The Performance of QSP Beamformer with Array Errors

Shuangning Shi¹, Yong Shang¹, Qinglin Liang¹, and Bin Liang²

¹School of Electronics Engineering and Computer Science, Peking University, 100871, China
²School of Electronics Engineering and Computer Science, Shihezi University, 832003, China

Abstract—There are errors and other uncertain factors in array processing, so the practical steering vectors deviate from the ideal steering vectors, which results in descent of the performance of the conventional beamformer. QSP beamformer is a robust beamformer based on the subspace approaches and it can achieve stable performance. The paper mainly discusses the performance of the QSP beamformer with array errors. The results of the simulation show that the effectiveness and output performance of the QSP beamformer is much better than that of the conventional beamformer.

DOI: 10.2529/PIERS061007112858

1. INTRODUCTION

Array signal processing is widely used in the field of radar, communications, sensor, medicine and so on. Beamforming [1, 2] is an important application of the array signal processing. Conventional beamforming can choose the optimum weight vectors and achieve the minimum-energy output without loss of the expected signal. In the condition of no error for the array, the conventional beamformer, such as MVDR Beamformer, has strong resolving power and high capability of disturbance rejection. In practical, the steering vector of the expected signal can not be accurately known, and the practical steering vector is not equal to the ideal steering vector, and then the performance of conventional beamformers, especially the output SINR of the array, will be degraded obviously [3–5]. To solve this problem, QSP beamformer, which is based on Quantum Signal Processing framework, is presented in this paper. The beamformer which is based on the subspace approaches is not sensitive to errors in the array and could obtain stable output performance no matter array errors exist or not.

The remainder of this paper is organized as follows. First we propose the basic signal model of array processing and mathematical expressions of the new beamformer. Then the simulations of QSP beamformer are carried out and the results are analyzed respectively. Finally, concluding remarks are drawn.

2. SIGNAL MODEL

Assume that there are J narrow-band stationary signals in far field \{\theta_j(t), j = 1, 2, \ldots, J\}, and the DOAs of the signals are \{\theta_j, j = 1, 2, \ldots, J\} respectively. At the receiver antenna array is composed of M elements. Suppose that the additive white Gaussian noises at each array element are \{e_k(t), k = 1, 2, \ldots, M\} with the same variance \sigma_e^2, then the steering vector of the signal whose DOA is \theta_j can be denoted as \mathbf{a}(\theta_j) = [1, \exp(-j\pi \sin(\theta_j)), \ldots, \exp(-j\pi(M - 1) \sin(\theta_j))]^T. The received signal of the kth array element can be denoted as follows:

\[ x_k(t) = \sum_{j=1}^{J} a(\theta_j) s_j(t) + n_k(t) \]  

(1)

where K is the number of the snapshots, k = 1, \ldots, K; \mathbf{n}(t) = [n_1(t), \ldots, n_M(t)]^T.

In real environment, there are lots of errors in the receiver, such as the errors caused by array band response and errors which lie in the amplitude-phase characteristics of the elements. The errors make the practical steering vector different with the ideal steering vector. To simplify the condition, we denote all the errors as a Gaussian random vector \mathbf{a}_e, which is an addition for the ideal steering vector. The mean of \mathbf{a}_e is 0 and the standard deviation is \sigma_e^2. So the practical steering vector is \mathbf{a}(\theta_j) + \mathbf{a}_e, and the practical sample data of the receiver after k snapshots can be denoted as

\[ \mathbf{x}_k(t) = \sum_{j=1}^{J} (\mathbf{a}_k(\theta_j) + \mathbf{a}_e) \mathbf{s}_j(t) + \mathbf{n}_k(t) \]  

(2)
3. QSP BEAMFORMER

The random vector errors mentioned above is mainly caused by the inaccurate steering vector $\mathbf{a}(\theta_j)$. The QSP beamformer based on signal subspace is proposed in [6] by the author. The new beamformer can overcome the unstable output performance caused by the inaccurate steering vector because it only needs the knowledge of the ideal DOAs.

The main principles of the QSP beamformer can be explained as follows: Assume that there are $J$ steering vectors $\{\mathbf{a}(\theta_j), 1 \leq j \leq J\}$ in Hilbert space $\mathcal{H}$, when a group of vectors $\{\mathbf{a}(\theta_j), 1 \leq j \leq J\}$ are given, we can construct a group of vectors $\{\mathbf{w}(\theta_j), 1 \leq j \leq J\}$ whose element is orthogonal to each other and which approach to the given group of vectors $\{\mathbf{a}(\theta_j), 1 \leq j \leq J\}$ under Least-Square constraint as close as possible. Furthermore, we introduce different impact factors $\{q_j, 1 \leq j \leq J\}$ for different signals, which satisfy $q_1 + q_2 + \ldots + q_J = 1$, then the vectors $\{\mathbf{w}(\theta_j), 1 \leq j \leq J\}$ should have the following equation minimum [7]:

$$
\varepsilon_{LS} = \sum_{j=1}^{J} q_j \left( \mathbf{a}(\theta_j) - \mathbf{w}(\theta_j), \mathbf{a}(\theta_j) - \mathbf{w}(\theta_j) \right)
$$

