Microwave Corona Breakdown in rf Devices

J. Rasch¹, D. Anderson¹, M. Lisak¹,
V. E. Semenov², and J. Puech³

¹Chalmers University of Technology, Goteborg, Sweden
²Institute of Applied Physics, Nizhny Novgorod, Russia
³Centre National d’Études Spatiales, Toulouse, France

Abstract — The main physical properties of microwave corona breakdown in gases in the presence of inhomogeneous electric fields are investigated using numerical calculations of the continuity equation for the density of free electrons. In particular, the interplay between diffusion and attachment in redistributing electrons from high field regions to low field regions and the concomitant effect on the breakdown threshold is studied for different examples of the variation of the electric field strength. The results give a clear physical picture of the dependence of breakdown electric field on pressure with two complementary limits; a high pressure limit with a localized breakdown plasma confined to the high field region and a low pressure limit determined by the properties of the low field region only and a breakdown plasma extending over the entire volume of the rf device. A comparison between results for the critical microwave breakdown field as obtained by numerical calculations and experimental results in a microwave cavity show very good agreement.

1. INTRODUCTION

Corona breakdown is a potentially serious failure mechanism in many gas-filled microwave devices like antennas, wave guides etc. since the technological development tends to constantly increase the power in the system and there are natural limitations to the electrical field intensity a system can withstand before failing. The basic physics involved in the corona breakdown process — the avalanche-like growth of the free electron density which creates a conducting (plasma) region in or around the rf system under the ionizing action of high energy electrons created by the microwave field — is rather well understood for homogeneous microwave fields, [1]. However, many rf components involve inhomogeneous fields due to mode structure and/or to the presence of e.g., tuning screws or other details introduced for constructive purposes (e.g., impedance matching) or simply defects in the device. When the electric field in a gas-filled microwave device is inhomogeneous in space, the interplay between the concomitant inhomogeneous ionization, which tends to create free electrons, and the diffusion and attachment mechanisms, which tend to decrease the free electron density, becomes complicated and depends significantly on the geometry of the device and on the gas pressure. In cases with locally enhanced ionization, the breakdown threshold is determined as an interplay between diffusion and attachment where diffusion transfers free electrons out of the localized region with high field into regions with weaker fields where attachment operates as a sink for the free electrons. For small pressures, diffusion is a strong electron redistributing effect and the influence of the high field region is small, but for large pressures, diffusion is weak and the breakdown threshold is set by attachment balancing the locally high ionization, which may significantly lower the breakdown threshold as compared to the corresponding homogeneous situation. Thus, in the case of inhomogeneous field profiles, the pressure becomes an important factor in determining the size and location of the conducting region.

Corona breakdown in gas filled rf devices has been analyzed for a number of different designs and microwave mode structures, including situations involving field singularities e.g., sharp corners or edges where the electric field strength (and the ionization) becomes locally very high. For inhomogeneous electric fields, the corresponding breakdown threshold can be calculated analytically only in a few special cases and usually resort must be taken to approximate and numerical methods, c.f. [2, 3]. However, many different and technically important situations still lack a complete understanding of the breakdown conditions.

In the present work we will illustrate the coupling between diffusion, attachment, and the inhomogeneous ionization by considering two specific situations where the inhomogeneity of the field plays an important role in determining the breakdown threshold: a rectangular resonant cavity excited in the TE110 mode, and a small high field region adjacent to a conducting surface surrounded by a low field region.
2. THE GENERAL PROCEDURE

The determination of the microwave breakdown threshold involves two steps: (i) determination of the electric field strength in the device (while neglecting plasma effects) and (ii) solving the equation for the plasma dynamics in this electric field, i.e., solving the continuity equation for the electron density, viz.

