An Accelerated Frequency Domain Ray-tracing Simulator for Ultra-Wideband Communications

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Abstract — Ray-tracing is an established technique for modelling wireless propagation over a large area in an efficient manner. Existing frequency domain ray tracing simulators incur severe computational burden when extended to model an Ultra-Wideband (UWB) channel of several Giga-Hertz. We propose an acceleration algorithm that improves the efficiency of such simulators by approximating the frequency domain response of individual ray at a reduced set of frequency points. Result show time savings up to 75% with a maximum relative error in the region of 0.8%.

1. INTRODUCTION

The sanctioning of Ultra-Wideband (UWB) as a wireless communications medium by the Federal Communication Commission (FCC), constitutes a potential for development of novel capabilities in wireless applications [1]. In particular the large bandwidth utilized by UWB offers unprecedented data-rates and enhancements to applications such as wireless geo-location [2]. With the arrival of wireless schemes such as IEEE 802.11b/g, it became apparent that propagation modelling was required for purposes of optimising performance and planning infrastructure in varied environments. Ray tracing is one such modelling method that has came to prominence [3]. While ray-tracing is an asymptotic method and may not be as computationally accurate as full wave methods, it offers a deterministic but efficient means of propagation modelling in large-scale environments at high frequencies.

Ray-tracing can be extended to model UWB wireless systems. However there are computation penalties incurred when using ray-tracing simulators based on frequency domain computations [4], due to the large bandwidths involved. Some ray tracing techniques rely on time-domain based methods to avoid having to calculate channel responses at every frequency [5]. These techniques do not consider the variations of material properties over the extensive UWB bandwidth. We present an acceleration algorithm that reduces the computation associated with frequency domain ray-tracing computation. This algorithm accounts for the effects of frequency dependant material properties. In addition, a comparison of a ray tracing simulation with empirical measurements for an indoor UWB channel is presented.

2. FREQUENCY DOMAIN RAY TRACING

Electric fields at specified receiver points are computed by a Ray-Tracing simulator. During such computations dominant paths by which energy propagates between the receiver and transmitter are identified. Subsequently the electromagnetic fields associated with these paths are calculated and summed to obtain the total electric field. At a point \( r \) the electric field can be written as,

\[ E(r, \omega) = \sum_{i=1}^{N} R_i(r, \omega), \]

where \( N \) is the total number of rays and depends on the highest order of scattering event that is allowed. The variation of the electric field with frequency in Equation (1) is due to the frequency dependency of each ray, which in turn can be classed into two categories.

- An substantial contribution to frequency dependence is due to the propagation distance of a ray. Electric lengths of a ray will differ at each individual frequency. Although this results in a phase component that varies rapidly with frequency, the effect can be easily computed.
Additional frequency dependent effects are attributed to interactions of ray with object in the propagation environment. Transmission and reflection coefficients of such object’s surfaces undergo variations due to the frequency dependence of the constituent material’s dielectric properties. Consequently, individual rays also exhibit frequency dependant variations. Such effects, while significant, vary slowly with frequency.

Our technique relies on separating these widely varying frequency dependencies for each ray. Fast variations are extracted and the remaining slowly varying effects are approximated using a low order polynomial. Using this method a channel response can be completely specified by analysing the ray at a reduced number of frequency sample points.

3. EFFICIENT FREQUENCY SWEEP

A channel response at a point \( r \) is given by,

\[
H(r, \omega) = \sum_{i=1}^{N} R_i(r, \omega),
\]

where \( H(r, \omega) \) is the component of electric field at the point \( r \) in the direction determined by the polarisation of the receiver antenna. Typically this quantity must be computed for many frequency points \( \omega_\gamma \) for \( \gamma = 1, \ldots, Q \), choosing \( Q \) large enough to ensure a satisfactory sampling rate. Hence a computation burden occurs when the huge bandwidths of UWB channels are simulated. The proposed acceleration technique reduces the amount of frequency samples that must be explicitly computed by separating the fast and slow varying components of each ray. For each point \( r \) we rewrite Equation (2) as (suppressing the \( r \) dependence for notational convenience)

\[
H(\omega) = \sum_{i=1}^{N} A_i(\omega) e^{-j\phi_i(\omega)},
\]

where we have made explicit the amplitude and phase frequency dependence of the ray contribution. This can be factorized as follows,

\[
H(\omega) = \sum_{i=1}^{N} A_i(\omega) e^{-j\tilde{\phi}_i(\omega)} e^{-j\phi_i^{(d)}(\omega)},
\]

where \( \phi_i^{(d)} \) is the phase behaviour associated with the free-space propagation of the ray and \( \tilde{\phi}_i \) is the residual phase behaviour associated with its interactions with materials.