Meanwhile subject to:

$$
\left( \mathbf{w}(\theta_j), \mathbf{w}(\theta_{j'}) \right) = c^2 \delta_{jj'}, j \neq j'
$$

where $c$ is a constant value greater than zero, $\delta_{jj'}$ indicates that the vectors should be orthogonal to each other and the constant $q_j$ denotes the impact factor of the $j$th signal. After calculation we can get:

$$
\mathbf{w}(\theta_j) = c\mathbf{a}(\theta_j)\mathbf{Q}\left((\mathbf{Q}^*\mathbf{A}\mathbf{Q})^{1/2}\right)^{\dagger} \{1 < j < J\}
$$

where $(\cdot)^\dagger$ denotes the Moore-Penrose pseudo-inverse of $(\cdot)$ and $\mathbf{Q}$ is a diagonal matrix $\mathbf{Q} = \text{diag}(q_1, q_2, \ldots, q_J)$.

According to Equation (5), different performance outputs can be obtained when different impact factors chosen. For the signal of which the DOA is $\theta_j$, if the value of $q_j$ is larger, $\mathbf{w}(\theta_j)$ is closer to the steering vector $\mathbf{a}(\theta_j)$, which means we can get high-SNR output after the signal passed the beamformer. This approach is suitable to the high noise situation. If the value of $q_j$ is smaller, $\mathbf{w}(\theta_{j'})$ will be closer to the steering vector $\mathbf{a}(\theta_{j'})$ because $q_{j'}$ gets larger. From Equation (5) we know that $\mathbf{w}(\theta_{j'})$ is orthogonal to $\mathbf{w}(\theta)$, so $\mathbf{w}(\theta_j)$ is nearly orthogonal to the steering vector $\mathbf{a}(\theta_{j'})$, and then we can get high-SNR output after the signal passed the beamformer. This approach is applied under the situation with high disturbance. Therefore the novel beamformer can achieve the optimal output gain and high SINR by configuring different impact factors flexibly under different circumstances.

4. SIMULATION RESULTS AND ANALYSIS

4.1. Simulation Situation

We utilize a four-element Uniform Linear Array (ULA) with the elements separated by half-wavelength interval and assume that the expected signal comes from the direction of 0 degree, and an interference signal comes from the direction of 15 degree, and SIR is 0 dB. The impact factor $q_1$ is given with 0.1. MVDR beamformer and QSP beamformer are adopted in the following simulation.

Simulation 1: In Fig. 1 curves, MVDR and QSP show the outputs of the two beamformers separately in the situation that the DOA of the signal is accurately known and there are no errors existing in the steering vector. We can see from Fig. 1 that the output performance of MVDR beamformer is good when the DOAs of all signals are stable and accurately known. As the SNR gets higher, the performance of the QSP beamformer is much worse than that of the MVDR beamformer. The curves MVDR R and QSP R in Fig. 1 show the outputs of the two beamformers separately in the situation where the DOAs of the signals have errors. The variance $\sigma_n^2$ is 0.01. It is obvious that the performance of MVDR beamformer is rapidly degraded when the errors exit in the steering vectors, especially when the SNR is high. Meanwhile, the performance of the QSP beamformer is only degraded slightly, and it is more stable than the performance of the MVDR beamformer.

Simulation 2: The parameters of this simulation are the same as that in Simulation 1 except that the $\sigma_n^2$ changes to be 0.1. In Fig. 2 curves, MVDR R and QSP R show the outputs of the
two beamformers separately in this situation. Compared with Fig. 1, the output SINR of the QSP beamformer is also more stable than that of MVDR beamformer when the variance of the error in the steering vector increases, but the output performance of the QSP beamformer is degraded slightly.

In a word, in the ideal situation, MVDR beamformer can achieve optimum output performance, while in practical situation various errors make the performance of MVDR beamformer degraded, and some errors, such as random errors of the steering vector, will badly deteriorate the performance of MVDR beamformer. However QSP beamformer can overcome the effect of the errors and achieve stable output performance.

5. CONCLUSION
In real communication system, random errors on the steering vector will badly affect the output performance of conventional beamformer. In this situation, QSP beamformer can achieve stable output performance when the errors are limited in a certain range. In a word the new beamformer is robust and can be applied in the environment mentioned above properly.

ACKNOWLEDGMENT
This work was sponsored by the following project: (1) Project No. 60302006 is supported by National Natural Science Foundation of China; (2) Project No. 60462002 is supported by National Natural Science Foundation of China.
REFERENCES