\[
\frac{\partial n(\vec{r}, t)}{\partial t} = D \nabla^2 n(\vec{r}, t) + \nu_i n(\vec{r}, t) - \nu_a n(\vec{r}, t) \tag{1}
\]

where \(n(\vec{r}, t)\) is the plasma (electron) density, \(D\) is the diffusion coefficient, \(\nu_i\) is the ionization frequency, and \(\nu_a\) is the attachment frequency. These parameters depend on the type of gas, gas pressure (\(p\)), and electric field strength. Whereas \(D\) and \(\nu_a\) depends primarily on pressure (scaling as \(D \propto 1/p, \nu_a \propto p\)), the ionization frequency in addition also depends strongly on the electric field strength and an often used empirical approximation is \(\nu_i \propto p E_{\text{eff}}^{\beta}\) where e.g., \(\beta = 5.33\) for air. Here the effective electric field, \(E_{\text{eff}}\), is defined by \(E_{\text{eff}}^2 = E_{\text{rms}}^2/(1 + \omega^2/\nu_c^2)\) where \(E_{\text{rms}}\) denotes the rms electric field, \(\omega\) is the microwave angular frequency, and \(\nu_c\) is the collision frequency between electrons and gas particles. The breakdown threshold is determined by the condition that the losses of electrons through diffusion and attachment is balanced by ionization, i.e., \(\partial n(\vec{r}, t)/\partial t = 0\). Eq. (1) can then be formulated as the eigenvalue problem

\[
\nabla^2 n(\vec{r}, t) + (\lambda s(\vec{r}) - q)n(\vec{r}) = 0 \tag{2}
\]

where \(n(\vec{r})\) must vanish on the boundary of the considered volume, \(\lambda = \max \nu_i/D, q = \nu_a/D\), and \(s(\vec{r}) = (E(\vec{r})/\max E(\vec{r}))^\beta\). The eigenvalue, \(\lambda\), determines the breakdown electric field as a function of the physical parameters of the gas and the geometry of the device.

3. BREAKDOWN IN A RESONANT CAVITY

Resonant cavities play an important role in many microwave applications. One relevant configuration is the rectangular resonant cavity in which the field (excited in its \(E_{110}\) mode) is given by

\[
\vec{E} = E_0 \sin \left(\frac{\pi x}{a}\right) \sin \left(\frac{\pi y}{b}\right) \hat{z} \tag{3}
\]

where \(a\) and \(b\) are the dimensions in the \(x\) and \(y\) directions. When this field is inserted into the eigenvalue equation, the \(z\)-dependence of the plasma density can be separated out by writing \(n(\vec{r}) = Z(z)n(x, y)\), where \(Z(z) = \sin(\pi z/L)\), and \(L\) is the height of the cavity. The remaining equation for the function \(n(x, y)\) then becomes

\[
\nabla^2 n(x, y) + (\lambda s(x, y) - \tilde{q})n = 0 \tag{4}
\]

where \(\tilde{q} = q + \pi^2/L^2\) accounts for the losses due to attachment and diffusion in the \(\hat{z}\) direction and \(s(x, y) = \sin^2(x/a)\sin^2(y/b)\). Eq. (4) is not tractable for exact analytical analysis, but a powerful and convenient approximate approach is to use direct variational methods based on the Ritz optimization procedure as demonstrated in [4].

An interesting aspect of breakdown plasma behavior, characteristic of situations involving inhomogeneous electric fields, is the interplay between attachment and diffusion in determining the electron losses and its dependence on pressure. As noted above, diffusion decreases with increasing pressure whereas attachment increases. Thus, the length an electron diffuses before it is lost by attachment is significantly smaller than the cavity dimensions at high pressures, and the breakdown plasma tends to become localized in the region where the ionization is larger than the attachment frequency. This means that as the pressure inside the cavity increases, the region occupied by the plasma becomes successively smaller. This is illustrated in a succinct way in Fig. 1 below. The plasma profile to the left in Fig. 1 is dominated by diffusion to the walls, it varies almost sinusoidally, extends over the entire cavity area, and smoothly approaches zero at the edges. This situation corresponds to low pressure and a breakdown plasma extending over the entire volume. On the other hand, the plasma density profile to the right in Fig. 1 has contracted into a needle-like profile due to the high pressure which decreases the diffusion out of the over-critical region in the center where ionization is larger than the attachment losses.

The breakdown plasma density will also have an extension in the \(z\)-direction according to the sinusoidal variation determined by the function \(Z(z)\), but in the \(x, y\)-plane the curves of constant
density describes concentric circles. In the figures shown above, \(a\) and \(b\) has been chosen equal. If the relative size of the cavity sides is changed, the density curves will describe concentric ellipses in the \(x, y\)-plane.

