - i) \sqrt{\frac{\omega_0 \mu_0}{8\pi \varepsilon}} \left( 1 + (1 + i) \frac{\delta}{\ln \frac{b}{a}} \right) \), \( \delta = \frac{c}{\sqrt{2\pi \omega a}} \) (c.f. [9–12] and Leontovich-Schukin boundary condition) is the longitudinal component of a surface impedance tensor for strong skin-effect, \( \frac{a \sqrt{\pi}}{b} > 1 \). In Eq. (3) we work under assumption \( b \gg a \) and the transfer to dilute Drude metal occurs due to the decrease of a carrier density and the increase of an effective carrier mass in plasma frequency as in [16].

4. INTERACTION BETWEEN MICROWIRES

For considering system of microwires the interaction between microwires should be taken into account [17]. The model for description of the interaction of a few microwires is the dipole-dipole interaction. In this paper a model where microwires are considered as the dipoles has been predicted. Let us consider a scalar magnetic potential of the cylinder characterized by a radius \( R \) and a length \( L \) with magnetization along the cylinder:

\[
U(r) = -\int_{V} \frac{\nabla' \cdot \vec{M}(r')}{|r - r'|} d^3r' + \oint_{S} \frac{\vec{M}(r')}{|r - r'|} dS \tag{4}
\]

Using the expansion of the reverse distance through the Bessel function [18] \( \frac{1}{|r - r'|} = \sum_{m=-\infty}^{\infty} e^{im(\varphi - \varphi')} \int_{0}^{\infty} dk J_m(k \rho) J_m(k \rho') e^{-k(z-z')} \) the potential of the wire in cylindrical coordinates have been obtained:

\[
U(r) = \pi M_z R^2 \left( \frac{1}{\sqrt{\rho^2 + (z - L)^2}} - \frac{1}{\sqrt{\rho^2 + z^2}} \right), \tag{5}
\]

where \( z \) — is the cylinder axis, \( \rho \) — is the axial radius of the cylindrical coordinates.

It is easy to calculate created magnetic field \( \vec{H}(r) = -\nabla U(r) \) from (5):

\[
\vec{H}(r) = \pi M_z R^2 \left\{ \frac{z - L}{\left( \rho^2 + (z - L)^2 \right)^{\frac{3}{2}}} - \frac{z}{\left( \rho^2 + z^2 \right)^{\frac{3}{2}}} \right\} \tag{6}
\]
If we want to take into account all neighboring microwires, we should count such an integral
\[ \hat{H}_{\text{neighboring}} = \frac{1}{R'} \int_{2R}^{\infty} \hat{H}(r) \, d\rho, \]
where \( R' \) — the average distance between microwires, \( c \) — concentration of microwires.

5. CONCLUSION
As follows from Eq. (1) in inhomogeneous NPVM the optical Magnus effect is reversed with respect to homogeneous inhomogeneous normal ones as well as the other light effects in homogeneous NPVM. Since the Umov-Poynting vector \( \vec{S} = \omega \cdot \vec{v} \) (\( \vec{v} \) — a group velocity, \( \omega \) — an electromagnetic energy density), the 1st term in square brackets coincides with equation for the Magnus optical effect \( \pm \nabla \ln n \times \vec{k} \) of Ref. [8]. According to Eq. (1), the linear polarized electromagnetic wave, incident on NPVM, should be split in two circular polarized waves propagating in different ways. For typical magnitudes of \( \varepsilon_{\text{eff}} \), \( \mu_{\text{eff}} \), Eq. (2), has been confirmed by experiment [19]. In case of a homogeneous \( \varepsilon_{\text{eff}} \) the inhomogeneity of \( n \) can be created by, e.g., gradient \( \nabla H_0 = (\nabla_x H_0, 0, 0) \). Putting the latter into Eq. (2), according to Eq. (1) one can derive the optical Magnus effect. Created magnetic field from each other microwires has been calculated. Created magnetic field from each other microwires has been taking into account \( \omega_0 = \gamma (H_0 + \hat{H}_{\text{neighboring}}) \).

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REFERENCES