Propagation of Partially Coherent Beams after a Source Plane Ring Aperture

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Abstract—The propagation properties of partially coherent beams passing through a source placed ring aperture are examined. The derivation is based on the lowest order general beam formulation, such that our results are applicable to a wide range of beams. In this study, our focus is on fundamental Gaussian, cosh-Gaussian, cos-Gaussian, sinh-Gaussian, sine-Gaussian and annular beams. The aperture consists of inner and outer parts, thus the middle hollow part appears in the form of a ring. The propagation environment is turbulent.

From the graphical outputs of the beams investigated, it is seen that despite the existence of the circular ring, during propagation, the beams tend to retain the basic profiles similar to the case of no aperture, but depending on the inner and outer radius dimensions, the propagated beams are reduced in intensity levels and become more spread. It is further observed that, when the inner part of the aperture has nonzero radius, ring formations are developed at the outer edges of the receiver plane intensities.

1. INTRODUCTION
In practical optical communication equipment, an aperture confinement of some type, usually a circular shape, is readily built into the transmission apparatus. Here we also introduce an inner part so that the eventual configuration is in the form of a ring consisting of an inner and outer radius.

In literature, many sources have investigated the propagation of laser beams with a source aperture confinement, usually coupled with the representation of the propagation environment as ABCD matrix. In these works, the propagation medium is mostly taken to be free space, rather than turbulent atmosphere. A collection of such studies is found in [1–8].

Recently there has also appeared a study concerning the propagation of beams in turbulent atmosphere incorporating a source aperture confinement [9].

Our goal in this paper has been to extend the aperture treatment to the case of selected general beams. Hence, this study is aimed at the in-turbulence propagation analysis of beams passing through a source aperture configuration. In particular, within the context of general beam, our graphical results cover cosh-Gaussian, sine-Gaussian and annular beams.

2. FORMULATION
As shown in [10], the source field of most popular beams contains a summation of two terms. In radial coordinates, the mutual coherence function of partial coherent version of such beams is given as

\[ \Gamma \left( s_1, s_2, \phi_{1s}, \phi_{2s} \right) = \exp\left\{ -\frac{0.25}{\sigma_s^2} \left[ s_1^2 + s_2^2 - 2s_1s_2 \cos(\phi_{1s} - \phi_{2s}) \right] \right\} \]

\[ \sum_{t_1=1}^{2} \sum_{t_2=1}^{2} C_{t_1}^* C_{t_2} \exp\left\{ -\left[ k\alpha_{t_1}s_1^2 + jV_{t_1}s_1(\cos \phi_{1s} + \sin \phi_{1s}) \right] \right\} \]

\[ -\left[ k\alpha_{t_2}s_2^2 - jV_{t_2}s_2(\cos \phi_{2s} + \sin \phi_{2s}) \right] \]  

\[ \sum_{t_1=1}^{2} \sum_{t_2=1}^{2} C_{t_1}^* C_{t_2} \exp\left\{ -\left[ k\alpha_{t_1}s_1^2 + jV_{t_1}s_1(\cos \phi_{1s} + \sin \phi_{1s}) \right] \right\} \]

\[ -\left[ k\alpha_{t_2}s_2^2 - jV_{t_2}s_2(\cos \phi_{2s} + \sin \phi_{2s}) \right] \]  

(1)

\( s_1, s_2, \phi_{1s} \) and \( \phi_{2s} \) refer to the two distinct source coordinates, \( \sigma_s \) is the partial coherence parameter. \( k = 2\pi/\lambda \) is the wave number with \( \lambda \) being the wavelength, \( \alpha = 1/(\alpha_0^2) + j/(2F_0) \) where \( \alpha_0 \) and \( F_0 \) respectively indicate radial Gaussian source size and focusing parameter. Via the amplitude
parameters \( C \), displacement parameters \( V \) and \( \alpha \) together with their \( t_1 \) and \( t_2 \) subscripts, it is possible to configure the general beam so that it will deliver fundamental Gaussian, cosh-Gaussian, 

cos-Gaussian, sinh-Gaussian, sine-Gaussian and annular beams [11]. We assume that prior to propagation, the source given by Eq. (1) passes through a ring aperture with internal radius of \( a \) and outer radius of \( b \) as shown in Figure 1.

