A Novel Design of Photonic Crystal Lens Based on Negative Refractive Index

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Abstract— In this study a concave lens based on two dimensional photonic crystal platform with negative refractive index is presented. The proposed lens is employed as a spot-size converter to facilitate coupling of the light from a single mode fibre with a large spot-size area into photonic crystal waveguide with a very small spot-size area, even smaller than the operating wavelength. Optimisation of the lens and its integration with the single mode fibre and photonic crystal waveguide into a single optical chip was performed by employing 2D Finite-Difference Time-Domain (FDTD). A significant reduction of the optical chip dimensions and high coupling efficiency have been achieved by optimizing each device (the lens and the photonic crystal waveguide).

1. INTRODUCTION

In 1968, Veselago \cite{1} proposed an artificial material whose permittivity and permeability are simultaneously negative. His study demonstrated that the electric and magnetic field vectors create a left-handed set of vectors with the wave vector. These artificial materials with both negative dielectric permittivity and negative magnetic permeability \cite{1, 2} are also known as metamaterials, or left-handed materials (LHM). Veselago \cite{1} first proposed this type of materials which obey Snell’s law with a negative refractive index. Later, it became apparent that such materials can be artificially constructed \cite{2, 3}. In recent years, many research groups around the world have proposed different ideas and suggestions for future applications of these materials, operating at optical and microwave frequencies. In this context, we have proposed a lens based on photonic crystal structure with negative refractive index which operates at optical frequency. Photonic crystal technology can permit strong light confinement in compact structures and can allow for innovative methods for manipulating the guided light. It is already know that conventional lenses are the most widely used to couple the light between various optical components.

However, there are disadvantageous of using conventional lenses since they need curved surfaces to form an image and the associated sub-wavelength alignment tolerances lead to high packaging costs. Also, when using such lenses to couple the light between various optical components, the complexity of the circuitry increases. In particular coupling of the light between conventional waveguides and photonic crystal waveguides remains a challenging issue due to the mismatch of the optical mode widths. In this regard photonic crystal lens based on negative refraction plays a significant role in light focusing and light coupling efficiency.

In this paper, we propose and optimise a photonic crystal lens with negative refractive index ($n = -1$) and its integration in a single optical chip, operating at optical frequency. We study integration of a single mode fibre, a photonic crystal lens, and a photonic crystal waveguide. In this optical chip, the light beam propagating through the single mode fibre enters into the photonic crystal lens where it is refracted and then focused at the focusing point. The photonic crystal waveguide is employed to collect the light at the focusing point.

2. NUMERICAL METHOD

Several computational methods have been employed to numerically characterise optical properties of photonic crystal devices. Waveguide components are commonly analysed by the beam propagation method, but such a method can only treat small angle bends and crossings, and weakly guided structures. In order to avoid such limitations, we have developed and employed the 2D finite difference time domain computational technique based on the Yee’s algorithm \cite{4, 5}. In this study, $E(x, y)$ the electric and $H(x, y)$ the magnetic, field components are calculated. We have considered the 2-D plane where the $z$ direction is normal to the $x$-$y$ plane of the grid. The electromagnetic
fields are calculated by solving the following time-dependent Maxwell’s equations in the FDTD scheme.

\[
\frac{\partial H_x}{\partial t} = -\frac{1}{\mu} \frac{\partial E_z}{\partial y} \quad (1)
\]

\[
\frac{\partial H_y}{\partial t} = \frac{1}{\mu} \frac{\partial E_z}{\partial x} \quad (2)
\]

\[
\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon} \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z\right) \quad (3)
\]

where \(\mu\), \(\varepsilon\), and \(\sigma\) are the permeability, permittivity and conductivity, respectively. In this method, we have considered the transverse magnetic (TM) mode. The TM mode has magnetic field components, \(H_x\) and \(H_y\), perpendicular to the \(z\)-axis, where \(H_z = 0\). Here, in the finite difference time domain scheme we have employed as absorbing boundary the perfect matched layer (PML) introduced by Berenger [6]. The key point of the Berenger PML absorbing boundary conditions is the creation of a non-physical absorber adjacent to the outer grid boundary by splitting the field components and introducing a new degree of freedom. The following spatial steps [7] is considered: \(\Delta x = \Delta y = a/40\) and \(\Delta t = 1/c((1/\Delta x)^2 + (1/\Delta y)^2)^{1/2}\), where \(c\) is the speed of light and \(\Delta x\) and \(\Delta y\) are meshgrid sizes in the \(x\)- and \(y\)-directions, respectively. It is worth mentioning that the finite difference time domain method represents one of the most powerful and computer efficient numerical techniques dealing with photonic crystal electromagnetic problems.

3. SIMULATED RESULTS

The photonic crystal lens considered in this work consists of air holes arranged in a triangular lattice, with a lattice constant \(a\), in a homogeneous dielectric medium. The photonic crystal structure parameters considered in this study are similar as follows: The air hole radius is \(r = 0.4a\) and the dielectric constant of the background is \(\varepsilon = 12.96\). First we performed band structure calculations by using plane wave expansion [8] in order to find out the frequency range where the photonic crystal structure exhibits a negative refractive index. Here we have considered only the TM modes (in-plane magnetic field). Calculated band structure results are presented in Fig. 1.

