Multi-band Chaotic Oscillator with Phase-locked Loop

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Abstract — The use of phase locked loops (PLLs) gives the opportunity to develop wideband chaotic oscillator with manipulated spectrum (electronic switching of frequency bands, electronic control of spectrum width). This paper contains results on numerical study of mathematical model of PLL and results of modeling with the use of specialized electronic design software. Considered models take into account peculiarities of modern components of phase locked loops (pulse phase discriminators, frequency dividers). Analysis of these models approves the possibility of control of spectrum width and possibility of electronic shifting of spectrum of chaotic phase-modulated signals in wide frequency range. Shift of the frequency range is obtained by means of changing coefficients of frequency dividers in phase locked loop. Additional control of the loop gain allows keeping the same dynamic properties of chaotic frequency modulation during spectrum shifting. As an example we have modeled generator of phase chaos with spectrum width about 500 MHz and ability of spectrum steering in the range 3–5 GHz.

1. INTRODUCTION

At present, research activity in the field of communications using chaotic signals switches from theoretical studies to the area of real engineering developments, which have perspectives of commercial application. It is worth mentioning that direct chaotic communication system, which was developed in IRE RAS [1–3], is included in the international standard for information technology 802.15.4a-2007, specifications for low-rate wireless personal area networks, as an optional solution for physical layer [4]. The majority of developed chaotic oscillator schemes for chaos communications, and also for direct chaotic communication system, produce chaotic signals with amplitude, varying irregularly in time. However for simple receiving system, which contains amplitude detector, it is preferable to use wideband signals with fixed amplitude and chaotically varying phase. In a number of theoretical [5–8] and experimental works [8–12], the possibility of applying phase locked loops to produce oscillations with chaotic phase was shown. The results presented in these works tell us that voltage controlled oscillator (VCO) with phase locked loop can produce wideband and ultra-wideband oscillations with homogeneous spectrum in different frequency ranges, including UHF range [12]. In this work, we show that PLL allows generating wideband phase-modulated oscillations with steerable spectrum. Steering range can be greater than the spectrum width for several times, therefore it is possible to realize in one device electronic switching of wideband UHF oscillations between non-overlapping frequency bands. We have accomplished modeling of PLL taking into account peculiarities of modern widespread microchips. Results confirm that chaotic wideband oscillations can be produced in wide domains of system parameters and spectrum of such oscillations can be controlled easily.

Let us consider standard PLL scheme, presented in Fig. 1, which contains voltage controlled oscillator, phase discriminator (comparator), loop filter, two frequency dividers, and additional amplifier in the control loop. Before describing the mathematical model (Section 3), we discuss some peculiarities of modern phase detectors in Section 2.

2. PHASE DISCRIMINATOR MODEL

The simplest phase detector (PD), used in modern PLLs, is an XOR gate. The average value of this square wave is the DC component that sets the VCO frequency. The square-wave changes duty-cycle in proportion to the phase difference resulting, after the filter, in the VCO control voltage. Its characteristics are very similar to the analog mixer for capture range, lock time, and low-pass filter requirements. Its model characteristic [9] is $2\pi$-periodical and close to sine-characteristic of analog mixer. That is why nonlinear dynamics (including chaotic regimes) of PLL with XOR phase detector is much similar to dynamics of classical PLL model, which assumes sinusoidal model characteristic of PD.
Another widespread type of PD is proportional phase-frequency detector (PPFD), which employs a charge pump that supplies charge amounts in proportion to the phase error detected (Fig. 2). Some have dead bands (Z in Fig. 2), which are areas where small changes in phase difference produce no correction to the VCO. If the inputs are slightly mismatched, either the up (H-level) or down (L-level) pulse will contain slightly more charge than the other and the PLL will be able to correct the offset.

Model characteristic of PPFD substantially differs from characteristic of analog mixer that is why we present it in Fig. 2. It has linear part (excluding dead-zone) on the interval \([-2\pi, 2\pi]\), but it is not \(4\pi\)-periodical. Lock-in range of PLL with PPFD equals to the holding range in contrast to PLL with analog mixer or XOR PD demonstrating hysteresis phenomenon on the boundary of synchronization regime.

Results of modeling presented below were obtained for both characteristics: XOR and PPFD. For modeling of PPFD in electronic design software, we considered parameters of PPFD close to parameters of MB15E03SL. Moreover, for simplicity we assumed that only one output of PPFD (for up-pulses or for down-pulses) is used in PLL-scheme, i.e., only one part of PPFD characteristic (Fig. 2) takes place.

### 3. PLL MODEL

Experiments in UHF range [12] confirm that PLL with low-pass RLC-filter in the control loop allows producing wideband oscillations with chaotic phase and homogeneous spectrum. Let us consider mathematical model of PLL with frequency dividers (Fig. 1) and second-order low-pass loop-filter [5, 10, 11]:

\[
\frac{d^3 \varphi}{d\tau^3} + \varepsilon \frac{d^2 \varphi}{d\tau^2} + \frac{d \varphi}{d\tau} + F(\varphi) = \gamma, \tag{1}
\]

\[
\varepsilon = \frac{\Omega}{n RC}, \quad \mu = \left(\frac{\Omega}{\omega_0}\right)^2 LC, \quad \gamma = \frac{n}{\Omega} \left(\frac{\omega_0}{n} - \frac{\omega_{\text{REF}}}{m}\right), \quad \varphi = \frac{\theta_{\text{VCO}}}{n} - \frac{\theta_{\text{REF}}}{m}, \quad \tau = \frac{\Omega t}{n}. \tag{2}
\]

