Rigorous Electromagnetic Analysis of Domestic Induction Heating Appliances

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Abstract — In this paper the developed analytical electromagnetic model of induction heating system is presented. The model was built up assuming equivalent electric and magnetic currents flowing in each planar element of the typical structure used for an induction heating system: the load disk represents the pan steel bottom, the copper inductor, and ferrite flux conveyor. A system of integral equations system was then obtained enforcing the boundary conditions on each element of the structure for the electric and magnetic fields, produced by the equivalent currents. The numerical solution of the system is a matrix equation with a known voltage vector in the left-hand side, and product of impedance coefficients matrix and unknown electric and magnetic currents vector in the right-hand side. Since the feeding voltage is known, and impedance coefficients are calculated using of geometry and material parameters, currents vector can be also calculated. Thus, the whole model is solved and it gives a detailed picture of currents distribution in the system, which in its turn allows to analyze heating process in the load. Each step of developing of the model was verified by appropriate experimental measurements. Achieved results give a possibility to analyze and develop improvements to increase efficiency, safety and to reduce the cost.

1. INTRODUCTION

Domestic induction cookers become more and more popular because of their high efficiency, safety and ease in use. Due to the state-of-the-art a present-day induction heating applicator consists of inductor, some ferrite bars below the coil screened by aluminum and the load (pot) above the coil (see Fig. 1). Inductor of the induction system is a spiral planar coil which is fed by a medium-frequency (20–100 kHz) power source. According to Faraday’s law the alternating magnetic field induces eddy currents in the metal pan and, additionally, in ferromagnetic materials produces magnetic hysteresis. Both phenomena heat up the pan. Ferrite is located under the coil as a shielding to protect the electronic control system which is usually placed under applicator.

There are different methods of analyzing and calculating some parameters of induction system which are widely described in existing literature. Most of them are numerical and based on finite elements method (FEM) and similar [1]. There are also some methods used for inductor impedance calculation such as the interpolation method [2] and squared-field-derivative for computation of losses [3]. More than that, since inductors of modern induction cooking systems are winded using multistrand wire with helix transposition of the strands — Litz-wire, there are some works dedicated to investigation of inductor impedance and its internal resistances according to peculiarities of construction of Litz-wire [4, 5].

The objective of the present work is to develop a rigorous analytical electromagnetic model of induction heating applicator which allows to analyze the magnetic coupling between system elements, their influence on the system operation and also to calculate circulating currents in each part of the system. Currents distribution in the load and their amplitudes give a possibility to value the heating efficiency.

2. ANALYTICAL MODEL

The key point of the system is the electromagnetic coupling among the source winding (inductor), the ferromagnetic pot bottom (load) and the flux conveyor (ferrite). Therefore, we focus the attention on the simplified geometry reported in Fig. 2. The model was built up assuming equivalent electric and magnetic currents flowing in each planar element of the structure (Fig. 4). The expressions for magnetic ($\vec{J}_m$) and electric ($\vec{J}$) current densities can be obtained from the Maxwell field equations in free space:

$$\begin{cases}
\vec{J}_m = j\omega \mu_0 (\mu_r - 1) \vec{H} \\
\vec{J} = \sigma \vec{E} + j\omega \varepsilon_0 (\varepsilon_r - 1) \vec{E}
\end{cases}$$

(1)
Applying these expressions for each element of the model with appropriate parameters of its material they can be written as follows:

\[
\begin{align*}
\tilde{J}_S &= \sigma_S \tilde{E} \\
\tilde{J}_{mS} &= j\omega\mu_0(\mu_rS - 1)\tilde{H} \\
\tilde{J}_C &= \sigma_C \tilde{E} \\
\tilde{J}_{mC} &= 0 \\
\tilde{J}_F &= \sigma_F \tilde{E} + j\omega\varepsilon_0(\varepsilon_rF - 1)\tilde{E} \\
\tilde{J}_{mF} &= j\omega\mu_0(\mu_rF - 1)\tilde{H}
\end{align*}
\]

where \( J_S, J_{mS}, J_C, J_{mC}, J_F, J_{mF} \) are electric and magnetic current densities in steel (load), copper (source) and ferrite (flux conveyor) respectively; \( \sigma_S, \sigma_C, \sigma_F \) are conductivities of steel, copper and ferrite respectively; \( \mu_rS, \mu_rC, \mu_rF \) are relative magnetic permeabilities, \( \sigma_0 \) and \( \mu_0 \) are conductivity and magnetic permeability of free space; \( E \) and \( H \) are electric and magnetic fields respectively. So, the solution of the problem is to find unknown currents in each planar element of modeled system.

