Light Scattering by Preferentially Oriented Ice Crystals

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Abstract — Scattered light for preferentially oriented ice crystals is divided into specular and diffuse components where the specular scattering is created by horizontally oriented facets of fluttering crystals. The specular component for a fluttering thin plate modeling these crystals is found analytically as a 2D convolution of a geometric optics pattern depending only on flutter and an independent diffraction function. The geometric optics pattern is explicitly expressed through the probability density for particle tilts and the diffraction function is taken in the Fraunhofer diffraction approximation. Certain possibilities to retrieve both flutter parameters and particle sizes from the specular patterns are discussed.

1. INTRODUCTION
Cirrus clouds consisting mainly of ice crystals play an important role in radiative budget of the Earth. Consequently, their radiative properties are needed to incorporate in up-to-date numerical models of climate prediction and change. These radiative properties have been calculated by a lot of authors in the assumption that the crystals are 3D randomly oriented. However, experimental data obtained from both the ground [1] and satellites [2–4] prove that the ice crystals often reveal their preferentially horizontal orientation. For such crystals, the radiative properties are poor studied yet both experimentally [1–4] and theoretically [5, 6].

Light scattered by preferentially oriented ice crystals reveals a specific property. Namely, it is divided on two qualitatively different parts called the specular and diffuse components (e.g., [7]). Therefore it is convenient to introduce the terms of specular and diffuse scattering by a crystal if the crystal obeys a sharp probability distribution over its spatial orientations. To some extent, the specular scattering is formed by reflection of light from those crystal facets that are horizontally oriented while the other facets create the diffuse component. Radiative properties of the specular component are simply expressed through the microphysical parameters of the crystals: Sizes, shapes and orientation. Such expressions are subjects of direct scattering problems. And vice versa, from the point of view of inverse scattering problems, this is the specular component that is mostly informative for retrieving the microphysical parameters from scattered light. A purpose of this paper is to present a simple and rather general theory for the specular scattering that can be effectively used for both direct and inverse scattering problems. It is worthwhile to note that such a theory can be applied not only to light scattering in the atmosphere but also to any scattering media consisting of aligned and large as compared to the incident wavelength particles. Examples of such media are special paints, biological structures and tissues, forest canopy, and so on.

Oscillations of orientation of the preferentially oriented ice crystals near the horizon are called flutter. In the atmosphere, the flutter is usually confined to a narrow cone with the maximum flutter tilt $T$ of about a few degrees. The value $T$ for cirrus clouds was studied from the ground by use of scanning polarization lidars [1]. Also, the value $T$ was estimated from the data obtained for the specular scattering component by the satellite radiometer POLDER [2, 3]. We show that not only the maximum flutter tilt $T$, but the tilt probability density can be, in principle, retrieved from certain specular scattering patterns.

2. SPECULAR AND DIFFUSE SCATTERING BY AN ICE CRYSTAL
In the problem of light scattering by an atmospheric ice crystal, it is expedient to consider the near zone at first where a lot of analytical expressions that are valid at arbitrary distances from the particle can be written down directly without any analytical or numerical calculations (e.g., [8]). Geometric optics description of the electromagnetic field in the near zone is a rather exact approximation. It is no wonder that practically all numerical calculations of this problem are made by means of the ray-tracing method corresponding exactly to geometric optics. Within the framework of geometric optics, it is obvious that the near-zone scattered light is produced by illuminated facets of the crystal. Every facet illuminated by a plane electromagnetic wave creates two plane-parallel
beams with known transversal shapes and propagation directions. One of them is the reflected beam which becomes directly a component of the near-zone scattered field desired. Another, i.e., refracted, beam propagates in the crystal until it meets other facets. Then every of these facets produce new components of the near-zone field because of refraction, and so on. The total near-zone scattered field consists of a lot of plane-parallel beams with different sizes, shapes, polarization, and propagation directions. It is worthwhile to note that such a facet-tracing computer code calculating scattering matrices for ice crystals was developed by us [9, 10] and it has been successfully exploring.

Let us go to horizontally oriented crystals. We say that a crystal has a perfect horizontal orientation if one or more of its facets are always horizontal when the crystal rotates about the vertical. For such a rotating crystal, a horizontal facet reflects the incident light in the same direction while non-horizontal facets smooth their reflected beams over changing propagation directions. As a result, the scattered light averaged over the crystal rotation consists of a bright dot and a fuzzy wan pattern corresponding to the specular and diffuse scattering, respectively.