The average intensity falling on a receiver plane which is \( L \) distance apart from the source plane, can be calculated in terms of the Huygens-Fresnel integral as shown below [10]

\[
\langle I_r(r, \phi_r) \rangle = \frac{k^2}{(2\pi L)^2} \int_{0}^{b/2\pi} \int_{0}^{2\pi} ds_1 ds_2 d\phi_1 d\phi_2 \Gamma_s(s_1, s_2, \phi_1, \phi_2) s_1 s_2 
\times \exp \left\{ \frac{jk}{2L} \left[ -2rs_1 \cos(\phi_r - \phi_1) + s_1^2 + 2rs_2 \cos(\phi_r - \phi_2) - s_2^2 \right] \right\} 
\times \exp \left\{ \frac{-1}{\rho_0^2} \left[ s_1^2 + s_2^2 - 2s_1 s_2 \cos(\phi_1 - \phi_2) \right] \right\} 
\tag{2}
\]

In Eq. (2), the diffraction phenomena is expressed by the first exponential, while the second exponential arises due to turbulence. In this second exponential, \( \rho_0 \) is the coherence length of a spherical wave propagating in the turbulent medium and under the conditions of Kolmogorov spectrum and quadratic approximation, given by \( \rho_0 = (0.545 C_n^2 k^2 L)^{-3/5} \), where \( C_n^2 \) refractive index structure constant expressing the turbulence strength. Finally \( r \) and \( \phi_r \) refer to the radial coordinates on the receiver plane.

Upon substituting for \( \Gamma_s(s_1, s_2, \phi_1, \phi_2) \) in Eq. (2) from Eq. (1), the integration in Eq. (2) can be solved with respect to \( s_1 \) and \( \phi_1 \) in the manner described in [12–14], leaving the following double integral.

\[
\langle I_r(r, \phi_r) \rangle = \frac{0.25k^2}{\pi L^2} \sum_{t_1=1}^{2} \sum_{t_2=1}^{2} \sum_{t=0}^{\infty} \frac{(0.25)^t}{(t!)^2} C_{t_1} C_{t_2} \left( k\alpha_{t_1} - \frac{jk}{2L} + 1 \right) + \frac{0.25}{\sigma_s^2} \left( t+1 \right) \sum_{p=0}^{2t} \frac{(2t)!!}{2^{p}(2t-2p)!!} 
\times \left\{ \left[ \left( k\alpha_{t_1} - \frac{jk}{2L} + 1 \right) + \frac{0.25}{\sigma_s^2} \right] a^2 \right\}^{t-p} \exp \left\{ - \left( k\alpha_{t_1} - \frac{jk}{2L} + 1 \right) + \frac{0.25}{\sigma_s^2} \right\} a^2 
\times \left\{ \left( k\alpha_{t_1} - \frac{jk}{2L} + 1 \right) + \frac{0.25}{\sigma_s^2} \right\} \left\{ b^2 \right\}^{t-p} \exp \left\{ - k\alpha_{t_1} - \frac{jk}{2L} + 1 \right\} \left\{ \sigma_s^2 \right\} b^2 
\times \int_{0}^{b/2\pi} \int_{0}^{2\pi} ds_2 ds_2 d\phi_2 d\phi_2 \exp \left\{ - k\alpha_{t_2} s_2^2 - jV_{t_2}^* s_2 (\cos \phi_2 + \sin \phi_2) \right\} 
\exp \left\{ \frac{jk}{2L} \left[ 2rs_2 \cos(\phi_r - \phi_2) - s_2^2 \right] \right\} 
\times \exp \left\{ - s_2^2 \left( \frac{1}{\rho_0^2} + \frac{0.25}{\sigma_s^2} \right) \left[ -2V_{t_1}^2 \frac{k^2 r^2}{L^2} + 4s_2^2 \right( 1 \right\} \frac{0.25}{\sigma_s^2} - 2V_{t_1} \frac{k r}{L} \cos(\phi_r + \sin \phi_r) \right\} 
\times \exp \left( \frac{4jV_{t_2} s_2 + jV_{t_2} \cos \phi_2 s_2 - \left( \frac{4jkr s_2}{\rho_0^2} + \frac{jkr s_2}{(L \sigma_s^2)} \right) \cos(\phi_r - \phi_2) + \frac{2s_2^2}{\rho_0^2 \sigma_s^2} \right)^t \right\} 
\tag{3}
\]

where \((n)!! = 1 \times 3 \times \ldots (n-1) \) or \((n)!! = \) \(2 \times 4 \times \ldots (n-1) \) depending on whether \( k \) is odd or even. As explained in [12, 13], it does not serve much use to continue analytically with Eq. (3), thus it is kept in the present form. Instead the routine mentioned in [12, 13] is employed for relatively rapid evaluation.

