Regularized and Blocky 3D Controlled Source Electromagnetic Inversion

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Abstract — Since the interpretation of controlled source electromagnetic (CSEM) data proves challenging in complex geological settings, 3D CSEM anisotropic resistivity imaging problem is formulated as an inverse problem. A least-squares misfit functional is minimized with a quasi-Newton algorithm to cope with the large number of unknowns. Furthermore, model and data weights are applied to speed up the convergence of the non-linear inversion. A-priori information obtained for instance from seismic interpretation can be included either in a blocky inversion, i.e., where the resistivity cube is parameterized with a small number of parameters or with a regularized inversion. Since earth resistivity contrasts can be high and spatially well defined, minimum norm support regularization terms are implemented. Although the blocky inversion proves quite powerful, we show that results can be misleading in complex geological settings. A combination of the blocky and regularized inversions may provide a more robust approach to interpret CSEM imaging results. We illustrate this with a deep-water example.

1. INTRODUCTION

Controlled source electromagnetic (CSEM) surveys consist of recording electromagnetic responses to an emitting electromagnetic source. In off-shore applications, a horizontal electric dipole source is towed over electromagnetic receivers [1]. This scanning technique helps detecting a thin and elongated resistive layer in a more conductive background. Because a reservoir charged with hydrocarbon can be several orders of magnitude more resistive than the same reservoir filled with brine, CSEM surveying is an important tool for oil and gas exploration. Crude interpretation of CSEM data based on anomalies detected after normalization of the data with computed or measured reference responses proves to be insufficient and even erroneous in complex geological settings in which other resistive bodies like, for example, hydrate, salt, or basalt, are present [2]. A more powerful approach consists of formulating the imaging problem as an inverse problem [3]. A least-squares misfit between observed data and computed synthetics is minimized with a quasi-Newton algorithm [4]. This local optimization approach, combined with a decoupling of the computational and inversion grids [5], allows us to deal with large 3D CSEM inverse problems containing typically between 5 and 20 millions of unknowns. Furthermore, model and data weights are added to speed up the convergence of the non-linear inversion [6].

Because of the diffusive nature of the electromagnetic waves in the conductive earth, the inverse problem is highly non-linear. To reduce the non-uniqueness and include a-priori information, we implemented two approaches: a blocky inversion scheme where the resistivity cube is described with a small number of parameters defining large homogeneous blocks and a regularized inversion. We did not use the classic square norm of the resistivity gradient as regularization term because this favours smooth results. To allow for large and spatially sharp resistivity contrasts, we preferred regularization terms based on the minimum norm support [7].

2. IMAGING APPROACH

Given the observed data, d, the imaging problem is formulated as an inverse problem:

\[
\text{Find } m^{\text{opt}} \text{ such that } J(m^{\text{opt}}) \leq J(m) \forall m.
\]

Here, m is the model parameter vector, \(m^{\text{opt}}\) is the optimal model parameter, and J is the least-squares misfit functional:

\[
J(m) = \frac{1}{2} \|W^d(d - c(m))\|^2 + \alpha R(m)
\]

where c are the synthetics solution of the Maxwell/Ohm equation with conductivity \(\sigma(W^m m)\), \(W^m\) some model weights, \(W^d\) some data weights and \(R\) a regularization term with \(\alpha\) the regularization term.
coefficient. In our implementation, the Maxwell/Ohm equations are discretized on a stretched computational grid with the finite integration technique and the associated linear system is solved with an iterative multi-grid based solver [8]. The computational grid is automatically defined from the minimum resistivity value and the frequency and is decoupled from the inversion grid.

We don’t directly invert for the conductivity on the computational grid because it would be expensive for multiple-frequency inversion. This approach however involves a mapping from the inversion grid to the computational grid that may introduce numerical inaccuracy. This decoupling of the computational and inversion grid also allows us to define a blocky and a regularized inversion.

In a blocky inversion, the domain is mainly divided in large regions/blocks and \( m \) is the vector constituted by the logarithm of the resistivity in each large region/block. A procedure to map this coarse discretization to a regular grid is then required to compute the electromagnetic fields with the finite integration technique.

In a regularized inversion, \( m \) is the vector constituted by the logarithm of the resistivity in each cell of the regular inversion grid. The inversion is stabilized with the minimum norm support functional:

\[
R(m) = \frac{\|m - m^{\text{ref}}\|^2}{\beta^2} + \|m - m^{\text{ref}}\|^2, \tag{3}
\]

with \( m^{\text{ref}} \) a reference model, deduced, for instance, from a-priori information. The coefficient \( \beta \) means that a difference between the current model and the reference model larger than roughly \( 3\beta \) is hardly penalized. In our numerical tests, \( \beta = 0.2 \). We also tested the gradient minimum norm support

\[
R(m) = \frac{\|\nabla m\|^2}{\beta^2 + \|\nabla m\|^2}. \tag{4}
\]

3. DEEP-WATER REAL DATA SET

To illustrate the behavior of blocky and regularized inversions, we inverted a data set recorded in deep water offshore Malaysia. The acquisition is a fairly standard line acquisition with 24 receivers spread along 20 km. We only processed the inline electric responses to a towed inline horizontal electric dipole. The geological setting is rather complicated because of large bathymetric variations, the presence of hydrates, an increase background resistivity in the thrusted zone, where the interpreted deep targets sit, and the possibility of stacked resistive layers corresponding to a shallow accumulation and two deeper targets. A structural interpretation based on seismic data is displayed in Figure 1.