4. BREAKDOWN AROUND A HEMISPHERICAL BOSS

Another important situation is the one where there is a small region of local field enhancement in an otherwise homogeneous field. We have investigated several different models for the ionization profile; a step-like profile, an exponential profile, and a profile corresponding to the field around a small hemispherical boss on an infinite conducting plane. The variation of the field profile was seen to play an important role for the size of the plasma region in the high pressure regime.

In the case of the hemispherical boss, the electric field around the boss is known in explicit form, and is given by

\[
E(r, \theta) = E_0 \sqrt{\left(1 + \frac{2a^3}{r^3}\right)^2 \cos^2 \theta + \left(1 + \frac{a^3}{r^3}\right)^2 \sin^2 \theta}. \tag{5}
\]

where \(r\) and \(\theta\) are spherical coordinates and the profile is rotationally symmetric. The main features of this field are that \(E \to E_0\) as \(r \to \infty\), and \(E \to 3E_0\) as \(r \to a\).

We have performed simulations of this problem in a cylindrical coordinate system. The boundary conditions were \(n(r = R) = 0\) and \(n(z = 0)(z = H) = 0\), where \(R\) and \(H\) were taken as different multiples of \(a\). For very low pressures, the effect of the field enhancement around the boss was found to be negligible, and the continuity equation, Eq. (2), can be solved using \(n(x, y, z) = N(r)Z(z)\), where \(Z(z) = \sin \left(\frac{z\pi}{H}\right)\) and \(N(r) = J_0(r)\) with \(J_0(r)\) being the zero order Bessel function. The eigenvalue can then be evaluated exactly as \(\lambda = q + \pi^2/H^2 + j_0^2/R^2\), where \(j_0 \approx 2.4\) is the first zero of \(J_0(r)\). In the pressure regime where this formula is valid, the breakdown plasma will fill the entire volume. However, when the pressure increases, the influence of the boss becomes stronger and eventually the electric field strength necessary to initiate breakdown becomes smaller than that without the field enhancement. The plasma also starts to contract and to become localized around the boss. For even higher pressures, the breakdown threshold will become determined by the balance of attachment and ionization in the very close vicinity of the boss surface. In the purely mathematical limit when the pressure becomes infinite, the plasma region will become infinitely small, and the breakdown threshold is given by the equality \(\nu_i(r = a) = \nu_a\). These qualitative considerations are illustrated in Fig. 2 below, which shows the shrinking of the plasma region as the pressure increases. The left figure shows the plasma filling the entire chamber, and the influence of the boss is relatively small, a situation corresponding to low pressures. Only half the plasma section is shown, corresponding to \(r \in [0, R]\) and \(z \in [0, H]\), the system being symmetric round the \(z\)-axis. The right figure shows the breakdown plasma for a higher pressure. Now the situation is completely dominated by the field around the boss, and the plasma region is very small and localized to the vicinity of the boss surface. The numerically obtained predictions for the breakdown thresholds in the rectangular cavity and for the hemispherical boss have been compared with experiments reported in [5] and show excellent agreement, see Fig. 3.
Figure 2: Illustration of the influence of pressure on the extension of the breakdown plasma around a hemispherical boss. Left, low pressure. Right, high pressure.

Figure 3: Comparison between experimental results (taken from [5]) and theoretical predictions for the rectangular cavity (left figure — taken from [4]) and for the hemispherical boss for two different boss radii (right figure).

5. CONCLUSION
The two systems analyzed above are very different, but have the common feature of involving inhomogeneous profiles of the electric field strength. In both cases, this makes the quasi-steady plasma region, described by the solution of the continuity equation, change size with changing pressure. With increasing pressure the plasma generally tend to contract around the region with highest electric field. An important implication of this behaviour for rf systems in general should be emphasized: At high pressures in combination with strongly inhomogeneous electric fields, the concomitant small plasma region does not necessarily have an important deleterious effect on the operation characteristics of the rf device. On the other hand, the creation of such a local quasi-steady plasma may lead to significant local absorption of rf power, which heats the surrounding gas and on a longer time scale may lead to important effects on the device, either by causing local melting of surrounding material or by lowering the global threshold so that full scale breakdown occurs. Thus, the stability and power absorption of the quasi-steady plasma should also be investigated before any general prediction about the breakdown threshold can be made.

REFERENCES