Field Propagation in Nanoporous Metal Waveguides

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Abstract—In this paper propagation of electromagnetic fields in a structure with regular arrays of pores in metal is considered. This structure is analyzed in a similar way as the arrays of dipoles. Now in this case, the surrounding medium is metal while the inclusions are empty spheres. The inclusions resonate at the certain frequency making the interaction between the inclusions very strong. The linear chain of such resonating particles forms a waveguide. Although the field is decaying outside the pores, these inclusions are so close to each other that there is interaction with the neighbouring pores. Near the resonance this interaction is strong enough that there exist guided wave modes along the array. Properties of these modes are investigated.

The allowed frequency range where the guided modes exist depend on the geometry, i.e., how close the pores are and what is the size of pores. There exist two propagating modes, transversal and axial polarizations. The transversally polarized field propagates as a forward wave and the axially polarized field as a backward wave.

1. INTRODUCTION

In recent years a lot of attention is paid for metal nanoscale structures because of new phenomena and potential applications for waveguide and antenna techniques. Especially in optical region new effects arise based on plasmon resonances. Optical properties of metals with nanopores are now of particular current interest [1–3]. Remarkable optical effects caused by excitation of plasmons in nanoporous metals such as extraordinary transmission [4] and total absorption of light [5] have been predicted theoretically. In [1] strong resonant dips in the reflectivity spectra of light were observed from nanoporous metal surfaces formed by periodic arrangements of close-packed spherical voids. In a recent paper [5] it was theoretically shown that this effect is caused by excitation of plasmons in voids (void plasmons). Arrays of regular resonant inclusions are theoretically studied in [6, 7].

In this paper we consider propagation characteristics in a wave guide structure which consists of regular linear array of pores in plasma. An analogous structure has been considered in optics by Weber and Ford [8] where a chain of silver nanospheres are operating at frequencies near the plasmon resonance of an individual sphere. If the spheres are polarized transversally with respect to the chain axis, and the chain is dense, there appears a frequency band in which each frequency corresponds to two modes: the forward mode and the backward mode. At the upper edge of this frequency band both these solutions join one another that correspond to zero group velocity but nonzero phase velocity. In the vicinity of this frequency the dispersion curve is almost flat that makes this structure suitable for a subwavelength imaging. In [9] similar dispersion was found in two coupled chains of silver spheres and it was proposed to use this effect for obtaining subwavelength images.

We expect that localized void plasmons in a thin layer of nanopore metal, interacting with surface plasmons, can be used instead the chain of metal spheres. In this paper we study wave propagation in the chain of spherical holes in a bulk metal using approximation of point-like dipoles as in [8, 9]. Actually the studied case is a dual problem to the chain of metal spheres in air and we compare features of spectra of modes for these two cases.

2. FIELD EXPRESSIONS

We consider metal structure with regular arrays of pores. The geometry is illustrated in Figure 1. This structure can be analyzed in a similar way as the arrays of dipoles. Now in this case, the surrounding medium is metal while the inclusions are spherical holes. Making a linear chain of such particles we have a waveguide. In optical region metals behave almost as a lossless plasma. In the beginning we model the spheres as dipoles with the polarizability α. In the expression of the polarizability it is seen that the inclusions resonate at the frequency where the relative permittivity of the surrounding medium is \( \epsilon_r = -\frac{1}{2} \).
We consider spherical empty cavities in isotropic plasma. The polarizability of the sphere in plasma is
\[ \alpha = \frac{3\epsilon_h(\epsilon_s - \epsilon_h)}{\epsilon_s + 2\epsilon_h} V \]
where the host medium is plasma \( \epsilon_h = \epsilon_o \epsilon_r \)
\[ \epsilon_r = 1 - \frac{\omega_p^2}{\omega^2} \]
and the permittivity inside the sphere is \( \epsilon_s = \epsilon_o \). The volume of the sphere is \( V = \frac{4\pi r_o^3}{3} \).

We are looking for propagating fields in a region where the permittivity of the host medium is negative which is the case below the plasma frequency. In the frequency range below the plasma frequency after inserting the permittivities into the expression for the polarizability we can write the inverse of the polarizability in the following form
\[ \frac{1}{\alpha} = \frac{1}{4\pi \epsilon_o r_o^3} \left( \frac{2|\epsilon_r| - 1}{|\epsilon_r|(1 + |\epsilon_r|)} \right) \]
The dipole moment of the pore is
\[ \vec{p} = \alpha \vec{E} \]
where \( \vec{E} \) is the local field. In a chain of dipoles the local field is obtained from the field expression for dipoles
\[ \vec{E} = \frac{1}{4\pi \epsilon} \left[ k^2 (\vec{u} \times \vec{p}) \times \vec{u} \frac{e^{-jkr}}{r} + [3\vec{u}(\vec{u} \cdot \vec{p})] - \frac{1}{r^2} \left( 1 + \frac{jk}{r^2} \right) e^{-jk \vec{r}} \right] \]
where \( \vec{u} \) is the unit vector indicating the direction from source dipole to the reference point.

