Application of a Modified Broyden’s Method in the Finite Difference Method for Electromagnetic Field Solutions

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Abstract — The method outlined in the paper is a modified Broyden’s method that offers an efficient approach to solve the nonlinear equation set obtained as a result of application of the Finite Difference Method (FDM) and is applicable for an anisotropic and nonlinear electromagnetic environment. As the classical Broyden’s method, the new method does not require the determination of the Jacobi matrix elements. An example presented in the paper illustrates the application of the proposed method to nonlinear magnetic circuit configurations described by FDM.

1. INTRODUCTION
The fundamental approach to solving nonlinear equation sets is the Newton-Raphson (Newton’s) matrix method. It requires time-consuming computations of the Jacobi matrix with evaluations of partial derivative functions. In order to avoid this computational expense, the Broyden’s method [1] is used that does not require the evaluation of the Jacobi matrix of partial derivatives. In general the Broyden’s method and its derivatives such as [2] are classified as an extension of the secant method of root finding to higher dimensions.

In the case of nonlinear and/or anisotropic materials, the basic Broyden’s method does not guarantee a successful solution. An alternative method could be the direct application of the simple iterative method that in the case of non-linear magnetic circuits also often fails to converge.

The method outlined in the paper is a modified Broyden’s method that offers an efficient approach to solve the nonlinear equation set obtained as a result of application of the Finite Difference Method (FDM) and is applicable for an anisotropic and/or nonlinear electromagnetic environment [3].

2. THE PROBLEM
The example used to illustrate the application of the modified Broyden’s method as applied to an anisotropic nonlinear magnetic environment is a solenoid consisting of a coil with a ferromagnetic core. There are three different magnetic permeability values in the system: that of the winding, the surrounding air and the ferromagnetic core (nonlinear field dependent function). The nonlinear equation system was created using the Finite Difference Method (FDM). The investigated solenoid’s volume was discretised using a rectangular non-uniform mesh of points. Mixed boundary conditions were used for this case, which were boundary conditions of Neumann and Dirichlet types specified on different subsets of the boundary. Neumann boundary conditions, that assume the continuity of the normal component of the magnetic flux density vector, were used for points situated on the border between environments with different material parameters. For the points far away from the magnetic field source, where the magnetic field is close to zero, Dirichlet boundary conditions were used. The object’s symmetry simplified the analysed configuration to a quarter of the complete system. The simplifying assumptions and resulting equations for the solenoid are presented in [3, 4].

3. THE ALGORITHM
The solution of nonlinear equation sets relies on the calculation of the update $s_i$ in consecutive iterations until a condition to terminate the calculation is met. In the quoted example, calculations have been also conducted by a simplified Newton’s method and by the simple iterative method. In the Broyden’s approach the update $s_i$ for the $i$-th iteration is calculated from Equation (1) [1, 2].

$$D \cdot s_i = -f \left( A^{(i)} \right)$$

(1)
where $f$ is the function describing the magnetic field in a point of a given discretisation mesh of the considered area, and $A$ denotes the magnetic vector potential.

The next iteration value is calculated from the formula:

$$A^{(i+1)} = A^{(i)} + s_i$$  \hspace{1cm} (2)

The matrix $D$ is a matrix that approximates the Jacobi matrix; initially it can be the unity matrix and its next values are calculated from the formula:

$$D_{i+1} = D_i + \frac{f(A^{(i+1)})}{s_i} s_i^T$$  \hspace{1cm} (3)

![Figure 1: The update distribution over the object mesh (one quarter of the object).](image)

The update in Fig. 1 illustrates the behaviour of the Broyden’s method in solving a nonlinear equation set. In this case, the solution is not convergent, but the Broyden’s update can be used to obtain, what we define here as a **useful update** $u$, which is calculated by solving the equation set below.

$$E(A^{(i)}) \cdot u_i = s_i$$  \hspace{1cm} (4)

where $E(A^{(i)})$ is the matrix of arguments, with elements calculated from the last approximation of $A^{(i)}$. The knowledge of the last solution enables us to calculate the respective function values of magnetic reluctivity $\rho(A)$, necessary to evaluate the matrix $E(A^{(i)})$ in the area of the ferromagnetic core. The next approximation of $A^{(i)}$ is calculated from:

$$A^{(i+1)} = A^{(i)} + u_i \hspace{1cm} (5)$$

The proposed new algorithm of the modification of the Broyden’s method can be described as follows.

**Step 1:** Evaluation of the reluctivity vector $\rho_0$, most frequently the same starting value is set for all vector elements.

**Step 2:** Calculation of $E_0$ elements for a given $\rho_0$.

**Step 3:** The equation system below is solved in order to determine the starting iteration point in the form of a vector $A^{(0)}$.

$$E_0 \cdot A^{(0)} = j$$  \hspace{1cm} (6)

where $j$ is the current density in the mesh points (different than zero if in the area of the winding)

**Step 4:** The function value for the current density approximation is calculated.

$$f^{(i)} = f(A^{(i)})$$  \hspace{1cm} (7)
Step 5: The initial matrix $D$ is set to the unity.