The phase term \( \phi_i^{(d)} \) has the form,

\[
\phi_i^{(d)}(\omega) = \frac{d_i \omega}{c},
\]

where \( d_i \) is the distance ray \( i \) travels in free space and \( c \) is the speed of light in a vacuum. By dividing out this linear phase, a slowly varying term \( A_i(\omega) \tilde{\phi}_i(\omega) \) remains. Consequently this residual can be approximated by \( P_i \), a low order polynomial of order \( m \),

\[
A_i(\omega) e^{-j\tilde{\phi}_i(\omega)} \approx P_i(\omega),
\]

yielding,

\[
H(\omega) \approx \sum_{i=1}^{N} e^{-j\phi_i^{(d)}(\omega)} P_i(\omega).
\]

\( P_i \) can be completely specified for each ray by computing exactly the channel response at \( m+1 \ll Q \) frequency points.

A received signal \( r(t) \) can be obtained by multiplying the channel response by \( S(\omega) \), the Fourier transform of the transmitted signal \( s(t) \), and taking an inverse Fourier transform,

\[
r(t) = \mathcal{F}^{-1}(S(\omega) H(\omega)).
\]
However, we present an alternative analytical approach that uses the polynomial approximation $P_i$ and the $\phi_i^d$ free-space phase term to rapidly compute $H(\omega_i)$ at each of the $Q$ frequency points $\omega_i$. To achieve this we exploit the following identities of Fourier transforms,

$$\mathcal{F}^{-1}\{f^g S(\omega)\} = \frac{d_g}{dl^g} s(t).$$  \hspace{1cm} (9)

$$s(t - \phi_i^d) = \mathcal{F}^{-1}\left\{S(\omega)e^{-j\phi_i^d(\omega)}\right\}.$$  \hspace{1cm} (10)

The received time domain signal $r(t)$ can be considered as a sum of individual ray contribution $r_i(t)$ where,

$$r_i(t) = \mathcal{F}^{-1}\left(S(\omega) P_i e^{-j\phi_i^d(\omega)}\right).$$  \hspace{1cm} (11)

By utilizing the identities of Equations (9) and (5) this time domain ray can be expressed as follows,

$$r_i(t) = \alpha_{0i} s_i \left( t - \frac{d_i}{c} \right) - \beta_{0i} H\left\{ s_i \left( t - \frac{d_i}{c} \right) \right\}$$
$$+ \sum_{g=1,3,5\ldots}^{m-1} (\sqrt{-1})^{g-1} \left( \alpha_{gi} \frac{d_g}{dl^g} H\left\{ s_i \left( t - \frac{d_i}{c} \right) \right\} + \beta_{gi} \frac{d_g}{dl^g} s_i \left( t - \frac{d_i}{c} \right) \right)$$
$$+ \sum_{g=2,4,6\ldots}^{m} (\sqrt{-1})^{g} \left( \alpha_{gi} \frac{d_g}{dl^g} s_i \left( t - \frac{d_i}{c} \right) - \beta_{gi} \frac{d_g}{dl^g} H\left\{ s_i \left( t - \frac{d_i}{c} \right) \right\} \right),$$  \hspace{1cm} (12)

where $H$ is the Hilbert transform, $c$ is the speed of light and $d_i$ is the propagation length of the ray. $\alpha_{gi}$ and $\beta_{gi}$ are the respective real and imaginary parts of the $g$th order coefficient of the polynomial $P_i$ of Equation (11). A distinct advantage of using the form in Equation (11) is that $s(t)$, $H(s(t)$ and their corresponding derivatives can be pre-computed prior to any simulations. This leaves the polynomial coefficients $\alpha_{gi}$ and $\beta_{gi}$ and the ray path delays $d_i/c$ to be computed during runtime. Assuming a prior known set of transmitted waveforms for $s(t)$, this offers an efficient means of computing the time domain received signal $r(t)$.