![Figure 1: A 3D structural interpretation.](image)

The amplitude of the data at 0.25 Hz normalized by the responses of a constant earth conductivity is plotted in Figure 2. Although a conventional analysis of anomalies provides some hints, the actual complexity of this problem renders this interpretation hopeless. Hence, we applied the isotropic inversion approach using four frequencies at 0.25, 0.75, 2.25, and 6.75 Hz. The initial guess is a constant earth conductivity at 2.0 Ohm-m.

4. BLOCKY INVERSION

Based on the seismic interpretation, see Figure 1, we carried out two blocky inversion. In the first blocky inversion, we did not consider the two possible deep targets. The model then consisted in
6 main blocks. In the inversion, we assumed a constant conductivity in each of the 4 large background layers. However, we allowed variable resistivity in the hydrates and shallow accumulation zones. In the second blocky inversion, the model consisted in 8 main layers. We however allowed variable resistivity in the hydrates, shallow accumulation and deep target zones. The results after 50 iterations are displayed in Figure 3.

Both resistivity models explain the data equally well, roughly up to the noise level. Unfortunately, those results give completely different images in the thrust zone where the deep targets sit, which prevent us from assessing the presence of the deep targets. This illustrates the limitations of a blocky inversion approach with the diffusive electromagnetic inversion. A sensitivity analysis shows the trade-off between the shallow accumulation, the first deep target and the background resistivity of the thrust zone since the depth difference between the shallow accumulation and the first target is of the order of the skin depth at 6.25 Hz. We then have a lack of resolution in the CSEM data and forcing a high resolution results with blocky inversions leads to ambiguous results. The sensitivity analysis also shows that the data are not really sensitive to the deepest target. The blocky inversion has proven to be a powerful tool to cointerpret seismic and CSEM data sets. Unfortunately, it also has the tendency to strongly bias the inversion. Some of the ambiguities may be
resolved when more data, like multiple acquisition lines, broadside data, and multiple components, are included in the inversion.

This blocky approach tends to constrain the inversion too much, especially in the shallow part. We already noticed this drawback as we were unable to successfully invert the data with constant resistivities in the hydrates, shallow accumulation, and target zones.

5. REGULARIZED INVERSION

To avoid the strong constraints of the blocky inversion, we carried out two regularized inversions, the first with the minimum norm support regularization and the initial guess as reference model and the second with the gradient minimum norm support regularization. We started the inversion with a strong regularization and relaxed it during the iterations. The results at iteration 75 are displayed in Figure 4.

![Figure 4: Optimal resistivity model in a logarithm scale of the two regularized inversions after 75 inversions.](image)

Both regularized results explain the data equally well and are, in fact, quite similar. The use of the minimum norm support functional in the regularization term is satisfactory because, at least near the seafloor, the resistivity jumps have not been smoothed. Due to the diffusive nature of the electromagnetic waves the resolution decreases with depth. This is why we preferred the minimum norm support regularization over the classic smoothing regularization term, $\|\nabla m\|^2$, that tends to smooth results. In the final images, we can clearly detect the hydrates with relatively high resistivity and the shallow accumulation. We also notice an increase of resistivity at depth. This increase roughly correlates with the thrust zone of the structural interpretation map, Figure 1.

We also carried out a regularized anisotropy inversion to check the validity of the isotropic inversions. The optimal vertical resistivity, Figure 5, is very similar to the one obtained with the isotropic inversion. We also noticed that the data are not really sensitive to the horizontal resistivity. For a line and deep water acquisition, this result was expected because the inline response to an inline horizontal electric dipole mainly depends on the vertical resistivity. Isotropic inversion gives the vertical resistivity.

6. REGULARIZED PLUS TARGETED BLOCKY INVERSION RESULT

The previous results show that the regularized inversion is able to retrieve roughly the first 700 m below the seafloor with a satisfactory resolution, and indicates a regional increase of resistivity. The
blocky inversion allows us to refine this result using a-priori information. Therefore, we decided to carry out a final partially blocky inversion. Using the optimal regularized inversion result as starting model, we inverted only the thrust region. We considered three main blocks: the background thrust zone and the two deep targets. We impose a constant background resistivity in the thrust zone and the resistivity can freely vary in the deep targets. Outside the thrust region, the resistivity is kept fixed and equal to the one retrieved with the regularized inversion. The final result, Figure 6, indicates an increase of resistivity of about 1 Ohm·m in the thrust zone and an uncharged first target. The result on the second target cannot be interpreted because the data are not sensitive to this target zone.

This result is in agreement with the well log resistivity drilled before this complete analysis and that indicated the presence of hydrates, a shallow accumulation, and an increase of resistivity in the thrust zone.

7. CONCLUSIONS

We have implemented a blocky and regularized CSEM inversion based on a finite-difference solution of the Maxwell/Ohm equations and forward and backward interpolation between the computational grid and the discretization used to represent the model in the inversion. This allows us to take into account a structural interpretation in the inversion. We have illustrated the relevance of the minimum norm support regularization to stabilize the inversion and preserve the resistivity discontinuities at least in the shallow part of the image where the multiple frequency data constrain the inversion relatively well. The use of a priori information is still crucial in the interpretation of
CSEM data. However, we showed that blocky resistivity inversions based on a seismic interpretation can be misleading or at least inconclusive. A careful combination of both regularized and blocky inversions leads to a more robust interpretation workflow as was illustrated with a real deep-water example.

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REFERENCES