In Equations (1)–(2): \(n\) — Frequency divider coefficient for VCO oscillations, \(m\) — Frequency divider coefficient for reference oscillations, \(R, L, C\) — Filter parameters (resistance, inductance, capacitance), \(\omega_0\) — Natural frequency of VCO, \(\omega_{\text{REF}}\) — Frequency of the reference signal, \(\theta_{\text{VCO}}\) and \(\theta_{\text{REF}}\) — Phases of VCO and reference oscillations. Parameter \(\Omega\) is the half of frequency holding.
rage:
\[ \Omega = \pi S G (U_H - U_L) \]  

Here \( S \) — Frequency tuning sensitivity of VCO, \( U_H \) and \( U_L \) — Maximum and minimum voltages on the output of phase discriminator, \( G \) — Gain of amplifier in the control loop of PLL (Fig. 1).

PLLs may feature various chaotic regimes in which the phase difference between response and master signals is either limited (quasi-synchronous chaotic regimes) or exhibits unlimited growth or decay (chaotic beats). It is known [8–12], that chaotic attractors which combine oscillatory and rotatory motions of phase difference (see Fig. 3) correspond to the oscillations with more wide and homogeneous spectrum than attractors corresponding to quasi-synchronous regimes or regimes of beats with pure rotatory dynamics of phase difference.

Chaotic regime presented in Fig. 3 exists in wide regions of parameters of model (1) which were studied for \( \mu, \varepsilon > 1 \) [5, 8]. The presence of divider coefficient \( n \) in the model leads to small \( \varepsilon \) and \( \mu \), close or even smaller than 1. It occurs because \( n \) can be large enough in case of big ratio of VCO frequency to the reference frequency. For small \( \varepsilon \) and \( \mu \) dynamics of (1) looks more complicated, nevertheless chaotic oscillations can be excited if \( \mu \gg \varepsilon \). This qualitative condition means high \( Q \)-factor of the loop filter: \( Q = (\mu)^{1/2}/\varepsilon \).

4. RESULTS OF MODELING

Here the topical result of electronic steering of the spectrum of wideband chaotic oscillations is presented. Modeling was carried out in specialized electronic design software and by numerical analysis of (1). In modeling PPFD phase, discriminator is used. We divide frequency range 3–5 GHz into four bands: 3–3.5 GHz, 3.5–4 GHz, 4–4.5 GHz, 4.5–5 GHz. To make electronic switching between bands we have to change natural frequency of VCO together with divider coefficient \( n \) for minimizing frequency difference between input signals at the input of phase discriminator. In this case, parameters \( \varepsilon \) and \( \mu \) depending on \( n \) substantially changes. It can cause changing of dynamic properties of chaotic oscillations or even hitting in regular regime. To prevent changing of selected

![Figure 3: Phase portrait, \( y = d\phi/\tau \) (left), spectrum of oscillations at the input of VCO (center), spectrum of modulated oscillations at the output of VCO (right). Results for model (1) with XOR phase discriminator.](image)

![Figure 4: Electronic steering of chaotic spectrum in PLL.](image)
chaotic regime it is necessary to keep constant all parameters in (1). It is possible if in addition to keeping $\omega_0/n = \text{const}$, we guarantee that $\Omega/n = \text{const}$. To fulfill the second condition, control of loop amplifier gain $G$ is required (see Eq. (3)).

In modeling parameters, $\omega_0$, $n$, and $G$ were been changing to switch frequency bands and to keep constant dimensionless parameters: $\varepsilon \approx 0.117$, $\mu \approx 1.52$, $\gamma \approx -1.58$ (variation of parameters is few percents of its values). Results of modeling (Fig. 4) demonstrate possibility of electronic switching of wideband chaotic oscillations between different frequency bands. Note, that manipulation of gain parameter $G$ leads to change of spectrum width, because it effects on amplitude of modulation in the control loop of PLL.

5. CONCLUSION

Electronic manipulation of the spectrum of wideband oscillations with chaotic phase, realized in one device — PLL, looks perspective to be applied in modern wireless communications. For instance, it can be used for additional frequency division multiplexing. PLL is widespread, robust system which can be easily implemented, it makes chaotic generator based on PLL simple and feasible.

Before applying oscillations with chaotic phase produced by PLL to direct chaotic systems we have to answer the key question about the speed of chaotic modulation, because it limits the maximum rate of information transmission. Mathematical model (1) does not include any delay or inertance of control element of PLL, so in modeling it is possible to achieve maximum frequency modulation with the rate comparable to the frequency of pulses (or oscillations) at the output of the phase discriminator. To study the inertance of VCO, we considered a simple VCO scheme based on transistor BFP405. We have found that rough retuning of VCO frequency from 6 to 7 GHz takes less than $\tau^* = 3\text{ ns}$. Therefore we can estimate the maximum rate of frequency modulation as 300 MHz (note that for smaller frequency deviation and for smooth retuning, maximum modulation rate can be much greater). If the length of chaotic pulses is ten times longer than $\tau^*$, then the transmission rate will be approximately 30 Mbps.

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REFERENCES