\section{Results and Discussions}

Although, the formulation in Eq. (3) can support a variety of beams, here due to space limitations, we restrict our results to cosh-Gaussian, sine-Gaussian and annular beams. Thus, in this section the intensity plots are offered for cosh-Gaussian, sine-Gaussian and annular beams, while they propagate in turbulent atmosphere after having passed though the circular aperture of the source.
plane. To permit cross-comparisons between the different beam types, for the cosh-Gaussian and sine-Gaussian beams, we use the common settings of, $\alpha_0 = 1$ cm, $F_0 \to \infty$, $V = 200$ m$^{-1}$, $\sigma_s \to \infty$, $C_n^2 = 10^{-15}$ m$^{-2/3}$. This means that collimated and coherent beams are examined in this study. For annular beams on the other hand, the above settings are modified in the following way, $\alpha_{01} = 1$ cm, $\alpha_{02} = 0.8$ cm, $V = 0$. Here $\alpha_{01}$ and $\alpha_{02}$ consecutively refer to primary and the secondary source sizes. The individual intensity illustrations are arranged to coincide with propagation lengths of $L = 0, 1, 2, 5$ km. The intensity graphs are drawn on the Cartesian transverse planes, where $s_x$, $s_y$ and $r_x$, $r_y$ will respectively correspond to the Cartesian counterparts of the source and receiver radial coordinates $s$, $\phi_s$ and $r$, $\phi_r$ encountered in Eqs. (1) and (3). The vertical axes of the plots are labelled as $I_{SN}$ or $< I_r N >$ meaning that all intensity levels are normalized with respect to the peak value of the source intensity obtained from Eq. (1) by setting $I_s(s, \phi_s) = \Gamma_s(s, s, \phi_s, \phi_s)$. It is

Figure 1: General layout of source and receiver planes and the source ring aperture configuration.

Figure 2: Progress of an cosh-Gaussian beam along propagation axis with aperture ring dimensions of $a = 0$, $b = 2$ cm.
worth pointing out that in order to obtain a problem-free (or reliable) intensity profile, as many as 26 terms had to be included in the most inner summation appearing in the first line of Eq. (3).

Figure 2 shows the progress of a cosh-Gaussian beam along the propagation axis with the radius of interior ring, \(a = 0\), that of the outer ring, \(b = 2\) cm. Judging by the source intensity plot (Plot 1), it is seen that an outer aperture radius of \(b = 2\) cm allows almost all the source intensity to be emitted from the source plane. In this manner, when Figures 2 and 3 are compared, where the latter shows the propagation of the same cosh-Gaussian beam with the outer radius of the aperture extending to infinity, hardly any noticeable difference is found. A closer joint examination of Figures 2 and 3 will reveal that, the finite aperture (i.e., Figure 2) nevertheless imposes more spreading on the beam, whilst also lowering its peak amplitudes. This is evident from the comparison of Plots 2, 3 and 4 of Figures 2 and 3.

In Figure 4, we introduce a finite interior part, that is \(a = 1\) cm, for the source aperture of the cosh-Gaussian beam. From Figure 4, it is seen that the existence of the nonzero inner part gives rise to the emergence of bigger side lobes, formation of outer rings and more beam spreading. It is interesting to note from Figures 2 and 4 that, the presence of an aperture at the source plane does not hinder the beam evolution phases. That is, the evolutions discussed and demonstrated in [15] for the unapetured propagation of cosh-Gaussian beam are valid here as well. Consequently even with an aperture confinement, as observed from Figures 2 and 4, the cosh-Gaussian beam will still transform into a cos-Gaussian beam upon propagation.

In Figure 5, the progress of the sine-Gaussian beam is displayed for and aperture configuration of \(a = 0, b = 1\) cm. Figure 5 proves that, this particular sine-Gaussian beam does not seem to be affected by the presence of the aperture, such that it conveniently transforms into a sinh-Gaussian beam after propagation. Considering the outer aperture size of 1 cm in relation foot-print of source beam from Plot 1 of Figure 5, it easy to see that an aperture opening of 1 cm is able to transmit the whole source beam intensity. Figure 6 shows the sine-Gaussian beam propagation for the case of \(a = 1\) cm, \(b = 2\) cm. According to Figure 6, sine-Gaussian beam will again yield a sinh-Gaussian beam after propagation, but due to the existence of the inner part of the aperture, outer rings will be formed as well.
Figure 4: Progress of a cosh-Gaussian beam along propagation axis with aperture ring dimensions of $a = 1$ cm, $b = 2$ cm.

Figure 5: Progress of a sine-Gaussian beam along propagation axis with aperture ring dimensions of $a = 0$, $b = 1$ cm.
Figure 6: Progress of a sine-Gaussian beam along propagation axis with aperture ring dimensions of $a = 1$ cm, $b = 2$ cm.

Figure 7: Progress of an annular beam along propagation axis with aperture ring dimensions of $a = 1$ cm, $b = 2$ cm.
Finally we provide the illustration of annular beam. To this end, Figure 7 shows that, when a source aperture confinement of $a = 1\, \text{cm}$, $b = 2\, \text{cm}$ is used, annular beam will generate an outer ring of a complete circle in addition to the central peak. This behaviour is similar to unapertured situations [16], except that the outer ring persists for greater distances.

4. CONCLUSION

We have analyzed the effect of a ring aperture placed on the source plane of the general beam propagating in turbulent atmosphere. From the illustrations presented, it is observed that the aperture does not have a major effect on the beam profile provided that its outer radius is in the range of beam foot-print and has no interior part. When the ring aperture contains an inner part however, it causes somewhat excessive spreading and the formation of outer rings in the received intensity profiles.

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