Estimation Error of Topographic Phase Based on RVoG Model Using POLinSAR Data

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Abstract — Topographic phase is a key parameter in the Random Volume over Ground forest model. Its estimation error influences the estimation accuracy of other forest parameters. Therefore, the error analysis and error control in topographic is necessary to obtain the accurate forest parameters. This paper analyzes the main error sources in topographic phase and provides a method to estimate the possible error range. To examine the proposed method, the error analysis is applied on the results of ESAR L band forest data.

1. INTRODUCTION

The estimation of biophysical parameters of forest from polarmetric and interferometric SAR data has been demonstrated recently [1]. The inversion is based on Random Volume over Ground (RVoG) model, which relates forest biophysical parameters with the observables. This model has been widely applied to the deciduous forest, conifer forest and rain forest [2]. Based the model, many techniques are proposed to inverse forest height and underlying ground height from the POLinSAR coherences [3].

Up to now, forest height error due to bad estimation of the topographic phase has been analyzed [4]. However, the estimation error of topographic phase has not been discussed. Since its errors would be passed to the results of other forest parameters, it is necessary to analyze and control the error.

This paper analyzes and estimates the estimation error according to the geometrical characteristics of coherence distribution. In the RVoG model, the topographic phase can be estimated from the intersections of the straight line, fitted by certain group of coherence samples, and the unit circle. According to the realistic coherence distribution, the factors leading to the estimation errors in topographic phase are analyzed. Using the geometrical characteristics of coherence distribution, this paper presents a method to estimate the possible estimation error. Experiments are applied to examine the effectiveness of the proposed error analysis.

This paper describes the RVOG model and its topographic phase inversion in Section 2. Three stages to analyze errors in topographic phase are presented in Section 3. The method to estimate possible error range in each stage is proposed, too. To examine the error analysis method, experiments are applied to the Traunstein forest data acquired by the L-band ESAR system. As the results in Section 4 shown, the proposed analysis method is effective. Finally, the applicability of the proposed method is drawn in Section 5.

2. RVOG MODEL AND TOPOGRAPHIC PHASE

RVoG model is a layered physical model for the forest. It assumes the forest as a homogeneous layer of randomly oriented scatterers covering on the ground. As shown in Figure 1(a), the thickness of the layer or the forest height is \(h_v\) and the scattering amplitude per unit volume scatterers is \(m_v\). The ground locates on the altitude \(z_0\) and its scattering amplitude is denoted as \(m_G\). When SNR decorrelation and temporal decorrelation are negligible, the interferometric coherence in the forest can be expressed as:

\[
\tilde{\gamma}(\vec{\omega}) \equiv \frac{\vec{\omega}^H \Omega_{12} \vec{\omega}}{\sqrt{\vec{\omega}^H \mathbf{T}_{11} \vec{\omega}}} = e^{j \phi_v} \tilde{\gamma} V + m(\vec{\omega}) + \frac{m(\vec{\omega})}{1 + m(\vec{\omega})}
\]  

where superscript \(H\) represents the conjugate and transposition operation. Vector \(\vec{\omega}\) is related to the polarization states. Matrix \(\mathbf{T}_{11}\), \(\mathbf{T}_{22}\) and \(\Omega_{12}\) are the outer products of the POLinSAR data:

\[
\Omega_{12} = \langle \hat{k}_1^H \hat{k}_2 \rangle, \quad \mathbf{T}_{11} = \langle \hat{k}_1^H \hat{k}_1 \rangle, \quad \mathbf{T}_{22} = \langle \hat{k}_2^H \hat{k}_2 \rangle
\]
where
\[
\vec{k}_1 = \frac{1}{\sqrt{2}} \left[ S_{1HH} + S_{1VV} \right] \quad \text{and} \quad \vec{k}_2 = \frac{1}{\sqrt{2}} \left[ \frac{S_{2HH} + S_{2VV}}{2S_{2HV}} \right]
\]

In (1), Symbol $\phi_0$ is the interferometric phase indicating the height of the ground. It is called topographic phase in this paper. $\tilde{\gamma}_V$ represents the complex coherence for the layer. It is closely related to the forest height and forest extinction coefficient $\sigma$. $m$ denotes the effective ground-to-volume amplitude ratio and it depends on the polarization state $\vec{\omega}$. Therefore, underlying topography can be estimated from the RVoG model [3].