Let \( E_0 \) the incident electric field induced by feeding voltage \( V_0 \). After enforcing the boundary conditions for the electric and magnetic field on each element of the structure, the system of equations which is the analytical description of electromagnetic model can be written as follows:

\[
\begin{align*}
0 + \tilde{E} (\tilde{J}_C) + \tilde{E} (\tilde{J}_S) + \tilde{E} (\tilde{J}_{mS}) + \tilde{E} (\tilde{J}_{mF}) &= \frac{\tilde{J}_S}{\sigma_S}; \\
\tilde{E}_0 + \tilde{E} (\tilde{J}_C) + \tilde{E} (\tilde{J}_S) + \tilde{E} (\tilde{J}_{mS}) + \tilde{E} (\tilde{J}_{mF}) &= \frac{\tilde{J}_C}{\sigma_C}; \\
0 + \tilde{H} (\tilde{J}_C) + \tilde{H} (\tilde{J}_S) + \tilde{H} (\tilde{J}_{mS}) + \tilde{H} (\tilde{J}_{mF}) &= \frac{\tilde{J}_{mS}}{j\omega\mu_0 (\mu_rS - 1)}; \\
0 + \tilde{E} (\tilde{J}_C) + \tilde{E} (\tilde{J}_S) + \tilde{E} (\tilde{J}_{mS}) + \tilde{E} (\tilde{J}_{mF}) &= \frac{\tilde{J}_F}{\sigma_F + j\omega\varepsilon_0 (\varepsilon_rF - 1)}; \\
0 + \tilde{H} (\tilde{J}_C) + \tilde{H} (\tilde{J}_S) + \tilde{H} (\tilde{J}_{mS}) + \tilde{H} (\tilde{J}_{mF}) &= \frac{\tilde{J}_{mF}}{j\omega\mu_0 (\mu_rF - 1)}.
\end{align*}
\]

We also assume that all currents flow uniformly along the wire and across its cross-section. As for directions electric currents flows along wire curve, but magnetic currents have two components — normal and radial:

\[
\begin{align*}
\tilde{J}_S &= J_S \hat{r}; \quad \tilde{J}_C = J_C \hat{r}; \quad \tilde{J}_F = J_F \hat{r} \\
\tilde{J}_{mS} &= J_{mS}\hat{r} + J_{mSz}\hat{z} \\
\tilde{J}_{mF} &= J_{mF}\hat{r} + J_{mFz}\hat{z}
\end{align*}
\]

After applying (4) in (3) the system (3) can be rewritten with detailed description considering direction of current densities vectors. Each field element of every equation is calculated using basic definitions of vector potential and magnetic vector potential.
3. NUMERICAL SOLUTION

Taking advantage from cylindrical symmetry we can represent the disk as a set of concentrical current paths (Figs. 3 and 4). Thus, each elementary current path should be considered as separate element which counteracts with the whole system: either with neighbor elementary paths or source turn or equivalent paths on ferrite disk.

The numerical solution of the analytical system of Equation (3) is the solving of linear matrix equation. The matrix Equation (5) includes all self- and mutual impedance coefficients which in its turn was grouped in matrices $M$ and $Z$, vector of fed voltage $V_0$ and column of unknown currents. Solving the Equation (5) gives us all currents known which allows to analyze its distribution in the load disk (Figs. 5 and 6).

\[
M_{23(N_a,N_a)} = \begin{bmatrix}
M_{i,j} & M_{i,j} & \cdots & M_{i,j} \\
M_{i,j} & M_{i,j} & \cdots & M_{i,j} \\
\vdots & \vdots & \ddots & \vdots \\
M_{i,j} & M_{i,j} & \cdots & M_{i,j}
\end{bmatrix}
\]

\[
R_{ilS3(i,j)} = -\frac{1}{4\pi} \int_{L_i} \int_{L_j} \frac{e^{-jk|\Delta|}}{|R_{ijk}|^3} t (\phi \cdot \phi') dldl' 
\]

where $R_{ilS3(i,j)}$ is mutual impedance of the $i$-th and $j$-th elementary currents, $R_{ijk}$ is the distance between active and passive points of integration (see vector potential definition), $N_a$ and $N_f$ are numbers of elementary current paths in load and ferrite disks respectively.

4. EXPERIMENTAL VALIDATION

For the experimental model a copper ring was taken of diameter $\Omega 15$ cm and diameter of cross-section $\Omega 1$ mm as a simple inductor. For the load and flux conveyor two iron disks were taken with external diameter $\Omega 20$ cm and internal $\Omega 10$ cm, thickness 1.4 mm ($\delta = 0.05$ mm skin-depth).

Figure 3: Model structure with elementary concentrical current paths.  
Figure 4: Schematic representation of the modeled system operation.
Table 1: Measured and calculated results.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Measured</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R (Ω)</td>
<td>X (Ω)</td>
</tr>
<tr>
<td>Single source turn (Ø15 cm)</td>
<td>0.01533</td>
<td>0.24183</td>
</tr>
<tr>
<td>Single source turn (Ø15 cm) + ring (Ø ext 20 cm, Ø int 10 cm)</td>
<td>0.04129</td>
<td>0.17358</td>
</tr>
<tr>
<td>Single source turn (Ø15 cm) + 2 ring (Ø ext 20 cm, Ø int 10 cm)</td>
<td>0.03139</td>
<td>0.16315</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

An analytical electromagnetic model to analyze induction heating applicator is presented. The system was described with potential balance equation based on electric and magnetic currents definition. The numerical solution was derived as the matrix equation based on equivalent circuit method. In order to compute self- and mutual impedance coefficients, a calculation of electric and magnetic fields is required. For this purpose basic definitions of vector potential and magnetic vector potential are used. Thus, when fed voltage and impedance matrixes are known, the currents are computed. The obtained values of currents and impedance give a possibility to analyze of induction system operation. The theoretical results were tested and verified experimentally. Measured and simulated results exhibit a good agreement.
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REFERENCES