### 3.1. Propagation through Walls

Transmissions through object with complex varying permittivities results in a ray undergoing an additional frequency dependant effect. Propagation through such objects results in an addition phase component $\phi_i^{\text{trans}}$ being added to the frequency domain response of a ray. This component is expressed as,

$$\phi_i^{\text{trans}} = d_i \beta_i,$$  \hspace{1cm} (13)

where $d_i$ is the propagation distance of the ray inside the wall and $\beta_i$ is the phase constant due to the material given by,

$$\beta_i = \omega \sqrt{\mu \varepsilon^e} \left\{ \frac{1}{2} \left[ \sqrt{1 + \left( \frac{p_\varepsilon}{p_c} \right)} + 1 \right] \right\}^{\frac{1}{2}}.$$  \hspace{1cm} (14)

It should be noted that we assume a relative complex permittivity of the form,

$$\varepsilon_{re}(\omega) = \varepsilon_r(\omega) - j \left( p_\varepsilon \varepsilon_r(\omega) \right),$$  \hspace{1cm} (15)

where $\mu$ is the permeability, $\varepsilon^e$ is relative permittivity and $p_\varepsilon$ is the loss tangent of the wall’s constituent material.

The total phase of a ray that propagates through an air medium and a wall can be express as

$$\phi_i^{\text{tot}}(\omega) = \frac{d_i \omega}{c} + d_i \beta_i$$  \hspace{1cm} (16)

where $d_i$ is the free space propagation length. While $\phi_i^{\text{tot}}(\omega)$ is not a linear phase response it can be approximated by an effective linear phase $\phi_i^e \approx \phi_i^{\text{tot}}$. This is possible as the portion of a typical ray that passes through object in an indoor environment will be significantly shorter than the free-space propagation length such that $d_i \ll d_i$. The effective linear phase $\phi_i^e$ can be used in Equation (4) instead of $\phi_i^d$ as a means of isolating the slowly varying residuals.
4. RESULTS

In order to evaluate the accuracy and efficiency of the proposed algorithm a ray tracing simulation was executed using a simple building environment based on the layout of the Sensor Laboratory of the Antennas and Electromagnetic Research Group at Queen Mary University London (Figure 1(b)). For the purposes of our simulations, we use the values of the relative complex permittivity for concrete as measured by Muqaibel et al. in [6] (Figure 1(a)). Rays were limited to two orders of reflection and the transmitted signal $s(t)$ consisted of a Gaussian doublet pulse waveform of Equation (17), as shown in Figures 2(a) and 2(b).

$$s(t) = 1 - \left(4\pi \left(\frac{t}{t_n}\right)^2\right) \exp\left(-2\pi \left(\frac{t}{t_n}\right)^2\right),$$

(17)

where $t_n = 0.780$ nanoseconds and thus determines the bandwidth of the pulse.

![Figure 1](image1.png)

(a) Variation of permittivity of concrete dashed line: Dielectric constant dotted line loss tangent; (b) Building environment.

![Figure 2](image2.png)

(a) Frequency response of Gaussian doublet pulse  
(b) Gaussian doublet pulse in time domain

Figure 2: Gaussian doublet transmitted UWB pulse.

A ray trace simulation was carried out over a full set of 800 frequency samples from 0.5 MHz to 10 GHz. Subsequently a simulation using a reduced frequency sweep of 24 samples was executed and the proposed algorithm was applied. The reduced frequency sweep consisted of 71 samples and a 11th order polynomials was used to approximate ray responses in the frequency domain. Figure 3(a) shows a section of the time domain received signal $r(t)$ obtained using the acceleration technique alongside the received signal computed using the original full frequency sweep ray-trace. From Figure 3(a) it is clear that the full frequency sweep simulation results match those of the accelerated reduced frequency sweep in the time domain. This is confirmed in Figure 3(b) where a less than 0.8% relative error is observed. By applying the acceleration algorithm a 75% time saving was achieved.

In order to access the accuracy of our simulator it was necessary to compare simulation results with measured data for UWB channels. This involved measuring the channel impulse response of a UWB channel in the building environment shown in Figure 1(b). A channel bandwidth from
3 GHz to 10 GHz was used in conjunction with a pair of Planar Inverted Cone Antenna (PICA) developed at Queen Mary University London [7]. This measured impulse response (dotted line) is shown in Figure 3 along with simulated response (dash line). The transmitter-receiver antenna pair was positioned 2.1 meters apart as in Figure 1(b).

![Figure 3](image)

Figure 3: (a) Simulated received signal $r(t)$; (b) Relative error expressed as % of maximum received signal; (c) Measured and simulated results for channel response.

5. CONCLUSION

We have presented a frequency domain based ray tracing acceleration technique that reduced number of frequency samples at which a ray trace has to be computed when simulating UWB propagation. Deploying this accelerated simulator in an indoor building scenario yields a time saving of 75%, with a maximum relative error of less than 0.8%. Additionally, validation of simulation result with empirical measurements has been performed.

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REFERENCES