The coherence in (1) can be rewritten as:
\[
\tilde{\gamma} (\vec{\omega}) = e^{j\phi_0} \left[ L(\vec{\omega}) (1 - \tilde{\gamma}_V) + \tilde{\gamma}_V \right] \quad \text{where} \quad L(\vec{\omega}) = \frac{m(\vec{\omega})}{1 + m(\vec{\omega})}
\]

$L(\vec{\omega})$ is the ratio and falls in $[0, 1)$. Figure 1(b) shows the distribution of complex coherences. They form a solid blue segment of the red dash straight line. Moreover, the complex topographic phase $e^{j\phi_0}$ is one intersection of the red line with the unit circle. Thus, the straight line and topographic phase can be estimated from any set of coherence samples, such as complex coherence in $HH$, $HV$ or $VV$ polarization state. In realistic, however, the coherence distribution in forest does not accord to a straight line strictly. To obtain the accurate topographic phase, many inverse techniques proposed to choose the reliable coherences sample. The coherences given by coherence optimization are widely used because they have the possible highest correlation [3].

3. ERROR ANALYSIS OF TOPOGRAPHIC PHASE

In real forest, the canopy scatterers are partly oriented and the volume scattering varies with the polarization states. The forest observations do not satisfy the assumption of RVoG model. The coherences in forest do not distribute along the straight line but disperse in a region. The shape of coherence region may be an ellipse or a triangle. The choices of coherence samples influence the estimation for the straight lines and topographic phase. Therefore, the error in topographic phase is analyzed based on geometrical parameter of the coherence region.

This paper analyzes topographic phase estimation $\hat{\phi}_0$ given by the optimum coherences. The whole analysis can be divided into three stages. Stage 1 analyzes the error when the coherence distribution has preferred orientation. Stage 2 gives the fine error analysis when the coherence distribution satisfies RVoG model. Stage 3 analyzes the error caused by SNR decorrelation when the estimate suffers from much less volume decorrelation.

3.1. Error Analysis in Stage 1

The coherence region is defined as the area within which all possible complex coherences disperse [5]. To describe its shape, a parameter named narrowness ratio is introduced. It is defined as the distance ratio:
\[
Na = \max \left\{ |\tilde{\gamma}_3 - \tilde{\gamma}_4|, (\tilde{\gamma}_1 - \tilde{\gamma}_2) (\tilde{\gamma}_3 - \tilde{\gamma}_4)^* \right\} \quad \max \left\{ |\tilde{\gamma}_1 - \tilde{\gamma}_2|, \tilde{\gamma}_3 \subset R, \tilde{\gamma}_4 \subset R \right\}
\]

where symbol $R$ consists of the coherence on the border of coherence region. The red ring in Figure 2 forms the $R$ and they are estimated as [5] did. The denominator in (6) is the length of longest
axis in the coherence region. It is shown as the long solid line in Figure 2. The numerator is the maximum length of the axis orthogonal to the longest axis. The narrowness ratio falls in the range [0, 1). The smaller the narrowness ratio is, the closer RVoG model is to the forest observations. The less possible estimation error occurs in topographic phase. Hence, 1 − Na can be regarded as the error risk indicator.

When the narrowness ratio is lower than 0.5, coherence region has obvious preferable direction and the estimation error can be analyzed according to RVoG model. According to the model, the line segment related to the topographic phase gives the possible largest distance. Hence, the longest axis gives the expected topographic phase $\phi_0$ shown as brown circle in Figure 2. Considering the reliability of RVoG model, the possible error in $\phi_0$ can be estimated from:

$$\phi_{e1} = \frac{1}{1 - Na} \left| \arg \left( e^{j\phi_e} e^{-j\phi_0} \right) \right|$$

Symbol $\arg()$ is to calculate the argument. When the RVoG model does not fit to the data, the estimation error $\phi_0 - \phi_0$ is amplified to large possible error. In this situation, no further error analysis can be done according to the RVoG model. When RVoG model is reliable, the possible error estimated in Stage 1 is very close to $\phi_0 - \phi_0$ and detailed analysis in Stage 2 can be applied.

### 3.2. Error Analysis in Stage 2

Besides the shape of the coherence region, the RVoG model can be examined according to the interferometric phase. Deduced from (1), the interferometric phase changes monotonically. The coherence samples with the maximum phase gradient can be used to estimate the expected topographic phase $\phi_0$. It is shown as the blue circle in Figure 3.

Since $\phi_0$ and $\bar{\phi}_0$ are estimations given by the RVoG model, their differences serves as a goodness indicator for the model and the expected ones. The small difference between $\phi_0$ and $\bar{\phi}_0$ indicates the expected topographic phase is reliable. Stage 2 analyzes the possible error when the phase difference between $\phi_0$ and $\bar{\phi}_0$ is less than 0.3 rad. In this situation, the deviation of estimated topographic phase from the expected ones can be taken as the estimation error:

$$\phi_{e2} = \max \left\{ \left| \arg \left( e^{j\phi_0} e^{-j\phi_0} \right) \right|, \left| \arg \left( e^{j\bar{\phi}_0} e^{-j\bar{\phi}_0} \right) \right| \right\}$$

When the phase difference is over the threshold 0.3 rad, it is hard to identify whether $\phi_0$ or $\bar{\phi}_0$ is the reliable. Maybe both $\phi_0$ and $\bar{\phi}_0$ suffer from the estimation errors. That is why the fine error cannot be estimated.