Step 6: The Broyden’s update is calculated from the equation system below.

$$D \cdot s_i = -f^{(i)}$$ (8)

Step 7: The reluctance vector $\rho_i$ is calculated from the current approximation.

$$\rho_i = \rho \left(A^{(i)}\right)$$ (9)

Step 8: Values of the matrix $E_i$ are calculated for the given approximation and the calculated vector $\rho_i$.

$$E_i = E \left(A^{(i)}, \rho_i\right)$$ (10)

Step 9: The useful update values $u_i$ are calculated using the equation system:

$$E_i \cdot u_i = s_i$$ (11)

Step 10: The next approximation is calculated.

$$A^{(i+1)} = A^{(i)} + u_i$$ (12)

Step 11: The function values are calculated for a given approximation.

$$f^{(i+1)} = f \left(A^{(i+1)}\right)$$ (13)

Step 12: The condition of the termination of iterations is checked using the square norm (the norm) below.

$$\sum_{j=1}^{w} f_j \left(A^{(i+1)}\right)^2 < \varepsilon$$ (14)

Step 13: If the condition (14) is met, the calculation process is terminated; if not a new matrix $D$ is calculated.

$$D = D + \frac{f^{(i)}s_i^T}{s_i^T s_i}$$ (15)

Figure 2: Distribution of the useful update (the proposed modification) in the first iteration step.
we set:
\[ f^{(i)} = f^{(i+1)} \quad \text{and} \quad A^{(i)} = A^{(i+1)} \]  
and return to the step 6.

In the considered example, calculating the update using the original Broyden’s method does not lead to the convergence. It is necessary to convert the Broyden’s update into the useful update (11). An example of a useful update distribution over the discretisation mesh is shown in Fig. 2. The process is convergent.

4. PARTIAL DERIVATIVES

In order to compare the proposed method with the Newton’s method, the simplified partial derivative formulae of Jacobi matrix have been introduced as shown below.

\[ \frac{\partial f_i}{\partial A_i} = e, \quad \frac{\partial f_i}{\partial A_{i+m}} = e, \quad \frac{\partial f_i}{\partial A_{i-m}} = e, \quad \frac{\partial f_i}{\partial A_{i+1}} = e, \quad \frac{\partial f_i}{\partial A_{i-1}} = e \]  

In (17) \( e, \ e, \ e, e1 \ \text{and} \ e1 \) are calculated for air, core and the horizontal and vertical boundary between the core and air, respectively \[3, 4\].

5. COMPARISON OF RESULTS

Calculations for the given magnetic configuration have been conducted using the simple iteration method, the Newton’s method, with the derivative values calculated from (17), and the proposed modification to the Broyden’s method, for low and high saturation conditions of the magnetic core \[4, 5\].

As expected, for low core flux densities — basically a linear magnetisation case — all three methods deliver solutions at comparable iteration numbers. In the case of the nonlinear magnetisation core conditions the norm values are diversified for the three methods (an example is shown in Table 1).

Table 1: Comparison of the norm for the three methods after 20 iterations

<table>
<thead>
<tr>
<th>Method</th>
<th>The value of the norm (14) after the last iteration</th>
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<tbody>
<tr>
<td>Simple iteration</td>
<td>5.3931 \times 10^9</td>
</tr>
<tr>
<td>Newton’s method</td>
<td>8.3102 \times 10^{10}</td>
</tr>
<tr>
<td>The proposed modified Broyden’s method</td>
<td>3.6472 \times 10^{-9}</td>
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Figure 3: Norm values in consecutive iterations. On the left the Newton’s method, on the right the proposed Broyden’s modification.
The starting point for $A_{s1}$ was calculated for all mesh points and all methods under the assumption that the initial reluctivity of the core is the same as that of air. The simple iteration method and the Newton’ method are non-convergent in this case. In other analysed cases the Newton’s method is convergent but requires more iteration steps that the proposed Broyden’s modified method.

The Fig. 3 and the Table 1 illustrate the convergence process for the same core high saturation case. Both Newton’s and the simple iteration methods (not shown in Fig. 3) fail to convergence and oscillate away from the solution. On the other hand the proposed modified Broyden’s method is convergent after several iterations.

6. CONCLUSIONS

A modification to a classical Broyden’s method that deals efficiently in solving the equations of electromagnetic configurations with material discontinuities (singular points) has been proposed. The proposed method was compared with the simple iteration and the Newton’s methods and demonstrated its superiority in terms of the existing convergence and the required iteration time. Calculations using the modified Broyden’s method for the solenoid were verified by means of the Final Element Method package Opera 3D by VECTOR FIELDS, with comparable results obtained.

REFERENCES