In a chain the fields are decomposed into transversal and axial components. From the above field expression we can identify the interaction components for transversal and axial polarizations. The interaction dyadic is
\[ \vec{\beta} = \beta_\perp I_t + \beta_\parallel \vec{\pi}_z \vec{\pi}_z \]
where the interaction coefficients are
\[ \beta_\perp = \frac{1}{4\pi \epsilon} \left[ \frac{k^2}{|z - z'|} - \frac{1}{|z - z'|^3} - \frac{jk}{(z - z')^2} \right] e^{-jk|z - z'|} \]
\[ \beta_\parallel = \frac{1}{2\pi \epsilon} \left[ \frac{1}{|z - z'|^3} + \frac{jk}{(z - z')^2} \right] e^{-jk|z - z'|} \]
Taking the reference dipole at \( z' = 0 \) and \( x' = 0 \), and because the structure is periodic in \( z \) direction \( (z = na) \), the dipole moments of each dipole can be written according to the Floquet theorem
\[ \vec{p}(n) = \vec{p}(0) e^{-j\pi n a} \]
The local field at the reference point is caused by all the dipoles in the array
\[ \vec{E} = \sum_n \vec{\beta} : \vec{p}(n) \]
In the summation the reference point, \( n = 0 \) is omitted.

In the frequency range below the plasma frequency the permittivity is negative and the wave number is imaginary \( k = -j|k| \). This means that the fields are decaying and only the neighbouring spheres interact. This effect is seen in the expressions of the interaction coefficients which are exponentially decaying functions. The total interaction coefficients for the two eigenpolarizations are

\[
C_{\perp} = \frac{1}{2\pi \epsilon_o a^3 |\epsilon_r|} \sum_{n=1}^{\infty} \left( \frac{k^2 a^2}{n} + \frac{1}{n^3} + \frac{|k| a}{n^2} \right) e^{-|k|na \cos q na}
\]

\[
C_{\parallel} = -\frac{1}{\pi \epsilon_o a^3 |\epsilon_r|} \sum_{n=1}^{\infty} \left( \frac{1}{n^3} + \frac{|k| a}{n^2} \right) e^{-|k|na \cos q na}
\]

3. DISPERSION RELATIONS

The eigenvalue equation for solving the propagation factor \( q \) along the array is obtained combining Equations (4), (9) and (10) which leads to two equations of the corresponding polarizations

\[
C_{\perp} = \frac{1}{\alpha}, \quad C_{\parallel} = \frac{1}{\alpha}
\]

Because the components in the summation of the interaction coefficients are fast decaying functions we can approximate the infinite summation by taking only the first term. This approximation gives us an analytic solution which is quite close to exact one

\[
\cos q_{\perp} a \approx \left( \frac{a}{r_o} \right)^3 \left( 1 - 3 \frac{\omega^2}{\omega_p^2} \right) e^{i|k|a} \frac{1}{1 + |k|a + |k|^2 a^2}
\]

and

\[
\cos q_{\parallel} a \approx \left( \frac{a}{r_o} \right)^3 \left( \frac{3}{2} \frac{\omega^2}{\omega_p^2} - 1 \right) e^{i|k|a} \frac{2(1 + |k|a)}{2(1 + |k|a)}
\]

In these results we can conclude that there exist guided waves in the frequencies close to \( \sqrt{\frac{2}{3}} \omega_p \).

In this special frequency the inclusions resonate, and it corresponds to the special permittivity value, \( \epsilon_r = -\frac{1}{2} \) of the medium. The other observation is that because the right-hand side of the above equations are of opposite sign, one of these eigenwaves is a forward wave and the other one is a backward wave. In Figure 2 are illustrasted the dispersion curves for transversally and axially polarized fields. These solutions are calculated using exact expressions of interaction coefficients.

Figure 2: Dispersion curves calculated for spheres \( a/r_o = 3 \).
It is seen that the transversally polarized field is the forward wave and the axially polarized field is the backward wave. These results can be compared to the results of the dual problem, regular array of plasma spheres in free space. In [8,9] it is shown that in the line of metal spheres the axially polarized field is a forward wave and the transversely polarized field has a backward wave component. Also the resonance frequencies are different, in our case \( \omega_r = \sqrt{\frac{2}{3}} \omega_p \) and in [9] \( \omega_r = \omega_p / \sqrt{3} \).

In Figure 3 the effect of geometry for the band width is illustrated. The dispersion curves are calculated for fixed period with different pore radius. For larger relative radius of pores the allowed frequency range for guided waves is larger than for small radius values.

![Figure 3: Dispersion curves, calculated for different ratios \( a/r_o = 4, a/r_o = 3 \) and \( a/r_o = 2.5 \).](image)

### 4. CONCLUSION

The dispersion characteristics of regular arrays of pores in plasma are considered. The structure is analyzed in a similar way as arrays of dipoles, now the surrounding medium is plasma and the inclusions are empty spheres. The inclusions resonate at the frequency where the relative permittivity of the plasma is \(-1/2\). Although the field is decaying outside the pores, these pores are so close to each other that there exists interaction with the neighbouring pores, which is strong enough near the resonance that there are propagating guided modes along the array. The linear array of such inclusions forms a waveguide whose dispersion relation is calculated. It is found that the transversely polarized field propagates as a forward wave and the axially polarized field as a backward wave. The effects of geometry for dispersion characteristics of propagating waves are also studied. The band width is broader when the inclusions are closer to each other and the interaction is stronger.

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### REFERENCES