### 3.3. Error Analysis in Stage 3

In RVoG model, the errors caused by the SNR decorrelation are neglected. However, the SNR decorrelation would dominate the phase error when the volume decorrelation effects are decreased. It is to say the phase difference between $\phi_0$ and $\bar{\phi}_0$ need to be less than 0.1 rad. The Stage 3 is to analyze the estimation error caused by the SNR decorrelation.

Taking the SNR decorrelation $\gamma_{SNR}$ into account, the forest coherence can be expressed as:

$$\tilde{\gamma}(\omega) = \gamma_{SNR} \cdot e^{j\phi_0} \left[ L(\omega) (1 - \gamma_V) + \gamma_V \right]$$

Figure 4 shows the effects of $\gamma_{SNR}$ on the coherence distribution. The observed segment marked by the brown solid line shrinks from the real green solid line. Therefore, the expected topographic phase $\phi_0$ is deviated from the real topographic phase $\bar{\phi}_0$. To get the phase information $\phi_0$, the SNR decorrelation $\gamma_{SNR}$ should be estimated. When $\gamma_{SNR}$ is known, the observed line segment can be shifted to the real segment. The intersection of real segment and the unit circle is the real topographic phase. Deduced from the coherence sample with the optimum coherence:

$$|\tilde{\gamma}_{1opt}| = |\gamma_{SNR}| \cdot |\tilde{\gamma}(\omega_{1opt})| \quad \Rightarrow \quad |\gamma_{SNR}| \geq |\tilde{\gamma}_{1opt}|$$

To estimate the maximum possible estimation error, the largest effect caused by $\gamma_{SNR} = |\tilde{\gamma}_{1opt}|$ is analyzed. The endpoints $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$ of the longest axis in the observed coherence region are
shifted to $\tilde{\gamma}_1/\gamma_{\text{SNR}}$ and $\tilde{\gamma}_2/\gamma_{\text{SNR}}$ respectively. The possible topographic phase $\tilde{\phi}_0$, farthest from $\hat{\phi}_0$, is obtained. The possible estimation error, accounting for $\gamma_{\text{SNR}}$, can be calculated from:

$$\phi_{e3} = \left| \arg \left( e^{j\hat{\phi}_0} e^{-j\tilde{\phi}_0} \right) \right|$$

(9)

Figure 2: Analysis in Stage 1. \hfill Figure 3: Analysis in Stage 2. \hfill Figure 4: Analysis in Stage 3.

4. EXPERIMENTS AND RESULTS

The proposed method is applied on the experimental results to demonstrate its effectiveness. The data were acquired by L band ESAR system in 2003 and they covered Traunstein Forest sites in Germany. The topographic phase estimations $\hat{\phi}_0$ given by the three optimum coherences are analyzed. Using the X band InSAR DEM results of the bare ground near the forest (10-pixel distance), the reference topographic phase $\phi_0$ in the forest can be calculated. Compared with the deviation $|\arg(e^{j\hat{\phi}_0} e^{-j\phi_0})|$, the estimation error given by the three stages is assessed:

1. Calculate $R$ and $Na$. If $Na < 0.5$, the coarse error can be estimated from (5) and go to Stage 2; or else, begin the analysis for the next result.
2. Calculate phase difference between $\phi_0$ and $\bar{\phi}_0$. If the phase difference is less than 0.3 rad, the error can be replaced with the error defined in (6) and step in Stage 3; or else, go back to Stage 1 to analyze next result.
3. If the phase difference is less than 0.1 rad, the error can be replaced with the result of (9); or else, go back to Stage 1 and analyze the next result.

Figures 5–7 compare the error estimations and the measured error in three stages respectively. From the distribution of the red samples, most error estimation in Stages 1 and 2 are larger than the measured ones. They can provide the coarse range of the estimation error. The estimated error in stage three gives more fine estimations because more percentages of estimated errors are close to the measured ones. It is potent to analyze the accuracy of topography estimations.

Figure 5: Analysis in Stage 1. \hfill Figure 6: Analysis in Stage 2. \hfill Figure 7: Analysis in Stage 3.
5. CONCLUSIONS
This paper analyzes the error in topographic phase estimated from RVoG model. According the
goodness of RVoG model, the possible error can be estimated in three different stages. The latter
stage can gives finer error estimation than that given in the front stage, but applicability of the
latter one is not as good as the front one.

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