In practice, every horizontally oriented crystal is exposed to the flutter or, equivalently, it reveals its preferentially horizontal orientation where the rotation axis oscillates slightly about the vertical. In this case, both the bright dot of the specular component and a fuzzy pattern of the diffuse component are expanded as compared to those of a perfectly oriented crystal. It is well known that the preferentially horizontal orientation of ice crystals in the atmosphere is inherent mainly to plate-like crystals. Here two basic facets of the plate-like crystals tend to be horizontal. Column-like crystals sometimes reveal such kind of orientation, too. In particular, the so-called Parry orientation for a hexagonal column means that the long axis of a column rotates in the horizontal plane about the vertical while two rectangular crystal facets are saved horizontally.

The split of the scattered electromagnetic field $E$ into the specular and diffuse components

$$E = E_s + E_d$$

is strictly defined as follows. Among the horizontally oriented facets of a crystal, it is easy to distinguish a main oriented facet that contributes directly to the specular component by reflection of the incident light. In addition to this plane-parallel beam reflected by the main oriented facet, there are a number of other beams leaving the crystal in the same direction at a given crystal orientation. By definition, superposition of all the beams leaving the crystal in the same direction as the beam reflected from the main oriented facet are called the specular component $E_s$. The rest part of the total near-zone scattered field corresponds to the diffuse scattering component $E_d$.

We should emphasize that the superposition of Eq. (1) is quite general. It retains its validity at arbitrary distance from the particle including the wave zone. Indeed, at any distance from the crystal each component of Eq. (1) can be obtained independently of the other by certain integral transform of the near-zone fields. By definition, the split of Eq. (1) is provided by appearance of the preferentially oriented facets of crystals. Otherwise, for example, in the case of 3D randomly oriented crystals, this split becomes meaningless.

3. SCATTERING BY A FLUTTERING PLATE

The main physical regularities inherent to the specular component can be obtained within a simplified problem of light scattering by a fluttering plate. Consider a thin plate with its orientation $\mathbf{N}$ illuminated by an incident plane electromagnetic wave propagating in the direction $\mathbf{i}$. In the near zone, the scattered field corresponding to the specular scattering is a plane-parallel beam propagating in the reflection direction $\mathbf{r}$. These three unit vectors $\mathbf{r}$, $\mathbf{i}$, and $\mathbf{N}$ are connected with each other by the following equations

$$\mathbf{r} = \mathbf{i} - 2(\mathbf{i} \cdot \mathbf{N})\mathbf{N} \quad \mathbf{N} = (\mathbf{i} - \mathbf{r})/|\mathbf{i} - \mathbf{r}|$$

A dot between two vectors means their scalar product.

At large distance from the plate, the reflected plane-parallel beam spreads about the reflection direction $\mathbf{r}$ because of diffraction. In the wave zone, this beam is transformed into a spherical diverging wave. Though polarization of the scattered field can be easily taken into account, the scalar approximation is used in this paper for brevity. Then the differential cross section of the spherical scattered wave $\sigma(s, \mathbf{i})$ is generally presented as

$$\sigma(s, \mathbf{i}) = A |\mathbf{I} \cdot \mathbf{N}| R(\mathbf{i} \cdot \mathbf{N}) F_i \mathbf{N}(s - \mathbf{r})$$
where $A$ is a plate area, $R$ is the reflection coefficient that is well known for any given polarization of incident light and the dimensionless function $F_{i,N}$ normalized to unity describes diffraction. In the Fraunhofer approximation [11] the function $F_{i,N}$ is the 2D Fourier transform

$$F_{i,N}(s-r) \approx (k/2\pi) \int \exp \left(-ik \left[ (s-r) \cdot \rho' \right] \right) S'(\rho') d\rho' / A'$$

(4)

where $k = 2\pi/\lambda$, $\lambda$ is the wavelength, and the function $S'(\rho')$ describing a shape of the near-zone reflected beam is equal to unity inside and to zero outside the beam in the plane $\rho'$ perpendicular to the reflection direction $r$. In particular, for a circle of the radius $a$, we get

$$F_{i,N}(s-r) = (\beta(ka)^2 / 4\pi) \left( 2J_1 \left[ k\sqrt{(w_||\beta)^2 + w_\bot^2} \right] \right)^2$$

(5)

where $J_1$ is the Bessel function and $\beta = (i \cdot N)$. Here we denote a projection of the vector $s-r$ on the plane $\rho'$ orthogonal to the reflection direction $r$ as the vector $w$ and its longitudinal and transversal components relative to the direction $[N - (N \cdot r)r] / |N - (N \cdot r)r|$ are denoted $w_||$ and $w_\bot$, respectively.