![Figure 1: Band structure calculation of the lens based on the photonic crystal structure.](image)

![Figure 2: Schematics of the photonic crystal lens.](image)

One can see that this photonic crystal structure in the second band in the range of 0.25 to 0.35 \(a/\lambda\) possesses bands with negative slope. It is already known that photonic crystal structures that own negative band slope [9] exhibit backwards electromagnetic (EM) wave propagation [1]. This backwards type of (EM) wave propagation can lead to negative refraction when the allowed mode distribution in the wave vector space is isotropic at a certain frequency. In such situations with the satisfaction of some additional restrictions specified in Ref. [9] the photonic crystal structure obeys Snell’s law with a negative refractive index. In this study, the operating frequency of the
photonic crystal structure is considered to be equal to $0.305a/\lambda$. At this particular frequency the value of the effective refractive index is negative ($-1$) [10].

Next, we have designed a concave lens by cutting the back-side of the photonic crystal structure into a halfcircular shape with the radius $R$, as shown in Fig. 2. The lens structure dimensions are $25 \times 16a$ (unit cells). In our simulations, we have considered 40 mesh grid points per unit cell. In order to obtain a better impedance matching, the normal to surface direction has been placed along the $\Gamma M$ direction.

In order to achieve high coupling efficiency of a realistic large Gaussian beam source to sub-wavelength size photonic crystal we have optimized the radius of the concave lens, $R$. This radius has been investigated to obtain the smallest focused spot, $S$, at a desired location, where the light can be focused and coupled to other devices.

Next, the incident Gaussian beam with the beam waist $10a$ has been launched at the free space on the left-hand side of the lens, illustrated in Fig. 3. The snapshot of the electric field is also presented in this figure.

On the right-hand side of the lens the incident beam has been focused at the point $P$, as shown in Fig. 3. The operating wavelength is $0.305a/\lambda$. This is considered to be the optimum behaviour of the plano-concave lens since the refractive index is negative ($n = -1$). One can see from this figure that when the light moves beyond the focusing point, $P$, its spot-size area increases rapidly.

Next, we optimised lens where for each value of the radius, $R$, we give the size of the focused spot, defined as the full width at half maximum (FWHM) of the beam intensity along $y$. Our simulations have indicated that the smallest spot-size can be achieved at a normalized radius equal to 2.1. Therefore, this value remained fixed throughout this study. Location of the focusing point $P$ has been determined by calculating the focal length which is direct related to the lens radius of curvature, given by $F = R/(n - 1)$, where $n$ is the refractive index [11]. Our numerical simulations indicated that the best focused spot can been achieved at $F/\lambda = 1.05$, which corresponds to $0.305a/\lambda$. In the case, $R = -2F$, where $R = 2.1\lambda$.

![Figure 3: Snapshot of the electric field patters, when the incident Gaussian beam is launched in free space.](image1)

![Figure 4: Evolution of the electric field from the SMF to the PCW through the PC lens at the frequency $f = 0.315a/\lambda$.](image2)

One can see that there is a small deviation from the value of 1, for the normalized focal length. This is due to the anisotropy in the mode dispersion in wave vector space. The radius of curvature is equal to $R = -2f$ for a photonic crystal with a negative refractive index equal to $-1$; however, this radius would be shorter in case when the refractive index is positive.

Next, the lens has been placed between the single mode fibre and an ordinary photonic crystal waveguide (PCW), as shown in Fig. 4. The PCW consists of a triangular lattice of dielectric cylinders placed in the air with $\varepsilon = 12.96$ and the radius of $r = 0.2a'$, where $a'$ is the lattice constant [26].

The propagation of the light from the SMF through the lens into the PCW is illustrated in Fig. 4. One can see that, the light is well confined in the PCW. It can be seen that compact integration of SMF, lens and PCW can be achieved by focusing the spot at the desired locations. Next, we investigate the effect of the lens in the light coupling efficiency. Our simulation indicated that in
the case when the lens is not used almost 90% of the optical power is lost. On the other hand, a maximum coupling efficiency of about 95% is achieved at the normalised frequency \(a/\lambda = 0.312\) when the lens is used, as shown in Fig. 5. In this figure variation of the coupling efficiency as a function of normalized frequency, \(a/\lambda\), when the lens is used is illustrated.

![Figure 5: Variation of the coupling efficiency as a function of the normalised frequency.](image)

One can see that the normalised frequency at this point is slightly different from the optimum focusing frequency of \(0.305 a/\lambda\). This is due to a slight mismatch between the frequency for the optimum performance of the lens, and the optimum performance for the PC waveguide alone. The study presented in this paper opens up the possibility to reduce the optical chip footprint to a few microns. This optical chip design can be significantly important for the effective integration of various optics devices in a very small area.

4. CONCLUSIONS

We have proposed a novel model of a lens which serves as a spot-size converter and is based on a twodimensional photonic crystal platform with a negative refractive index. A numerical method based on twodimensional Finite-Difference Time-Domain is developed and employed to design the photonic crystal lens. The configuration geometry of the PC lens is designed, optimized, and integrated into a single optical chip. The proposed PC lens is deployed to effectively couple the light from a SMF with large core size into a PCW with very small structure dimensions. A significant reduction in the device compactness and coupling efficiency is demonstrated by optimizing the proposed PC lens and the PCW.

REFERENCES