We have to average the scattering differential cross-section given by Eq. (3) over a probability density of plate orientations $p(N)$. Denoting this averaging by angular brackets, we get finally after certain calculations

$$\langle \sigma(s,i) \rangle = (A/4) \int R(i \cdot N(r)) F_{i,N}(s-r) p(N(r)) dr$$

(6)

To clarify a physical meaning of Eq. (6), let us ignore diffraction by replacing the diffraction function $F_{i,N}(s-r)$ by the Dirac delta-function $\delta(s-r)$. In this case, Eq. (6) is reduced to the geometric optics differential cross section

$$\langle \gamma(s,i) \rangle = (A/4) R(i \cdot N(s)) p(N(s))$$

(7)

Equation (7) reveals a remarkably simple physical meaning. It means that the differential cross section of a fluttering plate within the framework of geometric optics at any scattering direction $s$ is equal (with the trivial factor of $AR/4$) to the probability density of flutter at the corresponding plate orientation $N(s)$. Main properties of the function $\langle \gamma(s,i) \rangle$ are studied in our recent paper [12].

Now Eq. (11) allows us to write down Eq. (10) as the physically obvious 2D convolution:

$$\langle \sigma(s,i) \rangle = \int \langle \gamma(s,i) \rangle F_{i,N}(s-r) dr$$

(8)

Thus, the problem of light scattering by a fluttering plate is strictly reduced to a 2D convolution of two functions. One of the functions is responsible for flutter and the other describes diffraction.

Figure 1: Phase functions of a fluttering circular disc for the uniform (left) and gaussian (right) tilt distributions.
If a scale of the diffraction function $F_{i, N}$ is less than a scale of the geometric optics pattern, the total specular scattering can be treated as a smoothing of a rather sharp geometric optics pattern by diffraction. In the opposite case, we can say that a diffraction pattern is broadened by flutter.

As an illustration, Fig. 1 presents the total phase function $\langle \sigma(s, i) \rangle$ calculated numerically. Here light is incident on a circular plate of $\lambda/a = 0.027$ fluttering with the maximum zenith angle $5^\circ$ relative to the vertical. The zenith and azimuth angles of the incident direction $i$ are $\theta_i = 70^\circ$ and $\phi_i = 0^\circ$, respectively. The reflection coefficient $R$ corresponds to an interface with the refraction index of 1.31 and unpolarized incident light. It is interesting to note that, in addition to trivial smoothing of contours of the geometric optics phase function, a kind of cumulative effect for the total phase functions in the center can appear as seen in Fig. 1. This cumulative effect is explained by the fact that the elongated diffraction functions $F_{i, N}$ of Eq. (5) are oriented predominantly to the center.

4. CONCLUSIONS

A conception of specular scattering by ice crystals with preferential orientations proves to be efficient for both direct and inverse scattering problems of optics of cirrus clouds. It is shown that the specular scattering component is a 2D convolution of the geometric optics scattering pattern and the diffraction function. Here the geometric optics pattern is determined by only flutter parameters while the diffraction function depends only on the ratio of wavelength/(crystal size). The geometric optics specular pattern is found analytically in our recent paper [12] as a mapping of a probability density of plate orientations into scattering directions. Therefore, in the inverse scattering problems, not only a maximum tilt of fluttering crystals but the tilt probability density can be retrieved if the specular patterns are not essentially distorted by diffraction. On the other hand, a distortion of the specular patterns by diffraction can be used for retrieving sizes of crystal if the specular patterns are measured at several wavelengths. Such an opportunity has been recently demonstrated in [7]. The results obtained in this paper are rather general and they can be also applied to various scattering media with aligned and large as compared to incident wavelength particles: Special paints, biological media, forest canopy, etc.

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